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classmate
Date
Page

PROF: CHITRA

Statistics
- Descriptive statistics

Probability

Probability Distribution

Operations Research

OR
- Linear Programming
- Transportation
- Assignment
- Queueing
- Simulation
- Inventory
- Decision Analysis

Analysis of numerical data - Statistics

Mathematics is exact science, but Statistics is based on real life data which is not exact. In real-life, data is ~~invariably~~ inherently invariabl

Statistics is the language of data.

'Average' is also called 'expected value'. Average tries to analyse the data in a numerical fashion. average is called the 'location of data'.

In order to convert the data into informat^n^, thre aspects need to be looked int

1) Location / central Tendency
2) Spread
3) Shape

Average is the measure of central tendency

Freq. distribution : Take the diff. between max. & m and divide them into classes.

Then, find out how many people fall into those classes.

This is how 'Freq. Distribution' is created.

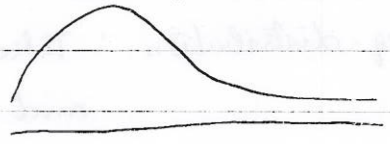
Discrete data is also called 'attribute data'. When data is collected by counting, it is 'discrete' (e.g. sh siz When data is collected by measuring, it is 'continuous data' (e.g. temperature). Continuous data is also called 'variable data'.

Anything that falls into categories is discreet & anything whose value is changeable - needs to be measured (even by counting) - is continuous.

Types of graphs:
Bar (discrete)
Pie (discrete)
Line (both continuous as well as discrete) - usually used in cases where x-axis has 'time'.*
Pictogram (discrete)
Histogram (continuous) - tells the shape of the data i.e. how d the data
Scatter (continuous) - used mostly for bi-variable data.

* Y-axis may be discrete or continuous values. Note.

If data is symmetric, data is scattered uniformly around the average.
If data is +vely skewed, the average (measure of location) lies on the right.



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te data'.
it is 'discrete' (e.g. shirt sizes)
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changeable –
g) – is

te) – usually used in cases where x-axis has 'time'?*

hape of the data i.e. how does the data look
for bi-variable data.

values.

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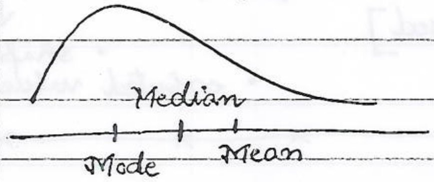
asure of location)
age, lies

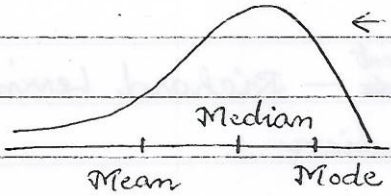
There are two other measures of central tendency:
- Mean
- Median
- Mode – most frequently occurring data value

Note If the data is symmetrical, Mean, Median & Mode all would lie at the same location (i.e. they all are equal).

For +vely skewed distribution:



← For –vely skewed distribution:



So, above conclusions give the arithmetic inference of data, however for best results draw a graph.

Note. Lesser spread tells that ~~m~~ the average is more reliable. Spread is also called 'variability'.

calculate the deviation w.r.t. the average i.e. how near or far the data pt. is from the centre/average.

$$\sum \frac{(X-\mu)^2}{N} = \text{Variance}$$

$$\sigma = \sqrt{\frac{\sum (X-\mu)^2}{N}}$$

Standard Deviation

↳ population size

Sample → $$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

↳ sample size

Q.) Take data pts. (50) from workplace & find out all (Mean, Median, Mode, draw a graph, etc.) & then comment on it
[Even Excel can be used]

↓
• shape
• expected value in future etc.

* * *

STATISTICS:

1.) Statistics for Manage~~s~~ment — Richard Levin & Rubin
2.) Aczel & Soundrapandian
3.) Anderson & Sweeney
4.) S.P. Gupta

Operations Research

QT in Management
1.) N. D. Vohra
2.) S. K. Kapoor
3.) Taha

nd out

comment on it

↓

shape

value in future etc.

evin & Rubin

Distribution of Marks

End-Term Exam — 60 marks

Assignment — 5 marks

Mid-Term Test — 20 marks

Project — 15 marks

(To apply concepts learnt to workspace) in any of the topics

Probability & Distribution constitute 40% of final exam

07-07-2011

Probability

Probability — ~~chance~~ chance of something happening ; assessment of something happening

A) $P(\text{Event}) = \dfrac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$ } Classical approach or a priori approach

The above approach could not be ~~possible~~ ~~to~~ used by managers as the total number of outcomes for a 'real' scenario cannot be known. Plus, the above process gives equal weigtage to all outcomes which is not practical.

To find probability, one can choose a sample & then draw an inference based on that.

Freqency — Past occurrence

Probability — Future possibilities

B) ~~Disadvantages of~~
Relative Frequency approach: 1) This* ~~be~~ method is only reliable if the sample size is very big.

2) It pre-supposes that whatever conditions were there in the process in past, would continue to be there in future as well

Advantage :

It is better than subjective assessment as it is data-driven & can be used wherever data is available & correctly recorded (e.g. for future predictions this method can't be used).

C) SUBJECTIVE APPROACH :

In case of new product launches etc, intuitive ig ; assessment

Classical approach or priori approach

~~possible to~~ used

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decisions are taken based on considering all the factors that occur to me.

However, it is judgemental and can vary from person to person. This method is widely used in business (based on individual experience, estimates etc)

Sample Space - set of all possible outcomes

It is collectively exhaustive (nothing more can happen out of the options available).

Probability is always a fraction.

P (Event) = 0 ⇒ impossible event

P (Event) = 1 ⇒ certain event

Combination - Various ways of choosing

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Q1) A committee of 12 MPs is to be selected from 100. If there are 44 from one party (in the 100), what is the probability that all 12 chosen will be from this party?

Ans.) $\frac{^{44}C_{12}}{^{100}C_{12}}$

[Note: If 6 need to be chosen from the par

$$\frac{^{44}C_{6} \times {^{56}C_{6}}}{^{100}C_{12}}$$

Q2.) Five people A, B, C, D, E have applied for 2 similar jobs. What is the probability that B gets selected?

Ans.) $P(B) = \frac{^{4}C_{1}}{^{5}C_{2}} = \frac{4}{10}$

Note :

Probability that B does not get selected, $P(B') = 1 - \frac{4}{10} = \frac{6}{10}$

Complement of an event = 1 − P(Event happened)

P(A) happens is also called single / unconditional / Marginal probability

PROBABILITY OF RELATED EVENTS

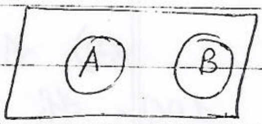
a.) P (A OR B) = Probability of A OR Probability of B OR ~~both~~ Probability of both A, B

Mutually Exclusive event : Occuring of one event precludes the occurrence of other (ie if one happens, the other cannot happen).

e.g. of 3 mutually exclusive events is transaction (Approved, Rejected, On Hold).

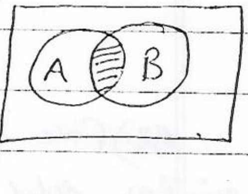
$$P(A \text{ OR } B) = P(A) + P(B)$$

if A & B are mutually exclusive



However, if A & B are non-mutually exclusive

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$



Q3.) A company employs 50 people 10 of who are

$1 - \frac{4}{10} = \frac{6}{10}$

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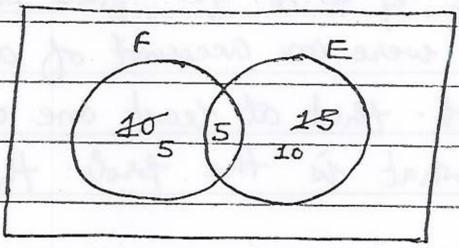
female. There are 10 male executives & 5 female executives. If a member of staff is selected at random, what is the probability that the person selected would be a female or an executive.

Ans) Using a Venn-diagram:

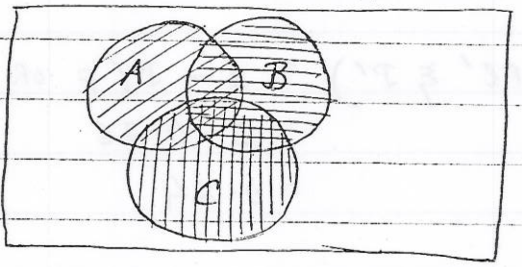


$$P(F \text{ OR } E) = \frac{5+5+10}{50} = \frac{20}{50} = 0{\cdot}4$$

using Formula:

$$P(F \text{ OR } E) = P(F) + P(E) - P(F \text{ AND } E)$$
$$= \frac{10}{50} + \frac{15}{50} - \frac{5}{50} = \frac{20}{50} = 0{\cdot}4$$

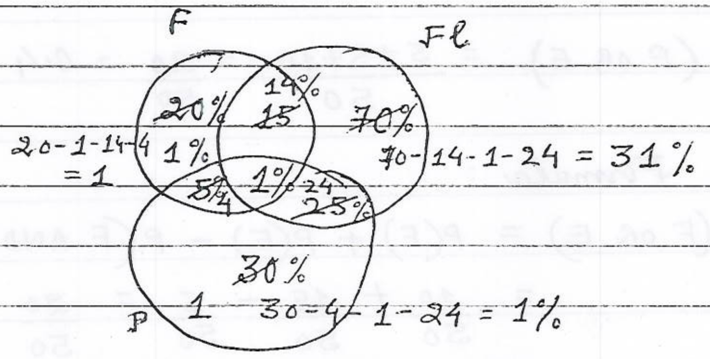
Extending the above concept to 3 events:



$$P(A \text{ OR } B \text{ OR } C) = P(A) + P(B) - P(A \text{ AND } B) + P(C) - P(A \text{ AND } C) - P(B \text{ AND } C) + P(A \text{ AND } B \text{ AND } C)$$
$$= P(A) + P(B) + P(C) - P(A \,\&\, B) - P(A \,\&\, C) - P(B \,\&\, C) + P(A \,\&\, B \,\&\, C)$$

Q4) A soft drink manufacturer found after market research that 20% of the drinks sold are chosen for the fizz, 70% for flavor & 30% for pricing. He also found that 25% of the drinks sold are chosen due to both flavor & pricing, 15% because of fizz & flavor & 5% for fizz & price. 1% of the drinks sold were on account of all three criteria. What is the prob. that at least one of the 3 criteria is met? Also, what is the prob. that none is met?

Ans.)



$\therefore (F \text{ OR } Fl \text{ OR } P) = (1 + 14 + 1 + 4 + 24 + 31 + 1)$
$= 76\%$

$\therefore P(F' \,\&\, Fl' \,\&\, P') = 1 - P(F \text{ OR } Fl \text{ OR } P)$
$= 1 - 0.76$
$= 0.24$

Using the Formula:

$P(F \text{ OR } Fl \text{ OR } P) = P(F) + P(Fl) + P(P)$
$- P(F \,\&\, Fl) - P(Fl \,\&\, P) - P(F \,\&\, P)$
$+ P(F \,\&\, Fl \,\&\, P)$
$= 20 + 70 + 30 - 15 - 25 - 5 + 1$
$= 76\%$ or $.76$

marginal probability — probability of a single event

JOINT PROBABILITY

$P(A \& B) = P(A) \times P(B)$ if events A & B are statistically independent

e.g. Is there a relationship between smoking & lung cancer?

This rule is said to be 'necessary & sufficient' condition. In managerial context, many of the decisions depend on the relationship between the variables.

To derive ~~to~~ a conclusion based on the method above, a contingency table needs to be drawn.

CONDITIONAL PROBABILITY

Given a particular condition, what is the probability of an event happening.

$P(A/B) = P(A)$ if B has already occurred ~~A & B are~~ & A and B are independent

95) A man has applied for a job in two companies A & B. He estimates a chance of getting through at A at 30% & at B as 45%. If the offer of jobs are independent events, what is the probability that (a) he gets an offer from both firms (b) he gets an offer from A but not from B (c) he gets at least one job offer (d) he does not get an offer from either companies?

Ans) $P(A) = 0.3$; $P(B) = 0.45$

$P(A \& B) = P(A) \times P(B)$ since A & B are statistically independent a.) $P(A \,\&\, B) = P(A) \times P(B)$

$= 0.3 \times 0.45 = 0.135$

b.) $P(A \,\&\, B') = P(A) \times P(B')$

$= 0.3 \times (1 - 0.45)$

$= 0.3 \times 0.55 = 0.165$

c.) $P(A \text{ OR } B) = P(A) + P(B) - P(A \,\&\, B)$

$= 0.3 + 0.45 - 0.135$

$= 0.615$

d.) $P(A' \,\&\, B') = 1 - P(A \text{ OR } B)$

$= 1 - 0.615 = 0.385$

(OR)

$P(A') \times P(B')$

$= 0.7 \times 0.55 = 0.385$

Q6.) 2 ambulances are kept in readiness in a hospital. Due to the demand on their time as well as maintenance, the prob. that a specific ambulance would be available is 90%. (a) In the event of a disaster, what is the probability that both ambulances would be available (b) If an ambulance is needed, what is the probability that the hospital would be able to send one?

Ans) $P(A) = 0.9$

$P(B) = 0.9$

$P(A \,\&\, B) = P(A) \times P(B)$ $\because A \,\&\, B$ are independent

$= 0.9 \times 0.9$

$= 0.81$

$P(A \text{ OR } B) = P(A) + P(B) - P(A \,\&\, B)$

$= 0.9 + 0.9 - 0.81$

$= 1.2 - 0.81 = 0.99 \Rightarrow$ So, 99% of the times the hospital is able to supply the demand.

Q7.) The health dept. routinely conducts 2 inspectⁿs of restaurants, with a restaurant passing only if both inspectors pass it. Inspector A is very exp. & passes only 2% of restaurants that have violated health rules. Inspector B is less exp. & passes 7% of restaurants with violatⁿs. What is the prob. that (1) A passes a restaurant given that B has found a violatⁿ? (2) B passes a restaurant given that A has also passed it. (3) Both pass a restaurant having violations?

Ans.)
$P(A) = 0.02$
$P(B) = 0.07$

1.) $P(A/B') = P(A)$ since A & B are independent events [B' is also independent]
$= 0.02$

2.) $P(B/A) = P(B)$
$= 0.07$

3.) $P(A \,\&\, B) = P(A) \times P(B)$
$= 0.02 \times 0.07$
$= 0.0014$ which is .14% chance that a faulty restaurant is passed by both & hence should be acceptable by the health dept.

Q8.) An ad agency has launched a campaign for a new clothing line. 3 hoardings having put up on a highway & the agency knows from exp. how much each one would be noticed by passing drivers.

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The prob. that the 1st hoarding is noticed is 80% while that for the second is 70% & for the third is 90%. What is the prob. that (a) all 3 are noticed by a passing car (b) the 1st & 3rd are noticed but not the 2nd (c) none are noticed (d) at least one is noticed (e) the first two are noticed?

Ans.) $P(A) = 0.8$

$P(B) = 0.7$

$P(C) = 0.9$

a.) $\therefore P(A \;\&\; B \;\&\; C) = P(A) \times P(B) \times P(C)$

$= 0.8 \times 0.7 \times 0.9$

$= 0.504$

b.) $P(A \;\&\; B' \;\&\; C) = P(A) \times P(B') \times P(C)$

$= 0.8 \times 0.3 \times 0.9 = 0.216$

c.) $P(A' \;\&\; B' \;\&\; C') = P(A') \times P(B') \times P(C')$

$= 0.2 \times 0.3 \times 0.1 = 0.006$

d.) $P(A \text{ OR } B \text{ OR } C) = 1 - P(A' \;\&\; B' \;\&\; C')$

$= 1 - 0.006 = 0.994$

e.) $P(A \;\&\; B) = P(A) \times P(B)$

$= 0.8 \times 0.7$

$= 0.56$

99:) The air traffic controller at an international airport has to follow the rule that if the prob. of 2 aircrafts meeting at the same pt. exceeds ·225, he has to divert one of the aircrafts. There are 2 flights scheduled to arrive 10 min apart. Flight 101 scheduled first has a history of being 5 min. late 20% of the time. Flight 201 scheduled next has a history of being 5 min early 25% of the time. (a) If he finds out flight 201 will definitely be early, should he divert 101? (b) If he finds out flight 101 be definitely be late, should he divert 201?

Ans.) $P(101 \text{ late}) = 0.2$
$P(201 \text{ early}) = 0.25$

$P(101 \text{ late} \mid 201 \text{ early}) = P(101 \text{ late})$ since 101 late & 201 early are independent events
$= 0.2 < 0.225 \Rightarrow$ no need to divert

$P(201 \text{ early} \mid 101 \text{ late}) = P(201 \text{ early})$
$= 0.25 > 0.225 \Rightarrow$ hence divert 201

DEPENDENT EVENTS

Happening of ~~one~~ second event would be dependent on happening or non-happening of first.

$$P(A \& B) = P(A) \times P(B/A)$$

$$\Rightarrow P(B/A) = \frac{P(A \& B)}{P(A)}$$

$$P(A \& B) = P(B \& A)$$ for statistically independent events

Q10.) The Work force of a firm employing 200 people consists of manual workers & supervisors as below:

	Male (M)	Female (F)	
(S) Supervisors	20	50	70
(W) Workers	100	30	130 ← Contingency Table (2 way table)
	120	80	200

If a person is chosen at random, find the prob (a) the person is a female (b) the person is a worker (c) the person is a female supervisor (d) the person is a male worker (e) the person is a male if the person selected is a supervisor (f) the person is a supervisor given that she is a female (g) are the events of gender & designatn statistically independent?

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← Contingency Table (2 way table)

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Note (i) In a joint prob., the denominator is always the total number from the contingency table.

(ii) ~~In~~ Marginal prob. is received from the body of the contingency table

a.) $P(F) = \frac{80}{200}$ This is marginal prob.

b.) $P(W) = \frac{130}{200}$

c.) $P(F \,\&\, S) = \frac{50}{200}$ This is joint prob.

d.) $P(M \,\&\, W) = \frac{100}{200}$

e.) $P(M/S) = \frac{20}{70}$ This is conditional prob.

$$\left[P(M|S) = \frac{P(M \,\&\, S)}{P(S)} = \frac{20/200}{70/200} \right]$$

f.) $P(S|F) = \frac{50}{80}$

g.) Check the Independence rule for this

If gender & de~~s~~signat^n are statistically independent

$P(M \,\&\, S)$ should be equal to $P(M) \times P(S)$

$$\Rightarrow \quad \frac{20}{200} \neq \frac{120}{200} \times \frac{70}{200}$$

∴ Gender & designat^n are not statistically independent

Q11.) A sample of 1000 households was selected & the respondents were asked whether they planned to purchase a HD TV. 12 months later, the same respondents were asked whether they actually purchased the TV. The results are summarized below:

Planned to purchase		Actually Purchased Yes (Pu)	No (Pu')	Total
(Pl)	Yes	200	50	250
(Pl')	No	100	650	750
		300	700	1000

What's the prob. that a person (a) plans to purchase a TV (b) purchases a TV (c) purchases a TV given that he has planned to purchase one (d) plans & purchases a TV (e) plans but does not buy a TV within a year (f) are planning to purchase & actually purchasing independent of each other?

Ans.) a.) $P(Pl) = \frac{250}{1000}$

b.) $P(Pu) = \frac{300}{1000}$

c.) $P(Pu|Pl) = \frac{200}{250}$

d.) $P(Pl \ \& \ Pu) = \frac{200}{1000}$

e.) $P(Pl \ \& \ Pu') = \frac{50}{1000}$

f.) If Pl & Pu are independent
$P(Pl \ \& \ Pu)$ should be equal to $P(Pl) \times P(Pu)$

$$\frac{200}{1000} \neq \frac{250}{1000} \times \frac{300}{1000}$$

Hence, planning & purchasing are not statistically ssmate

y Purchased
) No (Pu')
50
650
700
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(c) purchases
purchase
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e) × P(Pu)
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Total
250
750
1000

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independent.

2nd step: This means planning & buying a HD TV is a long term option. Hence, understanding this would help

Q12.) Credit card companies make agressive efforts to solicit new accounts from professionals. A sample of 200 professionals gave the foll. info:

Owning Credit card		Male (M)	Female (F)	Total
C	Yes	60	60	120
C'	No	15	65	80
		75	125	200

(a) If a professional is a male, what is the prob. he has a credit card? (b) If the professional does not have a card, what is the prob. of the person being a female? (c) are the two events of gender & ownership of cr. card statistically independent?

Ans.) (a) $P(C|M) = \frac{60}{75}$

(b) $P(F|C') = \frac{65}{80}$

(c) If gender & credit card ownership are statistically independent

$P(M \,\&\, C)$ should be equal to $P(M)$ & $P(C)$

$$\frac{60}{200} \neq \frac{75}{200} \times \frac{125}{200}$$

Hence, gender & ownership are related (OR) they are not statistically independent

Q13.) A courier company is worried about the likelihood of strikes by some of its employees. It has estimated the prob. of strike by its pilots as 0.75 & the prob. of strike by drivers as 0.65. Further, if the drivers strike, there is a 90% chance that the pilots will strike in sympathy. (a) what is prob. of both grps. striking (b) If the pilots strike, what's the prob. that the drivers would also go on strike?

Ans.) $P(P) = 0.75$

$P(D) = 0.65$

$P(P|D) = 0.90$

a.) $P(P \,\&\, D) = P(P) \times P(D|P)$

$= \cancel{0.75}\; P(D) \times P(P|D)$ [since we have $P(P|D)$ data with us]

$= 0.65 \times 0.90$

$= 0.595$

b.) $P(D \,\cancel{\&}|P) = \dfrac{P(D \,\&\, P)}{P(P)}$

$= \dfrac{0.595}{0.75}$

★ As more obj. data comes in, the 'prior probabilities' (which could even be subjective probability), refine into 'posterior probabilities'.

BAYES THEOREM

Allows to refine probabilities★ (as more & more results come in)

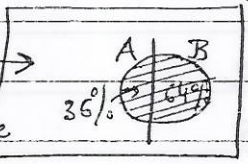
This is simple multiplication rule

$$\frac{P(A \,\&\, B)}{P(B)} = P(A|B)$$

Bayes Theorem requires that entire sample space should be divided into n ~~consist of~~ mutually exclusive & collectively ~~independent~~ exhaustive events.

		Prior Probability	$P(S\|B_r)$	Joint prob. $P(S \,\&\, B_r)$	Post. prob. $P(B_r\|S)$
e.g.	A	·6	·3	·18 [·6×·3]	·36 [·18/·5]
	B	·4	·8	·32 [·4×·8]	·64
all events →		1		P(S) 0.50	

The circle is the total sales →



The rectangle is the total quantity of tea available

S → Tea sold
B_r → Brand of tea
P(S) → Prob. of sales

Bayes Theorem ~~be~~ works on 'Posterior Probability'. to make it a better estimate.

Note: There are pros & cons of using Bayeson Theorem.

Imp. If a prob. is such that P(A|B) is given & it is asked to find out P(B|A), then it is to be done through 'Bayes Theorem'.

Q1) A hardware store purchases light bulbs in bulk from 3 suppliers A, B & C. They supply 60%, 30% & 10% of the store's requirements on an avg.; the proportion of defective bulbs supp. by each of them is 2%, 5% & 8% resp. If the manager of the store chooses a bulb at random & finds its defective, what's the prob. it came from C?

Ans.) $P(A) = 0.60$
$P(B) = 0.30$
$P(C) = 0.10$

Also,
$P(D|A) = 0.02$
$P(D|B) = 0.05$
$P(D|C) = 0.08$

we need to find out $P(C|D) = ?$

There are 3 approaches to solve problems by BAYES theorem:

1.) Tabular Method: [most compact method]

P(Sup.) (Prior Prob.)	P(Def | Sup.) (Condit^n Prob.)	P(Def. & Sup.) (Joint Prob.)	P(Sup | Def.) (Posterior
$P(A) = 0.60$	$P(D/A) = .02$	$.02 \times 0.6 = .012$	$12/35 = 0.$
$P(B) = 0.30$	$P(D\|B) = .05$	$.05 \times 0.3 = .015$	$15/35 = 0.$
$P(C) = 0.10$	$P(D\|C) = .08$	$.08 \times 0.1 = .008$	$8/35 = 0.$
		$P(D) = .035$	

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P (Suf / Def.)
(Posterior prob.)

12/35 = 0.34
15/35 = 0.43
8/35 = 0.23

So though B is supplying only 30% of stock, he is supplying 43% of defective stock.

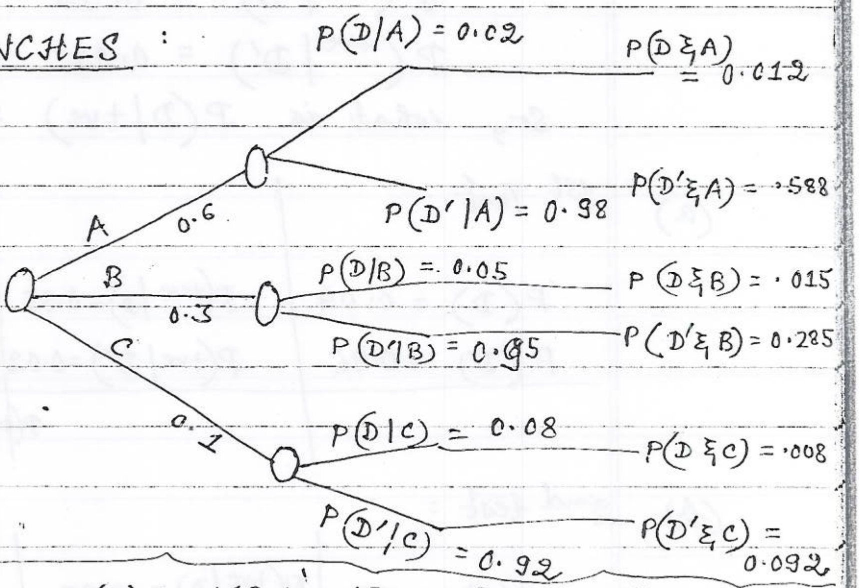
So, either Manager would ask B to improve quality (OR) reduce his share & increase that of A.

2) Contingency Table:

Supp.	D	D′	
A	·02 × ·6 = 0.012	0.588	0.60
B	·05 × ·3 = 0.015	0.285	0.30
C	·08 × ·1 = 0.008	0.092	0.10
	·035	0.965	

$$\therefore P(C|D') = \frac{0.092}{0.965}$$

3) Using BRANCHES:



Here, $P(D) = \cdot012 + \cdot015 + \cdot008 = 0\cdot035$

$$\therefore P(C|D) = \frac{P(D \,\&\, C)}{P(D)} = \frac{0\cdot008}{0\cdot035}$$

Q2.) It is known that 4% of the populatⁿ suffers from a particular disease. There is a clinical test used by doctors for diagnosing the disease but the test is not fool-proof. From past data it is established that the test gives correct +ve results 95% times when the person has the disease. The test also shows a wrong +ve result 2% of the times a person doesn't have the disease.

(a) If a test result is +ve, what is the prob. that the person has the disease. (b) If a second test also returns a +ve result, what is the prob. the person has the disease?

Ans.) Given $P(D) = 0.04$; $\therefore P(D') = 0.96$

$$P(+ve \mid D) = 0.95$$
$$P(+ve \mid D') = 0.02$$

So, what is $P(D \mid +ve) = ?$

(a) 1st Test :

		$P(D$ & $+ve)$ Joint prob.	$P(D \mid +ve)$
$P(D) = 0.04$	$P(+ve \mid D) = 0.95$	0.0380	$.038/.0572 = 0.66$
$P(D') = 0.96$	$P(+ve \mid D') = 0.02$	0.0192	$.0192/.0572 = 0.34$
		$P(+ve)$ 0.0572	

(b) 2nd test :

		$P(D$ & 2 +ves$)$	$P(D \mid 2\ +ve)$
$P(D) = 0.66$	$P(+ve \mid D) = 0.95$	0.6270	$.627/.6338 = 0.99$
$P(D') = 0.34$	$P(+ve \mid D') = 0.02$	0.0068	$.0068/.6338 = 0.01$
		$P(2 \mid +ve) = 0.6338$	

So, after the 2nd test, the doctor is 99% sure.

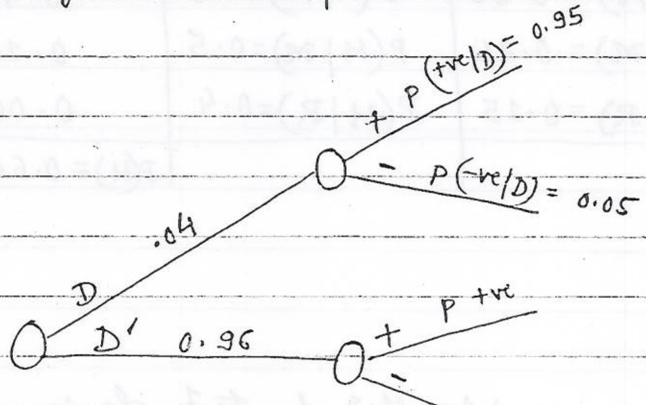
By Using Contingency Table :

(a)

	+ve	-ve	
D	·95 × ·04 = 0·038	0·002	0·04
D′	·02 × ·96 = 0·0192	0·9408	0·96
	0·0592	0·9428	1

By Using BRANCHING METHOD :

(a)



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P(D|+ve)
38/.0572 = 0.66
192/.0572 = 0.34

(D|2 +ve)
27/.6338 = 0.99
68/.6338 = 0.01

classmate
Date
Page

Q3.) An advertising agency wants to decide which of the 3 media to use for a certain product. The prob. they choose TV is 60%, magazines is 25% & radio is 15%. Based on past exp, the probalities of high coverage under each are 0.8, 0.5 & 0.4 respectively. After making the choice the agency determines they did achieve high coverage. Given this informatn. what is the prob.

(a) TV was chosen?

(b) magazines were chosen?

(c) Radio was chosen?

Ans.) Given :

		P(H & medium)	P(M/H)
$P(TV) = 0.60$	$P(H/TV) = 0.8$	0.480	$.48/.665 = 0.72$
$P(M) = 0.25$	$P(H/M) = 0.5$	0.125	$.125/.665 = 0.19$
$P(R) = 0.15$	$P(H/R) = 0.4$	0.060	$.060/.665 = 0.09$
		$P(H) = 0.665$	1

Q4.) A managerial aptitude test designed to separate officers into promising & non-promising officers for promotn. resulted into foll: Among the officers who had 'Excellent' confidential reports, 80% passed the test. Among 'Average', 40% passed. If it is known that only 60% of the officers are given

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060/.665 = 0.09
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'Excellent', what is the prob. that an officer who has passed has an 'Excellent' rating?

Ans.)

		P(P & Rating)	
$P(E) = 0.60$	$P(P/E) = 0.8$	0.48	$.48/.64 = 0.75$
$P(A) = 0.40$	$P(P/A) = 0.4$	0.16	$.16/.64 = 0.25$
		$P(P) = 0.64$	1

Q4.) A chemical company is planning to raise funds for expansion. The management believes that there is a 70% chance of getting a loan from financial institutⁿs. They also believe there is 90% chance of attaining the capital through a public issue provided the financial report is favourable. If however, the financial report is not favourable, the prob. of raising funds through public issue drops to 50%. The management estimates the chances of getting a favourable financial report as 60%. Should the corporatⁿ go for public issue or go to the FIs for loan?

Ans.)
$P(FI) = 0.70$
$P(PI) = 0.30$
$P(+ve) = 0.60$
$P(-ve) = 0.40$

P(PI/+ve)
P ~~(+ve/PI)~~ = 0.90

P(PI|-ve)
= 0.50

1.) Cumulative Binomial Distribution
2.) Poisson
3.) Normal

$P(\text{report})$	$P(PI/\text{report})$	$P(PI \ \& \ \text{Report})$
$P(+ve) = 0.60$	$P(PI/+ve) = 0.90$	0.54
$P(-ve) = 0.40$	$P(PI/-ve) = 0.50$	0.20

$\therefore P(PI) = 0.74 > P(FI)$

Hence, go for Public Issue (PI).

BINOMIAL DISTRIBUTION

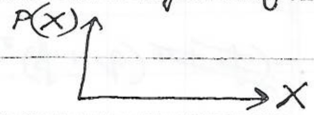
PROBABILITY DISTRIBUTION :

No. of defects X	Prob P(X)	Probability Distribut^n
0	1/8	GGG GDD
1	3/8	GGD DGD
2	3/8	GDG DDG
3	1/8	DGG DDD

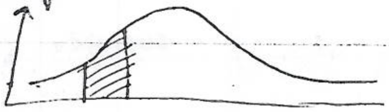
GGG GDD
GGD DGD
GDG DDG
DGG DDD

For an experiment, if all the random values are taken and each of their probabilities found out, all the probabilities would add to 1 and such a distribution would be know as 'Probability Distribution'.

If the probability of every value is known, the distribut^n is known as 'discrete' distribut^n. In a discrete distribution the height of the graph is the probability.



In a continuous distribution, the area under the curve gives the probability.



→ Discrete Probability distributⁿ

Binomial/Bernouli's Distribution :

Probability finding based on 'Binomial Distributⁿ' is based on 3 assumptions :

1.) Every trial has only two outcomes (convention: success & failure).

2.) The probability of success in each ~~outcome~~ trial (p) is constant.

∴ probability of failure $(q) = 1 - p$

3.) The trials are independent (The outcome of one trial doesn't affect the success or failure of the other)

e.g. Salesman has probability of success 60%. If he visits 3 houses, what is the prob. that he has exactly two orders?

$\boxed{1}$ 0.6 $\boxed{2}$ 0.6 $\boxed{3}$ 0.4

Here $n = 3$
$p = 0.6$
$q = 0.4$

$$\therefore P(\text{2 orders}) = 3 \times 0.6 \times 0.6 \times 0.4$$

↳ Multiplicatⁿ because the trials are independent

Formula :

$$P(r \text{ successes}) = {}^nC_r \cdot p^r q^{n-r}$$

$$~~(p+q)~~ (q+p)^n = q^n + {}^nC_1 q^{n-1}$$

Binomial distributⁿ has got a lot of practical usage.

Note : In Discrete distribut^{n}s
<7 means upto 6
≤ 7 means upto 7

Q1. In a 20 quest^n 4 answer multiple choice test, what is the prob. a student gets (i) exactly 7 ans correct (ii) at least 7 ans correct (iii) at most 7 ans correct (iv) all correct, if he answers randomly

Ans. The assumptions of Binomial distribution are satisfied.
'p' value is const, as every quest^n has 4 options.
trials are independent as student answered randomly

Here, $n = 20$
$p = 0.25$
$q = 0.75$

$P(X=7) = {}^nC_r \, p^r q^{n-r}$
$= {}^{20}C_7 \, (0.25)^7 \, (0.75)^{13}$

(i) $P(X=7) = P(X \leq 7) - P(X \leq 6)$
$= 0.8981 - 0.7858$
$= 0.1123$

(ii) $P(X \geq 7) = P(7) + P(8) + P(9) \ldots\ldots P(20)$
$= 1 - P(X \leq 6)$
$= 1 - 0.7857$
$= 0.2143$

(iii) $P(X \leq 7) = 0.8981$

(iv) $P(X = 20) = P(X \leq 20) - P(X \leq 19)$
$= 1 - 0$ (approx.)
∴ Probability is almost impossible.

Q2.) The likelihood that someone who logs on to a particular site in a shopping mall on the web will purchase an item is 20%. If the site has 10 people accessing it in a particular minute, what is the probability that (a) none purchase anything, (b) exactly 2 will purchase (c) less than 2 will purchase (d) not less than 2 will purchase an item (e) what is the no. of people you'd expect to purchase an item ?

Ans.) First, let's check what distributⁿ the questⁿ follows.

I.) Two outcomes; (II) will buy / not buy ⇒ hence 'p' constt 20%

~~II.)~~

III.) Trials are independent ; Hence, binomial distribution

$n = 10$

$p = 0.2$

$q = 0.8$

a.) $P(X=0) = 0.107$

b.) $P(X=2) = P(X \leq 2) - P(X \leq 1)$

$= 0.6778 - 0.3758$

$= 0.302$

c.) $P(X<2) = P(X \leq 1)$ $[P(0) + P(1)]$

$= 0.3758$

d.) $P(X \not< 2) = P(X \geq 2)$ $[P(2) + P(3) + \ldots P(10)]$

* $S.D. = \sqrt{npq}$

S.D. helps to pin-point what are rare occurrences & what are regular occurrences

$= 1 - P(X \leq 1)$

$= 1 - 0.3758$

$= 0.6242$

e.) Expected value $= \overset{*}{\text{Mean}} = np$

$= 10 \times 0.2 = 2$

Q3.) If 60% of TV users watch a particular program what is the prob. that in a sample of 5 at least half will be watching the prog.?

Ans. I.) Only Two outcomes watching / not watching

II.) p is 60%

III.) Trials are independent

Hence, Binomial Distribution

$n = 5$

$p = 0.6$

$q = 0.4$

$P(X \geq 3) = 1 - \underset{\downarrow}{P(X \leq 2)}$ $[P(3) + P(4) + P(5)]$

This is when $p = 0.6$ but we can't use the tables for $p = 0.6$. Hence the def/meaning of success must be changed.

So, the questⁿ becomes: what is the probability that at most or 2 are not watching.

$= P(2) + P(1) + P(0)$

$= P(X \leq 2)$ when $p = 0.4$

$= 0.6826$

Imp. **Note** : When p value is greater than 0.5, to use the tables, switch/re-define the meaning of success & failure.

Q4.) A quality control system selects a sample of 3 items from a Productⁿ line. If 1 or more is def., a second sample is taken, also of size 3; and if 1 or more of this is def., the Productⁿ line is stopped. Given the prob. of a defective item is 5%, find the prob. that (a) a 2nd sample is taken, (b) the Productⁿ line is stopped.

Ans.) (a) $P(\text{second sample}) = P(X \geq 1 \text{ in first sample})$

Here, Binomial distribution is satisfied

$n = 3$

$p = 0.05$

$q = 0.95$

~~a)~~ $P(X \geq 1) = P(1) + P(2) + P(3)$

$= 1 - P(0)$

$= 1 - {}^3C_0 (.05)^0 (.95)^3$

$= 1 - 0.857$

$= 0.143$

So, 14% of the time second sample would be taken.

b.) $P(\text{line stopped}) = P(X \geq 1 \text{ in 2nd sample})$

$= P(\text{second sample}) \times P(X \geq 1 \text{ in 2nd sample})$

$$= 0.143 \times 0.143$$
$$= 0.02$$

So out of 100 samples, in 2 samples the line would be stopped.

Q5.) A garage examines cars for defective tyres and finds defective tyres in 1 in every 5 cars examined. If 80 cars are examined daily:
(a) what is the average no. of cars with def. tyres (b) what is the S.D. of cars with def. tyres?

Ans.

$$\text{Avg.} = np$$
$$= 80 \times 0.2$$
$$= 16$$

$$S.D. = \sqrt{npq}$$
$$= \sqrt{80 \times 0.2 \times 0.8}$$
$$= 3.58$$

Q6.) An electrical products manufacturer guarantees that he will replace free of charge any product that is found def. within 1 yr. of purchase. Past exp. suggests that 10% of the products sold come back for replacement. What is the prob. that out of 25 units sold in a week, more than 2 would be replaced under guarantee (b) If the avg. cost of replacement is 540 Rs. per unit, what is the avg cost of providing the guarantee?

Ans.) $n = 25$

$p = 0.1$

$q = 0.9$

a.) $P(X>2) = P(3) + P(4) + \dots\dots P(25)$

$= 1 - P(X \leq 2)$

$= 1 - 0.5379$

$= 0.4629$

b.) Expected no. of items to be replaced under guarantee $= np$

$= 25 \times 0.1$

$= 2.5$

$\therefore$ Cost of guarantee $= 2.5 \times 540$

e.) Here, $\lambda = 2.5$ calls/half hr.

$$P(X=0) = \frac{e^{-2.5} \cdot (2.5)^0}{0!}$$

$$= 0.0821$$

Q2.) Between 8:00 & 9:00 AM, 30 workers arrive at the machine shop with an average time between arrivals of 2 min. What is the prob. that

a) 6 min will elapse with no workers arriving

b) that during a 6 min. interval, at most 3 workers arrive?

Ans

Since average number of occurrences is given, its Poisson distribution.

λ is the no. of occurrences (not the gap between occurrences).

a.) $\therefore \lambda = 30/\text{hr}$

Here, we need to find out $P(X=0$ during 6 min$)$

So, $\lambda = 3/6\text{min}$.

$P(X=0) = \cancel{P(X \leq 1)} = P(X \leq 0)$

$= \cancel{0.19915}\ 0.04979$

b.) $P(X \leq 3) = 0.64723$

Imp. Poisson graph is always +ve ly skewed.
As λ increases, the skewness decreases.

In Binomial distributions, graph's shape depends on the value of p.

Q3.) During peak traffic hrs. accidents occur on a certain road @ 2/hr. The morning peak lasts for $1\frac{1}{2}$ hrs & the evening is 2 hrs.

(a) On any given day, what's the prob. there would be no accidents during morning peak

(b) prob. of 2 accidents during evening peak

(c) 4 or more accidents during morning peak

(d) On any 1 day, what's the prob. there won't be ANY accidents at all?

Ans. λ for morning peak = 3/ morning peak

λ for evening peak = 4/ evening peak

a.) $P(X=0) = 0.04979$

b.) $P(X=2) = P(X \leq 2) - P(X \leq 1)$

$= 0.23810 - 0.09158$

$= 0.14752$

c.) $P(X \geq 4) = P(4) + P(5) \ldots\ldots$

$= 1 - P(X \leq 3)$

$= 1 - 0.64723$

$= 0.3528$

d.) P(X=0 in the morning) & P(X=0 in the evening)

= P(X=0 in morning) × P(X=0 in evening)

$= 0.04979 \times 0.01832$

(OR)

2nd method : Take ~~A~~ peak = $7\frac{1}{2}$ hrs.

Q4) A man has 4 cars to hire. The avg. demand for cars on a week day is for 2 cars. Estimate the no. of week days for which demand exceeds supply. Assume 312 week days a year. Would you suggest that he invests in another car?

Ans.) This is Poisson distribution.

Unit of measurement = per week day

$\lambda = 2$ cars/ week day

$P(X>4) = P(5) + P(6) + P(7) \text{------}$

$= 1 - P(X \leq 4)$

$= 1 - 0.94735$

$= 0.05265$

$\therefore$ No. of days demand exceeds supply $= 0.05265 \times 312$

$= 16.42$ days

Thus, only on 5% of week days, demand exceeds supply, therefore there is no need to invest in another car.

Approximations: When n is very large & p is very small, Binomial distribution calculations can be approximated to Poisson distribution calculations

Use Poisson approximatⁿ in place of Binomial if

$n \geq 20$ AND $p \leq 0.05$

Q5.) An electrical manufacturer claims that 2% of all appliance breakdown are caused by failure to follow instruct^{n}s. Find prob. that amongst 100 breakdowns, more than 5 were caused due to this reason?

Ans.) This is Binomial situation.

$$n = 100$$
$$p = 0.02$$

But since $n > 20$ & $p < 0.05$, use Poisson.

$$\therefore \text{Average}, \lambda = np$$
$$= 100 \times 0.02$$
$$= 2$$

$$\therefore P(X > 5) = P(6) + P(7) + \cdots\cdots + P(100)$$
$$\simeq 1 - P(X \leq 5)$$
$$= 1 - 0.98344$$
$$= 0.01656$$

Inf.

	Binomial	Poisson
Mean	np	λ
S.D.	$\sqrt{npq}$	$\sqrt{\lambda}$

Q6.) ~~Q7.)~~ An insurance company has a policy base of 1 lakh policy holders. If claims relating to death from a rare disease are received in 0.01 % of the policies, what's the prob.

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that in a year no more than 4 claims are received due to this cause ?

Ans. $n = 1,00,000$

$p = 0.0001$ (ie. 0.01%)

$q = 0.9999$

Since n is very large & p very small, use Poisson approximatⁿ.

$$\lambda = np$$
$$= 1,00,000 \times 0.0001$$
$$= 10$$

$$\therefore P(X \not> 4) = P(X \le 4)$$
$$= 0.02925$$

V. Imp

NORMAL DISTRIBUTION

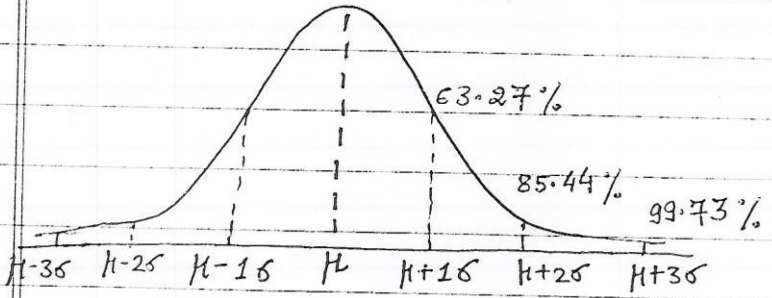
Differences between Normal (Continuous) & Discrete distributⁿs.

a) Discrete is counting distributⁿ
Normal is measuring distributⁿ

b) Poisson & Binomial are Mathematical models.
Normal is Empirical in nature (no assumptions ; based on verification).

c) Normal is known as 'Mother' of all distributⁿs
because : i) Many naturally occurring phenomena are based on Normal distributⁿ.
i.e.
~~ii)~~ If a system has only chance causes occurring on it, the situation follows Normal distributⁿ.

ii) If ~~if~~ the populatⁿ is not normally distributed, the sample distributⁿs of the populations would be normally distributed (In other words, at some level all discrete distributⁿ could be approximated to Normal).



Characteristics of Normal Distributⁿ :

1.) Symmetrical distribution (ie. Mean = Median = Mode)
2.) Bell-shaped (probabilities rise & fall gradually)
3.) Distributⁿ approaches X-axis but never

touches it (asymptotic).

4.) Distribut^n defined on two parameters :

Mean
i) μ defines the central posit^n of the distribut^n
ii) σ determines the deviat^n from centre ie.
S.D. Spread of the distribut^n

Greatest adv. of Normal Distribution : Enables you to make limits where most of your values would ~~like~~ lie.

Note: If normality exists in a sample of size N, it will definitely exist in the size of 2N; ie increasing the sample size won't affect normality in the distribut^n.

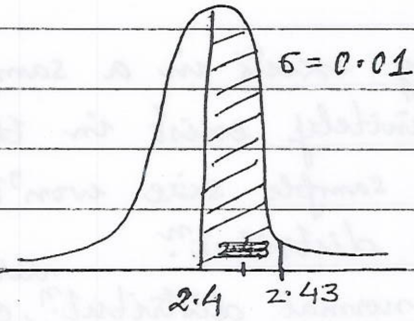
Also, any normal distribut^n variate could be converted into standard normal distribut^n variate

Normal Distribution is empirical (ie. either Normality exists or it does not).

Standard Normal Distribution, $z = \dfrac{X-\mu}{\sigma}$

Q1. The diameters of ball bearings are normally distributed with a mean of 2·4 inches and a S.D. of 0·1 inch. Determine the % of ball bearings with a diameter (a) between 2·4 and 2·43 inches (b) greater than 2·43 inches (c) between 2·38 and 2·43 inches (d) between 2·38 and 2·39 inches

Ans.



$$Z_1 = \cancel{+}\frac{X-\mu}{\sigma} = \frac{2\cdot4 - 2\cdot4}{\sigma} = 0$$

Hence, Z_1 is not req.

a.) $\therefore\ Z = \frac{X-\mu}{\sigma} = \frac{2\cdot43-2\cdot4}{0\cdot01} = 3$

$$P(2\cdot4 < X < 2\cdot43) = P(0 < Z < 3)$$
$$= 0\cdot4987$$

b.) $Z = 3$

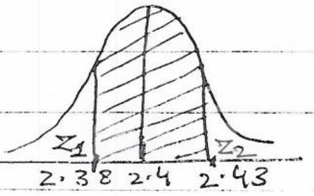
$P(X > 2\cdot43)$

$= P(Z > 3)$

$= 0\cdot5 - 0\cdot4987$

$= 0\cdot0010$

c.)



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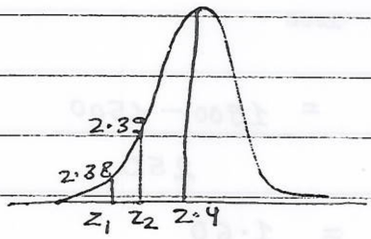
$$z_1 = \frac{X-\mu}{\sigma} = \frac{2.38-2.4}{0.01} = -2$$

$$z_2 = 3$$

$$P(2.38 < x < 2.43) = P(-2 < z < 3)$$
$$= P(-2<z<0) + P(0<z<3)$$
$$= 0.4772 + 0.4987$$
$$= 0.9759$$

d.)



$$z_1 = \frac{2.38-2.4}{.01} = -2$$

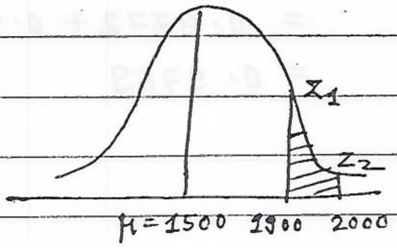
$$z_2 = \frac{2.39-2.4}{0.01} = -1$$

$$\therefore P(2.38 \leq X \leq 2.39) = P(-2 < z < -1)$$
$$= P(-2<z<0) - P(-1<z<0)$$
$$= 0.4772 - 0.3413$$
$$= 0.1359$$

Q2.) A large departmental store has 4500 accounts receivables. The amount in these accounts is known to be normally distributed with a mean of 1,500 & a S.D. of 2,50 Rs. What is the probability (a) that an account is between 1,900 & 2,000

(b) between 1150 & 1650 (c) between 1900 & 1250 (d) How many accounts are less than 1000 or more than 2000 (e) what is the value of the amount so that 10% of the accounts exceed this amount?

Ans.



(a)

$$z_1 = \frac{x-\mu}{\sigma} = \frac{1900-1500}{250}$$

$$= 1.60$$

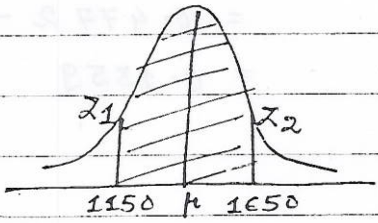
$$z_2 = \frac{2000-1500}{250} = 2$$

$$\therefore\ P(1900<x<2000) = P(1.6<z<2)$$

$$= P(0<z<2) - P(0<z<1.6)$$

$$= 0.4772 - 0.4452$$

(b)



$$z_1 = \frac{1150-1500}{250} = -1.4$$

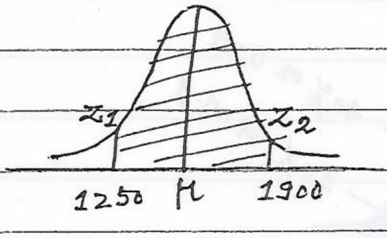
$$z_2 = \frac{1650-1500}{250} = +0.6.$$

1250
1000
value
accounts

)
<z<1.6)
2

$$\therefore P(1150 < X < 1650) = P(-1.4 < z < 0.6)$$
$$= P(-1.4 < z < 0) + P(0 < z < 0.6)$$
$$= 0.4192 + 0.2257$$
$$= 0.6449$$

(c)

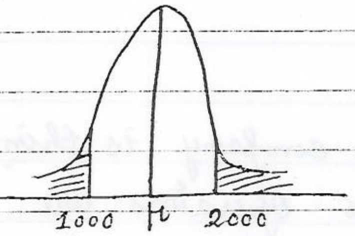


$$z_1 = \frac{1250 - 1500}{250} = -1$$

$$z_2 = \frac{1900 - 1500}{250} = 1.6$$

$$P(1250 < X < 1900) = P(-1 < z < 1.6)$$
$$= P(-1 < z < 0) + P(0 < z < 1.6)$$
$$= 0.3413 + 0.4452$$
$$= 0.7865$$

(d)



$$z_1 = \frac{1000 - 1500}{250} = -2$$

$$z_2 = \frac{2000 - 1500}{250} = 2$$

$$\therefore P(X < 1000 \text{ OR } X > 2000) = P(X < 1000) + P(X > 2000)$$
$$= P(z < -2) + P(z > 2)$$

$$= 1 - 2P(0 < Z < 2)$$ (can be done separately & added as well)

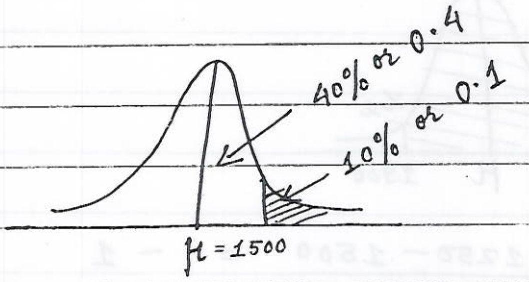
$$= 1 - 2 \times 0.4772$$

$$= 1.9544$$

$$= 0.0456$$

∴ No. of accounts < 1000 OR $> 2000 = 0.0456 \times 4500$

(e)



$$\therefore P(0 < Z < Z_1) = 0.4$$

$$Z_1 = 1.28$$

$$\frac{X - \mu}{\sigma} = 1.28$$

$$\Rightarrow \frac{X - 1500}{250} = 1.28$$

$$\therefore X = 1$$

Q3.) A Data ~~In a~~ processing company is thinking of laying off some operators. Operators are known to have an average data entry speed of 52 words/min. with a S.D. of 8 words/min. The company has decided that operators having speed below 46 words/min. will be laid off. If the company has 200 operators how many would be laid off? (b) suppose, company wants to lay off only 10 operators, where should e

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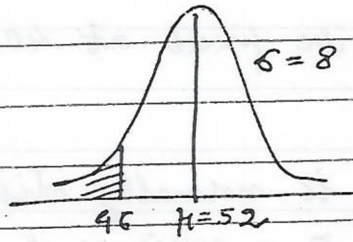
they fix the cut-off speed. The distribut^n of data entry speed is normal.

Ans.(a)



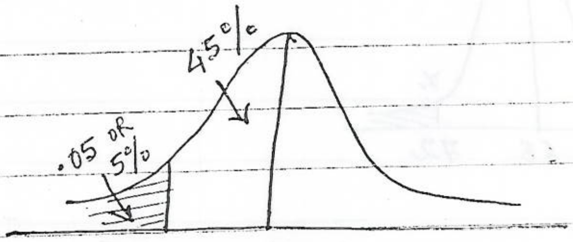
$$z = \frac{X-\mu}{\sigma} = \frac{46-52}{8} = \frac{-6}{8} = -0.75$$

$$\therefore P(X<46) = P(z< -0.75)$$
$$= 0.5 - P(-.75<z<0)$$
$$= 0.5 - 0.2734$$
$$= 0.2266$$

$\therefore$ 22.66% of the operators would be laid-off

i.e. 0.2266×200
$= 45.2 \simeq 45$

(b.) % laid off $= \frac{10}{200} = 0.05$



$\therefore P(z<z_1) = .05$ & $P(z_1<z<0) = 0.45$ in the table

$z_1 = -1.65$ (search for 0.45 & the value which is closest to it is 1.65)

$\Rightarrow$ ~~μ~~ $\frac{X-\mu}{\sigma} = -1.65$

$$\Rightarrow \frac{X-52}{8} = -1.65$$

$$\therefore X = -13.2 + 52$$

$$= 39.8 \sim 40$$

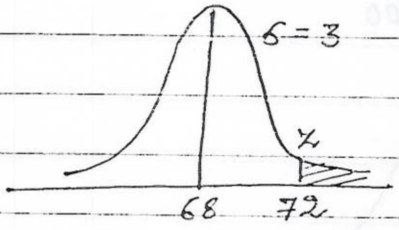
∴ cut-off speed should be fixed at 40 words/min

ques 4) The heights of soldiers is normally distributed with a mean of 68" & a variance of 9 sq. inches. What is the probability that a soldier picked up at random (a) is taller than ~~7~~72" (b) between 63 & 66" (c) what would be the height such that only 30% of the soldiers are ~~taller~~ shorter? (d) what should be the ht. of the door such that 70% of the soldiers can enter without bending?

Ans) ∵ Variance = 9 sq. inches

∴ S.D = 3 inches

(a)



$$z = \frac{X-\mu}{\sigma} = \frac{72-68}{3} = 1.33$$

$$P(X>72) = P(z>1.33)$$

$$= 0.5 - P(0<Z<1.33)$$

words/min

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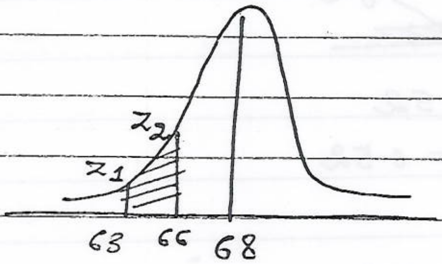
$= 0.5 - 0.4082$

$= 0.0918$

(b)



$$z_1 = \frac{63-68}{3} = -1.67$$

$$z_2 = \frac{66-68}{3} = -0.67$$

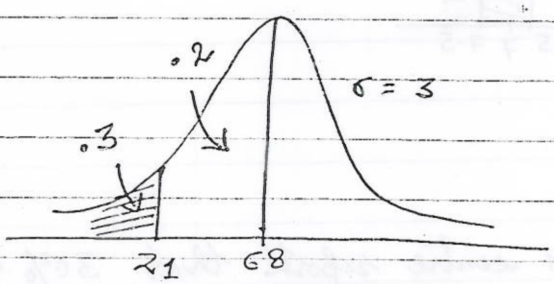
$\therefore P(63 < X < 66) = P(-1.67 < z < -0.67)$

$= P(-1.67 < z < 0) - P(-0.67 < z < 0)$

$= 0.4525 - 0.2486$

$= 0.2089$

(c)



$\because P(z < z_1) = 0.3$

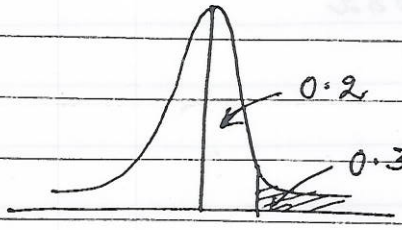
$\therefore P(z_1 < z < 0) = 0.2$

Since, $z_1 = -0.52$

$$\frac{X-68}{3} = -0.52$$

$\therefore X = 66.44$

(d)



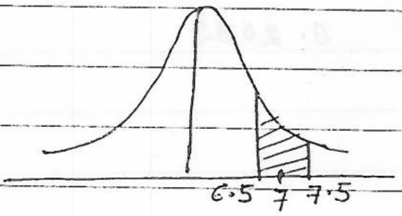
$$z_1 = 0.52$$

$$\text{or} \quad \frac{x - 68}{3} = 0.52$$

$$\therefore x = 69.56$$

Note : If both np & nq are equal to or greater than 5, then approximate Binomial distribution with Normal distribution (ie np & $nq \geq 5$)

ie. $P(X = 7)$ in Binomial $= P(6.5 < X < 7.5)$ in Normal



Ques 5.) A business centre reports that 30% of all small businesses are owned by women. What is the probability that a random sample of 50 small businesses will show (a) at least 42 are owned by men (b) less than 30 are owned by women (c) between 30 & 42 (both inclusive) are owned by men ?

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$\geq 5)$

$< 7.5)$
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n 30
& 42 (both

Note: In continuous distribution, equal to doesn't matter but in discrete it matters

Ans. This is a case of Binomial Distribution with

$n = 50$

& $p = 0.3$

Since n is very large, we find out :

$$\left.\begin{array}{l} np = 50 \times 0.3 = 15 \\ nq = 50 \times 0.7 = 35 \end{array}\right\} \geq 5$$

$\because$ both np & $nq \geq 5$, we will use Normal approximation to Binomial problem

(a.) $p = 0.7$ (Here success is probability of businesses owned by men)

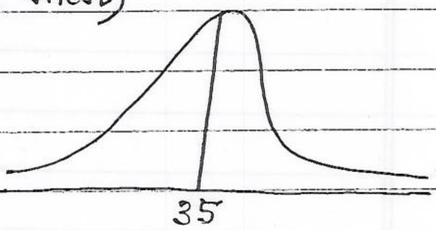
$q = 0.3$

$n = 50$

$\mu = np = 35$

$\sigma = \sqrt{npq}$

$= 3.24$



Binomial $P(X \geq 42) = P(42) + P(43) + \text{-----}$

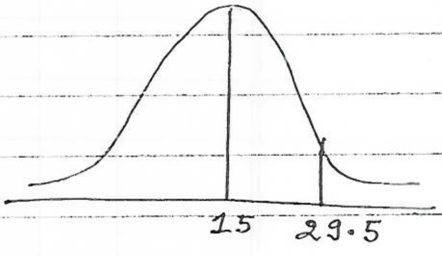
$= \text{Normal } P(X \geq 41.5)$

(b) $p = 0.3$

$q = 0.7$

$\mu = np = 50 \times 0.3 = 15$

$\sigma = \sqrt{npq} = 3.24$

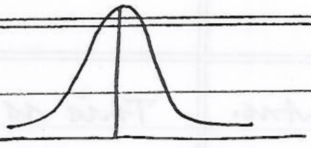


Binomial $P(X < 30)$

$= P(0) + P(1) + P(2) + \text{-----} P(29)$

$= \text{Normal } P(X \leq 29.5)$

$= 0.5 + P(0 < X < 29.5)$



(C) Binomial ~~Prob~~ $P(30 \leq X \leq 42)$

$= P(30) + P(31) + \text{-----} P(42)$

$= \text{Normal } P(29.5 < X < 42.5)$

→ Analytical method to arrive at an optimal solution

DECISION ANALYSIS

In decision analysis, there are:

- strategies
- States of nature (factors not in my hand)
- Pay offs and ~~The combinat^n of these two~~
- Chances (for every state of nature)

Given the above, how can I choose an optimal decision.

Decisions can be of two types :

- one-shot decision and
- sequence of decisions

Decision under uncertainty means there is no estimate of the probabilities of the diff. states of nature.

Decision under risk means the estimates of the probabilities of diff. states is known. If we are able to assign a probability to the likehood of the states of nature, then that is decision under risk.

Against strategies, there are states of nature which are not in your hand. For Every combinat^n of strategy & state of nature, there is a payoff (the money that would be involved ie. profit or loss).

Decisions under risks involve a sequence of decisions

Contingency Table – A combinat^{n} of Strategy &
states of demand.

Q1 XYZ company summarizes international reports & makes forecasts which are purchased weekly by investors. The demands for the reports is limited to a max of 30 units. The possible demands are 0, 10, 20 & 30 reports per week. The profit per report sold is Rs. 30 & the loss per unsold report is Rs. 20. No product^{n} of extra reports during the week is possible. Further there is a penalty of Rs. 250 for not meeting the demand. Unsold reports can't be carried on to next week.

Prepare a pay-off table & determine the optimal number of reports to produce under various criteria.

Ans. We will prepare contingency table (ie Pay-off table)

	Strategies			
States of nature	0	10	20	30
0	0	-200	-400	-600
10	-250	300	300-200 = 100	300-400 = -100
20	-250	300- 250 = 50	~~600-250~~ = 600	600-200 = 400
30	-250	300- 250 = 50	600-250 = 350	900
Avg. pay$_{off}$	-187.5	50	162.5	125

Profit/report sold = Rs. 30
Loss/report unsold = Rs. 20
Loss for unmet demand = Rs. 250

actual no. of occurrences is very small

POISSON DISTRIBUTION

Parameters in Poisson Distribution:

n is very large, p is very small

e.g. No. of calls received in a call centre in a day, no. of pin-holes in (say) 1 m. of steel sheet. [No. of calls could be counted but the probability of a call coming can't be]

The assumptions in Poisson Distribution:

a.) The prob.s of the actual occurrences couldn't be found out

b.) * Potential no. of occurrences is very large, but actual occurrences is small

c.) The average number of occurrences per unit of measurement is known.

d.) The trials are independent.

Note: Poisson, just like Binommial, is discrete distribution.

Random values can be from 0 to infinity

λ = average no. of occurrences / unit* of measurement

$$P(X=r) = \frac{e^{-\lambda} \cdot \lambda^{r}}{r!}$$

Imp: Use Poisson Distribution only if :-
1.) The value of n is very large/infinite
2.) you want to find prob. wrt a unit of measurement.

Q1. If calls arrive at random & an executive receives 5 calls on avg. in one hour, what's the prob. (a) he receives fewer than 5 calls (b) exactly 5 calls (c) more than 3 calls in the next hr (d) 3 calls in the next 2 hrs (e) no calls in the next half an hour.

Ans.) Step 1: Identifying the type of distribution.
a) The potential number of calls is very high
b.) The calls are independent
Here, $\lambda = 5$ calls/hr.

a) $P(X<5) = P(0)+P(1)+P(2)+P(3)+P(4)$
$= P(X \leq 4)$
$= 0.44049$

b.) $P(X=5) = P(X \leq 5) - P(X \leq 4)$
$= 0.61596 - 0.44049$
$= 0.17547$

c.) $P(X>3) = P(4)+P(5) \ldots\ldots$
$= 1 - P(X \leq 3)$
$= 1 - 0.26503$

d.) $P(X = 3$ in 2 hrs.)
Here, $\lambda = 10$ calls/2 hrs.
$P(X=3) = P(X \leq 3) - P(X \leq 2)$
$= 0.01034 - 0.00277$
$= 0.00757$

Now, we need to apply one of the criteria:

1.) La Place criterion : All the states of nature are equally likely is the assumption used in this criterion. The max. of average pay-offs need to be chosen

Under La Place criterion → Max. (average pay-off).
selected ~~unit~~ strategy → 20 copies

2.) Maximin (Pessimist) strategy : For every strategy, find out the max. of the minimum values. This is used by people who want to play very safe.

Min	-250	-200	-400	-600

Under ~~the~~ Maximin strategy → Max. (min. pay-off)
selected strategy → 10 copies

3.) Maximax (Optimist) strategy

Max	0	300	600	900

Under Maximax criterion → Max. (max. pay-off)
selected strategy → 30 copies

4.) Optimism Index : Just decide on an α-value based on the level of optimism you have. Let's say α is 60% or 0.6 optimism.

$\alpha \times$ Max. $+ (1-\alpha)$ Min.	-100	100	260	$\boxed{390}$

Under Hurwicz criterion → Max. ($\alpha \times$ Max. pay-off + $(1-\alpha) \times$ Min. pay-off)

Selected strategy → 30 copies

5. Regret criterion: Based on the pay-off table (Savage criterion of minimax regret)

Regret (Loss) = Best decision for state of nature − Pay-off of strategy

(The values are taken along the row)

Regret Table

States of nature	Strategies 0	10	20	30
0	0	0 − (−200) = 200	0 − (−400) = 400	0 − (−600) = 600
10	300 − (−250) = 550	0	300 − 100 = 200	300 − (−100) = 400
20	600 − (−250) = 850	600 − 50 = 550	0	600 − 400 = 200
30	900 − (−250) = 1150	900 − 50 850	900 − 350 = 550	0
Max.	1150	850	$\boxed{550}$	600

Under Regret criterion:

Min (max. regret)

Selected strategy → 20 copies

Q2.

Solution

8 or 9 cop

6 copies

11 copies

11 copies

Savage Regret criterion won't undergo any change if instead of estimate or probability, cost is provided.

Regret Table

	Strategies					
States of nature	6	7	8	9	10	11
6	0	180-150 =30	60	90	120	150
7	30	0	30	60	90	120
8	60	30	0	30	60	90
9	90	60	30	0	30	60
10	120	90	60	30	0	30
11	150	120	90	60	30	0
Max^m	150	120	90	90	120	150

Under Regret criterion : Min (max^m regret)

So, selected strategy → 8 or 9 copies

Q2. A stockist of a particular commodity makes a profit of Rs. 30 on each sale made within the same week of purchase otherwise he incurs a loss of Rs. 30 on each item. The no. of items sold within the same week range between 6 and 11. Find the optimum no. of items the stockist should buy to maximise his profits.

Solution

Pay-off table

	Strategies					
States of nature	6	7	8	9	10	11
6	180	180-30 = 150	180-60 = 120	180-90 = 90	180-120 = 60	180-150 = 30
7	180	~~120-60~~ = 210	180	150	120	90
8	180	210	240	210	180	150
9	180	210	240	270	240	210
10	180	210	240	270	300	270
11	180	210	240	270	300	330

Profit/item sold = Rs. 30

Loss/item unsold = Rs. 30

8 or 9 copies ←	Avg.	180	200	210	210	200	175
6 copies ←	Maximin	180	150	120	90	60	30
11 copies ←	Maximax	180	210	240	270	300	330
11 copies ←	Hurwicz with $\alpha = 0.7$	180	192	204	216	225	240

Now, we will create the Savage table

Prob. distribution	Freq.	States of Nature	6	7	8	9	10	11
·15	9	6	180	150	120	90	60	30
·20	12	7	180	210	180	150	120	90
·40	24	8	180	210	240	210	180	150
·15	9	9	180	210	240	270	240	210
·10	6	10	180	210	240	270	300	270
0	0	11.	180	210	240	270	300	330
1·00	60	EMV	180	201	$\boxed{210}$	195	171	141

1. Max. Likelihood Principle : Amongst all states of nature, I choose which is the most likely & then I would ~~choose~~ find out the optimum/best decision for that i.e. Max (most likely state of nature)

2. Expectation Principle : calculate the expected monetary value (E.M.V.) for every strategy & find out the value with the max. EMV.

Expected Profit under Perfect Information (EPPI) :

This is the max. amount of profit that could be made.

EPPI (for 6 items) $= \cdot 15 \times 180 + \cdot 20 \times 210 + \cdot 40 \times 240 + \cdot 15 \times 270 + \cdot 10 \times 300 + 0 \times 330$

$= 235 \cdot 5$

without perfect info, I can only make 210.

Probabilities applied to Regret table are expected Losses. We need to find out the min. value of expected losses.

The diff. between the two is known as Estimated Value of Perfect Information (EVPI)

EVPI = EPPI − EMV

= 235.5 − 210

= 25.5

Note: Let's try to solve the above questⁿ using Regret table

Regret Table

		~~Strategies~~	strategies					
Prob. ~~Sta~~	Freq.	States of nature	6	7	8	9	10	11
.15	9	6	0	30	60	90	120	150
.20	12	7	30	0	30	60	90	120
.40	24	8	60	30	0	30	60	90
.15	9	9	90	60	30	0	30	60
.10	6	10	120	90	60	30	0	30
0	0	11	150	120	90	60	50	0
1.00	60	Exp. Loss	$\overset{*}{55.5}$	34.5	25.5	40.5	64.5	94.5

∴ EVPI = min(Exp. Loss)

So, selected criterion → 8 copies

EVPI is Expected Loss of optimal decision)

ie EVPI = EL (optimal decision)

* EL = 0 × .15 + 30 × .20 + 60 × .40 + 90 × .15 + 120 × .10 + 150 × 0 = 55.5

DECISION TREE

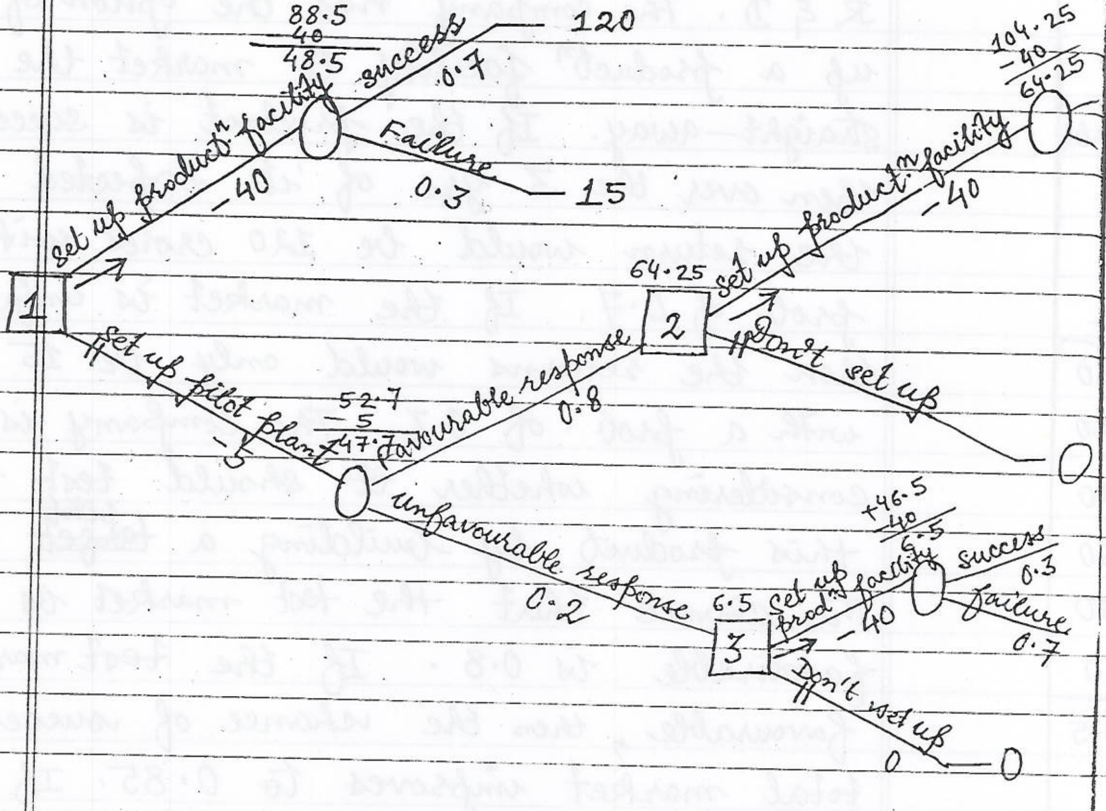
Q1. A company has produced a new product in R & D. The company has the option of setting up a productⁿ facility to market the product straight-away. If the product is successful, then over the 3 yrs. of its expected life, the return would be 120 crores with a prob. of 0.7. If the market is unfavourable then the returns would only be 15 crores with a prob. of 0.3. The company is considering whether it should test market this product by building a ~~target~~ pilot ~~market~~ plant. The chance that the test market is favourable is 0.8. If the test market is favourable, then the chance of successful total market improves to 0.85. If the ~~test~~ market is poor, the chance of success in the total market is only 0.3. As before, the returns from a successful market would be 120 cr. & from unsuccessful market 15 cr.

The installatⁿ cost to produce for the total market is 40 cr. & the cost of the pilot plan is 5 cr. Draw a decision tree & determine the optimal decision.

Convention used

□	decision node
○	chance node
→	selected branch
‖	branch not selected

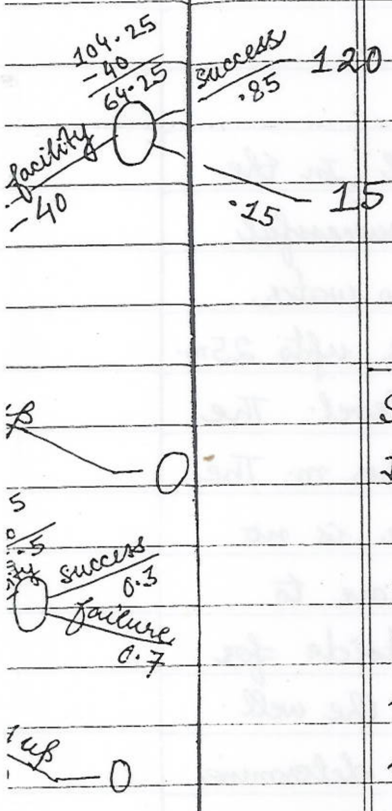
Note: A decision tree always starts with a decision node. So, for this quest^n:



Every chance node, evaluate

[Decision node : Choose between the branches]

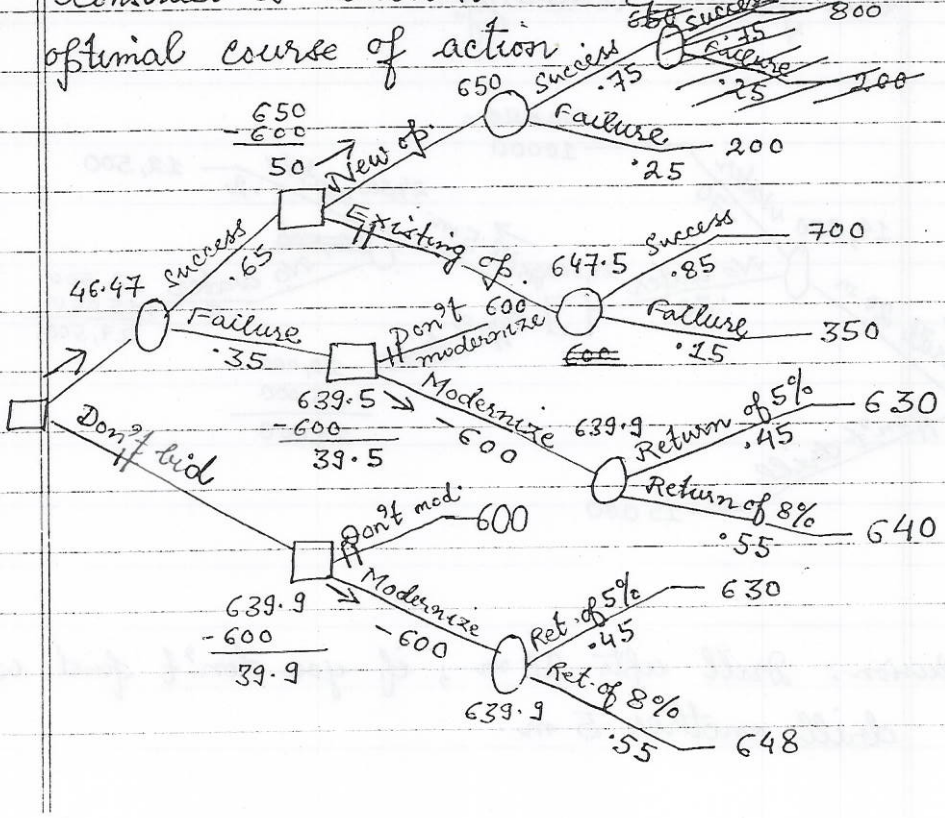
Every decision node, choose



Q2. An oil company is debating whether to go for an offshore drilling contract. If they bid, the value would be 600M ₹ with 65% chance of winning. It may set up a new drilling setup or move an existing setup to a new site. The prob. of success & expected ~~returns~~ revenue is as below:

	New Opn.		Existing Opn.	
	Prob.	Exp. ~~Ret~~ Rev. (M ₹)	Prob.	Exp. ~~Ret~~ Rev. (M ₹)
Success	.75	800	.85	700
Failure	.25	200	.15	350

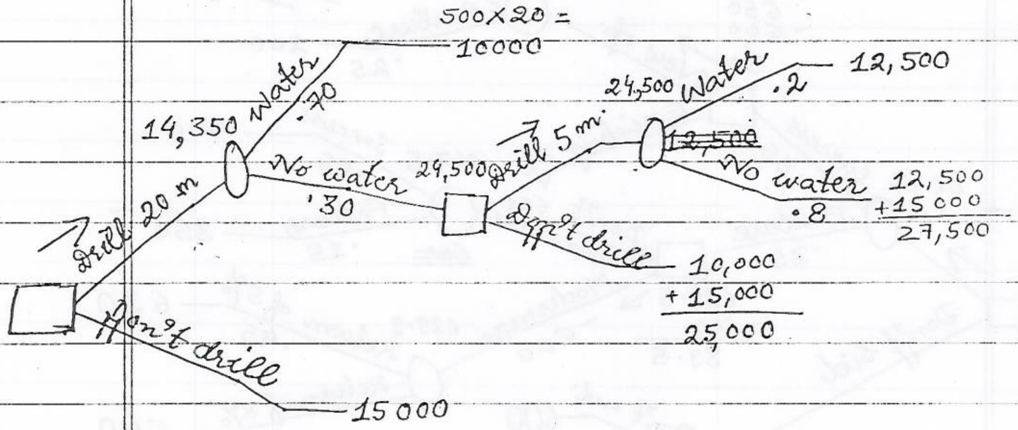
If the company does not bid or if it loses the contract, it can use the 600 M to modernize the operat^ns. This would result in a return of 5% or 8% ~~of~~ on the sum invested with probabilities .45 & .55 resp. Construct a decision tree & determine the optimal course of action.



Conclusion: So, bid for the contract. If its a success, set up a new operat^n; if its a failure, modernize.

Q3. A company is considering drilling a well. In the past only 70% of wells drilled were successful at 20 m. depth. Moreover, on finding no water at 20 m, some persons drilled further upto 25m. but only 20% struck water at that level. The prevailing cost of drilling is 500 ₹ per m. The company estimates that in case there is no water in its own well it would have to pay 15000 ₹ to buy water from outside for the same period of using water from the well.

Draw an appropriate decision tree & determine the optimal strategy.



Conclusion: Drill upto 20 m; if you don't find water drill another 5 m.

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Q4. An oil company has recently acquired rights in an area to conduct surveys & test drillings leading to drilling oil in commercial quantities. At the outset, the company has the choice to conduct further geological tests or to start drilling immediately. From the known conditions the company estimates that there is a 70-30 chance of further tests showing a success. Whether the tests show success or not or even if no tests are undertaken, the company could still drill or alternatively consider selling its right to drill in the area. Thereafter, if it carries out the drilling program, the likelihood of final success or failure is as below:

- If successful test conducted, the expectatn of success in drilling would be 80-20.
- If the test indicates the failure then the expectatn would be 20-80.
- If no test have been carried out at all, the expectatn of success in drilling would be 55-45.

Costs & revenues have been estimated & the NPV of each situatn is as below:

Outcome	Rs.(m)
Success:	
With Prior tests	100
Without Prior tests	120

Decision Theory – Aczel & Soundrapandian

Baysian Theorem & Simplex Method won't be asked in exams.
Also, no theory

Failure

with prior tests −50

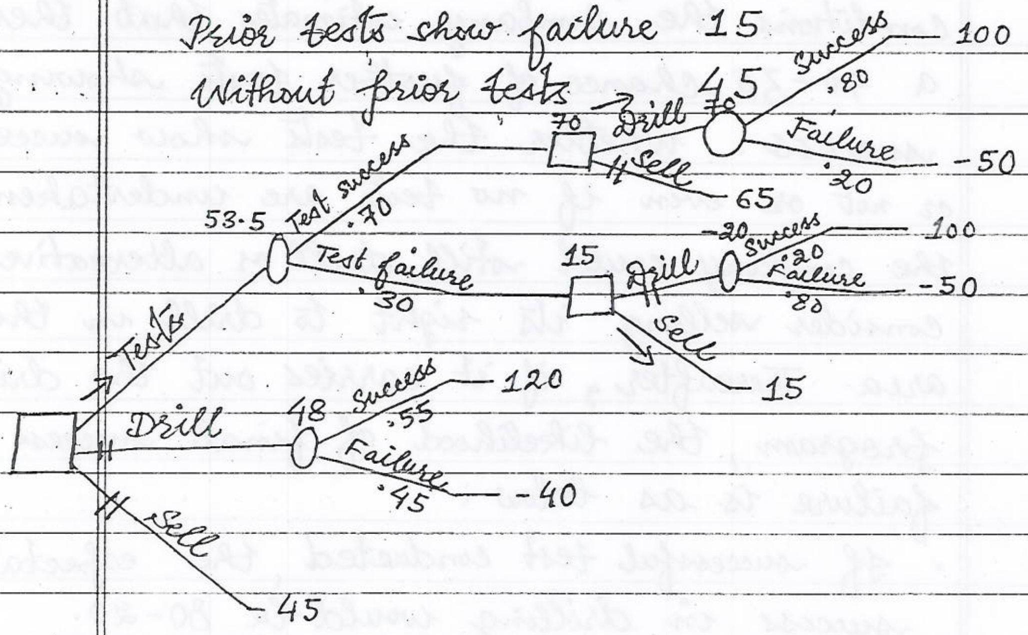
without prior tests −40

Sale of rights :

Prior tests show success 65

Prior tests show failure 15

Without prior tests 45



Conclusion: Perform tests ; if the tests are successful, drill ; if the tests are failure, sell.

Q

Ans.

Prob.

.001

.0005

.9985

Q The 8-lakh property of XYZ company has 1/10th of 1% chance of catching fire that may cause damage to the extent of ₹ 1 lakh & a 1/20th of 1% chance of catching fire that will completely destroy the property. XYZ is reviewing two alternative insurance policies :

a.) A policy with ₹ 50,000 deductable ; the annual premium for this policy is 1/10th of 1% of the value of the property.

b.) A no deductⁿ policy with full compensatⁿ having an annual premium of ₹ 1000.

Make a pay-off table & decide the optimal strategy.

Ans.

		Strategies	
		Cost - 800	Cost - 1000
Prob.	States of nature	Deductable policy	No ded. policy
·001	Partially	49,200 *	99,000
·0005	Completely	7,49,200 *	7,99,000
·9985	No fire	(-800)	(-1000)
	EMV	~~1222·60~~ -375	-500

Loss Incurred	1,00,000	8,00,000
- First 50,000	50,000	50,000
- ~~For~~ Cost	800	- 800
	49,200 *	7,49,200 *

∴ optimal strategy is to go for deductable policy

Note: EVPI = EPPI - EMV

$$= \left[(99000 \times \cdot 001) + (799000 \times \cdot 0005) - (800 \times \cdot 9985) \right] - (-375)$$

$$= -300 \cdot 3 - (-375) = 74 \cdot 7$$

LINEAR PROGRAMMING (LP) Q1.

Used where resources are limited but there are multiple situations where the resources are to be applied.

LP problems contain :

1. Objective (eg. to maximise profit or minimise cost) should be identifiable and measurable.

2. Activities (e.g. no. of units or no. of points to which the resources can be applied) should be identifiable and measurable. Ans.

3. Resources should be identifiable & measurable and resources should be limited. I:)

4. Relationships should be linear (In economies of scale, linearity doesn't exist, neither in volume discounts).

5. Finite number of alternative solutions should be present in a LP model.

II:)

III:)

IV:)

Q1. A firm produces 2 products A & B. Each unit of A requires 2 kg. of raw materials & 4 labour hrs. Each unit of B requires 3 kg. of raw materials & 3 labour hrs. Every week the firm has available 60 kg of raw materials & 96 labour hrs. ~~A~~ 1 unit of A sold gives a profit of 40 ₹ & 1 unit of B sold gives 35 ₹ as profit. All the products produced can be sold. Formulate the problem as an LP problem.

Ans.

	A	B	Availability
R/M (kg)	2	3	60
Labour hrs	4	3	96
Profit/unit (₹)	40	35	

I:) Identify decision variable

Decision variables are those variables whose values we want to solve. So, here:

X = no. of units of A to produce

Y = no. of units of B to produce

II:) Find out the objective function in the problem.

Maximise $Z = 40X + 35Y$

III:) subject to

$$\text{constraints}\begin{cases} 2X + 3Y \le 60 \\ 4X + 3Y \le 96 \end{cases}$$

(≤ because some resources unutilized is OK if profits are maximized)

IV:) $X \ge 0,\ Y \ge 0$

Non-negative constraint; important so that LP does not have to search unnecessary values (to make feasible region finite)

Q2. An agricultural institute suggests that farmers should spread at least 4800 kg. of a phosphate fertilizer & not less than 7200 kg of a nitrogen fertilizer per hectare to increase crop productivity. There are two sources to obtain these : mixtures A & B. Both these are available in 100 kg bags & cost 40 ₹ & 24 ₹ resp. Mixture A contains phosphate & Nitrogen in the ratio 20-80 while B contains these in equal proportions.

Write these as an LP problem.

Ans.

	A	B	Requirements
Phosphates (kg)	20	50	4800
Nitrogen (kg)	80	50	7200
Cost per bag	40	24	

I.) X_1 = No. of bags of A to buy

X_2 = No. of bags of B to buy

II.) Minimise $Z = 40X_1 + 24X_2$

III.) subject to

$$20X_1 + 50X_2 \geq 4800$$

$$80X_1 + 50X_2 \geq 7200$$

IV.)

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Note : Any no. of variables & any no. of constraints are possible in an LP problem.

Q3. A manufacturer makes 4 styles of purses. A 3 compartment bag taking 45 min. to assemble, a ~~sold~~ shoulder bag taking 1 hr., a tote bag needing 45 min. & a pocket purse needing 30 min. There are 32 hrs of assembly time available per day. The profit contributions are 16 & 25 ₹ on the 1st two items & ₹ 12 each on the remaining 2. A ~~cp~~ kind of fancy pin is used in the pocket purse of which 30 pieces are available per day. A diff. type of pin is used in the other types of which only 70 are available. Enough raw materials are available for 60 pocket ~~purposes~~ purses & tote bags which need the same quantity. A manufacturer estimates a min. demand of 6 pocket purses & 10 ~~solder~~ shoulder bags. Formulate it as an L.P.

Ans.

	3 comp.	Shoulder bag	Tote	Pocket Purse ~~Purpose~~	Availability
Assembly Time (min.)	45	60	45	30	32×60
Profit/unit (Rs.)	16	25	12	12	
Pins/~~used~~ unit	–	–	–	1	30
Pin II/unit	1	1	1	–	70
Raw materials	–	–	1	1	60
Demand	–	min. 10	–	min. 6	–

I.)
X_1 = No. of 3 comp. bags to make
X_2 = No. of shoulder bags to make
X_3 = No. of tote bags to make
X_4 = No. of pocket purses to make

II) Maximise $Z = 16X_1 + 25X_2 + 12X_3 + 30X_4$

III) subject to

$$45X_1 + 60X_2 + 45X_3 + 30X_4 \leq 1920$$

↖ Don't simplify

$$X_4 \leq 30$$

$$X_1 + X_2 + X_3 \leq 70$$

$$X_3 + X_4 \leq 60$$

$$X_2 \geq 10$$

$$X_4 \geq 6$$

IV) $X_1 \geq 0$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

ce

30 X4

≤ 1920
simplify

Q4. XYZ company has collected info. for advertising its products. The info. is given below:

Constraints	Medium	No. of families expected to cover	Cost/ad (Rs.)	Max. availability	Exp. exposure (units)
Max. 4	TV (30 sec)	3000	8000	8	80
	Radio (15 sec)	7000	3000	30	20
Min 2	Sunday Paper (1/4 page)	5000	4000	4	50
	Mag. (1 page)	2000	3000	2	60

Other info: The advertising budget is 70,000. At least 40,000 families should be covered. At least 2 insertions must be given in the Sunday paper but not more than 4 ads on TV.

Draft this as an LP problem to maximise the expected exposure.

Ans.

Max. availability		70,000		
Min. requirement	40,000			

Note: The constraint column put on left in the above table

I.)
X_1 = No. of ads on TV
X_2 = No. of ads on Radio
X_3 = No. of ads on Sunday paper
X_4 = No. of ads on Mag.

II.) Maximise $Z = 80X_1 + 20X_2 + 50X_3 + 60X_4$

III.) Subject to

$$3000X_1 + 7000X_2 + 5000X_3 + 2000X_4 \geq 40,000$$
$$8000X_1 + 3000X_2 + 4000X_3 + 3000X_4 \leq 70,000$$
$$X_1 \leq 8$$
$$X_2 \leq 30$$
$$X_3 \leq 4$$
$$X_4 \leq 2$$

$X_1 \leq 4$

$X_3 \geq 2$

IV.) $X_1 \geq 0$

$X_2 \geq 0$

$X_3 \geq 0$

$X_4 \geq 0$

Q5.) A mutual fund has 20 crores available for investment. It is considering investment in Govt. bonds, blue chip stocks, speculative stocks & bank deposits. The annual exp. return & risk factors are given below:

	Annual exp. return	Risk Rating (0-100)	Min requirement	Max
Govt bonds	14%	12		
Blue chip stocks	19%	24		
Speculative stocks	23%	48		20% of amt. invested
Bank deposits	12%	6	2	
Maximum		42 (avg.)		

The MF is req. to keep at least 2 crores in Bank deposits & not to exceed an average risk factor of 42. Speculative stocks can be at most 20% of the total amt. invested.

Formulate this as an LP problem. Decide how the MF should invest so as to max. its total expected return.

Ans. I.) X_1 = amt to be invested in Govt. bonds

X_2 = amt. to be invested in blue chip stocks

X_3 = amt. to be invested in speculative stocks

X_4 = amt. to be invested in bank deposits

II.) Maximise $z = .14X_1 + .19X_2 + .23X_3 + .12X_4$

III.) subject to

$$X_1 + X_2 + X_3 + X_4 \leq 20$$

$$\frac{12X_1 + 24X_2 + 48X_3 + 6X_4}{X_1 + X_2 + X_3 + X_4} \leq 42$$

$$X_4 \geq 2$$

$$X_3 \leq .2(X_1 + X_2 + X_3 + X_4)$$

IV.)

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

Assumptions in L.P.:

1. Finite choices
2. Proportionality (ie. linearity)
3. Additivity (ie there is no interaction between the variables)
4. Certainty (ie. the parameters to the variables are known for sure. LP is not probabilistic model, it is deterministic model)
5. Continuity (ie. decision variables can be fractional values)

Note: LP provides the best optimal solution among all the options including Integer Prog. options.

6. An LP can have only one objective (either maximise or minimise, not both). Use 'goal programming' to achieve multiple objectives.

★ At corner pts, at least two constraints are met/satisfied.

SOLUTION OF L.P.

Graphical Method : can be used only when 2 variables are present.

Q. Let's take the first question we solved for L.P.

Maximise $Z = 40X_1 + 35X_2$

Subject to

$2X_1 + 3X_2 \leq 60$

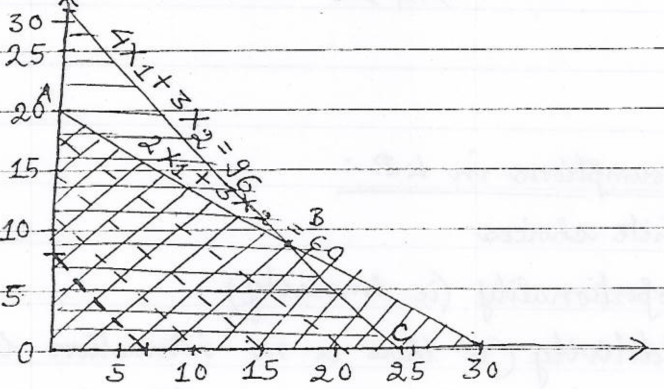
$4X_1 + 3X_2 \leq 96$

& $X_1, X_2 \geq 0$

1. Identify the area in the graph which satisfies the constraints. Do this by equating the equations to 0.

$2X_1 + 3X_2 = 60$ (0, 20) & (30, 0)

$4X_1 + 3X_2 = 96$ (0, 32) & (24, 0)



2. Now, choose the solution that gives the max profit. Do this by noting the values of corner* pts. O, A, B & C in the region OABC

		Z
O	0, 0	0
A	0, 20	700
B	18, 8	1000
C	24, 0	960

Graphical Method II : Better method than I.

Draw a line parallel to the objective function (ie. the slope of this is equal to the objective funct^n)

This is done by equating the objective function to an arbitrary value (obtained by taking LCM (here 280).

$$40X_1 + 35X_2 = 280$$

$$(0, 8) \quad (7, 0)$$

The last pt. of exit of the ~~isometric~~ isoprofit line from the feasible region would be the optimal solution.

Note: In case of maximise problems, the line moves outward.

In case of minimise problems, the line moves inward (so the arbitrary value chosen should be sufficiently large).

Binding constraint : A binding constraint forms the boundary of the feasible region.

Conditions of multiple optima :

i) Ob. function should be parallel to a constraint.

ii) The constraint to which the ob. funct^n is parallel should be ~~Ob. function should have~~ a binding constraint

iii) Constraint is in the direction of the movement of the objective function.

Unbounded problem : A problem in which the movement of the objective funct^n doesn't hit a constraint.

Infeasible regions – The regions exist but don't exist nearby.

Q1. A travel agent is planning a chartered trip to a major sea-port. The 8D 7N package includes the fare for the round trip, surface transportatⁿ, food & lodging & selected sight-seeing. The trip is restricted to 200 persons. The problem for the tr. agent is to determine the no. of deluxe, standard & economic packages to offer. These 3 plans differ acc. to service levels & quality of cabins. The foll. table summarizes the estimated prices for the 3 packages & the tr. agent's expenses. The agent has hired a ship for a fee of 2 lakhs for the trip. While planning, the foll. should be taken into a/c.

At least 10% of the packages should be deluxe

At least 35% but not more than 70% must be of the standard type.

At least 30% must be economy type.

The max no. of deluxe rooms available in the ship is 60.

The hotel desires that at least 120 tourists should be on a deluxe & standard packages taken together. Formulate this as an LP problem.

*Contribution ~~Ans~~		Price	Hotel cost	Boarding & other exp.	Min.	Max.	
2250	Deluxe	10,000	3000	4750	10%	60	} 120
2300	Standard	7,000	2200	2500	35%	70%	
2400	Economy	6,500	1900	2200	30%		

* Contribution = Price - Hotel cost - Boarding & other expenses

Ans I.) X_1 = No. of deluxe packages to offer
X_2 = No. of standard packages to offer
X_3 = No. of economy packages to offer

II.) Maximise $Z = 2250X_1 + 2300X_2 + 2400X_3 - 2,00,000$

III.) subject to

$$X_1 \geq .1(X_1 + X_2 + X_3)$$
$$X_2 \geq .35(X_1 + X_2 + X_3)$$
$$X_3 \geq .3(X_1 + X_2 + X_3)$$
$$X_1 \leq 60$$
$$X_2 \leq .7(X_1 + X_2 + X_3)$$
$$X_1 + X_2 \geq 120$$
$$X_1 + X_2 + X_3 \leq 200$$

IV.)
$$X_1 \geq 0$$
$$X_2 \geq 0$$
$$X_3 \geq 0$$

Q2. A refinary makes 3 grades of petrol A, B, C from 3 crude oils D, E, F. Crude oil F can be used in any grade but the others must satisfy the foll. specifications:

Grade	Selling price/l	Specs
A	18	Not less than 50% D Not more than 25% E
B	16.5	Not less than 25% D Not more than 50% E
C	15.5	No restriction

Grade	Capacity avail (k.l)	Price/l
D	500	19.5
E	500	14.5
F	360	15.1

Formulate this as an L.P.

Ans. I.) X_{ij} = quantity of i^{th} crude used in j^{th} product

II.) $$\text{Maximise } Z = 18(X_{DA} + X_{EA} + X_{FA}) + 16.5(X_{DB} + X_{EB} + X_{FB}) + 15.5(X_{DC} + X_{EC} + X_{FC}) - 19.5(X_{DA} + X_{DB} + X_{DC}) - 14.5(X_{EA} + X_{EB} + X_{EC}) - 15.1(X_{FA} + X_{FB} + X_{FC})$$

III.) subject to

$$X_{DA} \geq .5(X_{DA} + X_{EA} + X_{FA})$$
$$X_{EA} \leq .25(X_{DA} + X_{EA} + X_{FA})$$
$$X_{DB} \geq .25(X_{DB} + X_{EB} + X_{FB})$$
$$X_{EB} \leq .50(X_{DB} + X_{EB} + X_{FB})$$

$$X_{DA} + X_{DB} + X_{DC} \leq 500\,000$$
$$X_{EA} + X_{EB} + X_{EC} \leq 500\,000$$
$$X_{FA} + X_{FB} + X_{FC} \leq 360000$$

IV.) $X_{ij} \geq 0$

Q 3. Evening shift doctors at a hospital work 5 consequitive days & have 2 consequitive days off. Their 5 days work can start on any day of the week & the schedule rotates indefinitely. The hosp. req. the foll. min. no. of doctors on each day :

	Min no. of days
S	35
M	55
T	60
W	50
Th	60
F	50
S	45

No. more than 40 doctors can start their 5 working days on the same day. Formulate this as an LP problem & determine the min. no. of doctors to be employed by the hospital.

Ans:

I.)
X_1 = No. of doctors starting on Sunday
X_2 = No. of doctors starting on Monday
X_3 = No. of doctors starting on Tuesday
X_4 = No. of doctors starting on Wednesday
X_5 = No. of doctors starting on Thursday
X_6 = No. of doctors starting on Friday
X_7 = No. of doctors starting on Saturday

II.) Minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$

III.) subject to

$$X_1 \leq 40$$

$X_2 \leq 40$

$X_3 \leq 40$

$X_4 \leq 40$

$X_5 \leq 40$

$X_6 \leq 40$

$X_7 \leq 40$

$X_1 + X_7 + X_6 + X_5 + X_4 \geq 35$

$X_2 + X_1 + X_7 + X_6 + X_5 \geq 55$

$X_3 + X_2 + X_1 + X_7 + X_6 \geq 60$

$X_4 + X_3 + X_2 + X_1 + X_7 \geq 50$

$X_5 + X_4 + X_3 + X_2 + X_1 \geq 60$

$X_6 + X_5 + X_4 + X_3 + X_2 \geq 50$

$X_7 + X_6 + X_5 + X_4 + X_3 \geq 45$

IV.) $X_i \geq 0$

Every $\leq$ constraint will have a slack variable
& Every row of the problem represents a constraint

SIMPLEX METHOD

No need to solve problems using Simplex Method, but it ~~st~~ is used for reading values from the table.

Q We will solve the same questⁿ/example we used in the graphical method.

$$\text{Max } Z = 40X_1 + 35X_2$$

subject to

$$2X_1 + 3X_2 \leq 60$$
$$4X_1 + 3X_2 \leq 96$$
$$\& \ X_1, X_2 \geq 0$$

1.) R.H.S. values should be +ve. If RHS is -ve, it should be multiplied by -1 throughout (The inequality signs will change correspondingly).

2.) The non-negative constraint is a must in Simplex

i.) For every $\leq$ we introduce a slack variable (for unutilized raw materials) S.

ii.) ~~Put the~~ Equate the equations to 0 after the introductⁿ of the slack variables.
This is known as standardizing the problem.

iii.) Then we construct the simplex table writing all the constraints in separate rows as we encounter them

Standardizing the L.P. :

$$\text{Max } Z = 40X_1 + 35X_2 + 0S_1 + 0S_2$$

Subject to

$$2X_1 + 3X_2 + S_1 = 60$$
$$4X_1 + 3X_2 + S_2 = 96$$
$$X_1, X_2 \geq 0$$
$$S_1, S_2 \geq 0$$

Note: At each iteration, only one variable can be 'Entering' & one can be 'Leaving'.

The colum corresponding to Entering variable is 'Pivot' column, row corresponding to Leaving variable is 'pivot' row. The intersection of the two is 'Pivot' element

Profit element	C_j	40	35	0	0	b_i	Replacement
C_j	Basis	X_1	X_2	S_1	S_2	RHS	Ratios
0	S_1	2	3	1	0	60	$\frac{60}{2}=30$
0	S_2	4	3	0	$\underline{1}$	96	$\frac{96}{4}=24$
opp. cost of making 1 unit of column value	Z_j	0	0	0	0		
Net profit after opp. cost for making 1 unit of column variable	$\Delta_j = C_j - Z_j$	40	35	0	0		

↑ Most +ve value known as 'Entering' value

S_2 will become the 'leaving' variable

$S_1 = 60$

$S_2 = 96$

$X_1 = 0$

$X_2 = 0$

$Z = 0$ ($\because X_1$ & X_2 are zero)

C_j	Basis	X_1	X_2	S_1	S_2	b_i
0	S_1	0	$3-2\times\frac{3}{4} = 3/2$	1	$0-2/4 = -1/2$	$60-\frac{96\times2}{4}$ 12
40	X_1	$\underline{1}$	$3/4$	0	$1/4$	24
	Z_j	40	30	0	10	960
	$\Delta_j = C_j - Z_j$	0	5	0	-10	

← After dividing the pivot row with pivot element

↑ Entering variable

$X_1 = 24$

$X_2 = 0$

$S_1 = 12$

$S_2 = 0$

} optimal mix

$Z = 960$

Ref. Ratios

$\frac{12}{3/2} = 8$

$\frac{24}{3/4} = 32$

	C_j	40	35	0	0	
35	X_2	0	1	$2/3$	$-1/3$	8
40	X_1	$\underline{1}$	0	$-1/2$	$1/2$	18
	Z_j	40	35	$10/3$	$25/3$	$Z = 1000$
	Δ_j	0	0	$-10/3$	$-25/3$	

Imp. of Δ_j:

a) Helps to decide whether we have reached optimal sol.

b:) It tells the impact on profit for each unit of corner variable straight-away ie. gives marginal impact on profitability.

c:) It also tells what min. ~~marginal wo~~ rental values for the resources should be charg

d:) Δ_j also tells us or helps us to impute (by working backwards) what is the max. profit the resources would generate for me).

be "Entering"
lumn
The intersection is "pivot" element
ment
S₁ = 60 → $S_1 = 60$
$_2 = 96$
$_1 = 0$
$_2 = 0$
= 0 (∵ X_1 & X_2 are zero)

4
0 } optimal mix
the pivot row
ivot element
2
0
60

2

. optimal sol.
f corner
al impact on
he resources should be charged or what I need to pay for getting one additional unit of the resources
(by working
resources

Note i) Every Table ~~has~~ entry would form 1 identity matrix
ii) Identity matrix identifies the Basic variables & vice-versa

Ques. 2) Minimise $Z = 40X_1 + 24X_2$

subject to

$$20X_1 + 50X_2 \geq 4800$$
$$80X_1 + 50X_2 \geq 7200$$
$$X_1, X_2 \geq 0$$

I) $20X_1 + 50X_2 - S_1 + A_1 = 4800$

S_1 → surplus variable

A_1 → artificial variable introduce to generate identity matrix

$$80X_1 + 50X_2 - S_2 + A_2 = 7200$$

So the problem becomes:

$$z = 40X_1 + 24X_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$

↓ cost having very high artificial variable which would be driven away (& never come back) in the Simplex algorithm

Note: The Dual table for the above Primal problem is as follows:

Cj	Basis	y_1	y_2	S_1	S_2	A_1	A_2	b_i
96	y_2	0	1	$-1/2$	$1/3$	$1/2$	$-1/3$	$25/3$
60	y_1	1	0	$1/2$	$-2/3$	$-1/2$	$2/3$	$10/3$
	Δj	0	0	18	8	M-18	M-8	$G = 96 \times \frac{25}{3} + 60 \times \frac{10}{3}$

$$= 800 + 200$$
$$= 1000$$

I)

Ques 3. A firm uses 3 machines in the manufacture of 3 products. Each unit of product A req. 3 hrs on m/c 1, 2 hrs on m/c 2 & 1 on m/c 3. Each unit of B req. 4 on m/c 1, 1 on m/c 2 & 3 hrs on m/c 3. Each unit of C req. 2 hrs on each of the machines. The contribution margins are 30, 40 & 35 per unit resp. The m/c hrs available are 90, 54 & 93 resp.

a) Formulate the above as an LP problem.

b) Obtain the optimal sol. using Simplex. Which of the 3 products will not be produced by the firm. Why?

c) Calculate the % capacity utilisat^n of the machines.

d) What would be the effect on the sol. for each of the foll:

i) Obtaining an order for 4 units of product A which has to be met

ii) An increase of 20% of capacity in m/c 1.

Ans. (a)

	m/c 1 (hrs)	m/c 2 (hrs)	m/c 3	Contribut^n Rs/unit (hrs) ~~Avail~~
A	3	2	1	30 ~~90~~
B	4	1	3	40 ~~54~~
C	2	2	2	35 ~~93~~
~~Contribution~~ (Rs/unit)	~~30~~	~~40~~	~~35~~	
~~Maximise z = 30A + 40B + 35~~				
Availability	90	54	93	

~~m/c 1 ≤ 90~~

~~m/c 2 ≤ 54~~

~~m/c 3 ≤ 93~~

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unit resp.
93 resp.
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m/c 1.

3	Contribut^n Rs/unit (hrs) ~~Avail~~
	30 ~~90~~
	40 ~~54~~
	35 ~~93~~

I) X_1 = no. of units of A to produce
X_2 = no. of units of B to produce
X_3 = no. of units of C to produce

Maximise $Z = 30X_1 + 40X_2 + 35X_3$

subject to :

$$3X_1 + 4X_2 + 2X_3 \leq 90$$
$$2X_1 + X_2 + 2X_3 \leq 54$$
$$X_1 + 3X_2 + 2X_3 \leq 93$$
$$X_1 \geq 0$$
$$X_2 \geq 0$$
$$X_3 \geq 0$$

~~Formulati~~ Standardisation of L.P.

$$3X_1 + 4X_2 + 2X_3 + S_1 = 90$$
$$2X_1 + X_2 + 2X_3 + S_2 = 54$$
$$X_1 + 3X_2 + 2X_3 + S_3 = 93$$
$$X_1 \geq 0, \ S_1 \geq 0$$
$$X_2 \geq 0, \ S_2 \geq 0$$
$$X_3 \geq 0, \ S_3 \geq 0$$

classmate

In order to make 1 unit of X_2 (column variable), I will have to give up 4 units of S_1, 1 unit of ~~X~~ S_2 & 3 units of S_3 (basic variables) [Read column wise]

	C_j	30	40	35	0	0	0	b_i	Replacement Ratios
C_j	Basis	X_1	X_2	X_3	S_1	S_2	S_3	R.H.S.	
0	S_1	3	$\boxed{4}$	2	1	0	0	90	$\frac{90}{4} = 22.5$ ← Leaving variable (min +ve replacement ratio)
0	S_2	2	1	2	0	1	0	54	$\frac{54}{1} = 54$
0	S_3	1	3	2	0	0	1	93	$\frac{93}{3} = 31$
	Z_j	0	0	0	0	0	0	$Z = 0$	
	$\Delta_j = C_j - Z_j$	30	40	35	0	0	0		

∴ $S_1 = 90$
$S_2 = 54$
$S_3 = 93$
& $X_1 = 0$
$X_2 = 0$
$X_3 = 0$
& $Z = 0$

↑ Entering variable (X_2 makes the max. output/profit after considering opportunity cost, hence use X_2 in place of S_1 in the next table)

slack variables will always be 0 at the beginning

C_j	Basis	X_1	X_2	X_3	S_1	S_2	S_3	RHS
40	X_2	3/4	1	2/4	1/4	0	0	22.5 ← Diving by pivot element
0	S_2	$2 - \frac{3}{4}\cdot 1$ $= 5/4$	0	$2 - 2/4$ $= \boxed{3/2}$	$0 - 1/4$ $= -1/4$	1	0	$54 - \frac{90\cdot 1}{4}$ $= 31.5$
0	S_3	$1 - 9/4$ $= -5/4$	0	$2 - \frac{3\cdot 2}{4}$ $= \frac{1}{2}$	$0 - 3/4$ $= -3/4$	0	1	$93 - \frac{3\times 90}{4}$ $= 25.5$
	Z_j	30	40	20	10	0	0	$Z = 900$
	Δ_j	0	0	15	-10	0	0	

Ref. Ratios
$\frac{22.5}{1/2} = 45$
$31.5 \times \frac{2}{3} = 21$
$25.5 \times 2 = 51$

S_2 is leaving variable

↑ Entering variable

∴ $X_2 = 22.5$
$S_2 = 31.5$
$S_3 = 25.5$
$X_1 = 0$
$X_3 = 0$
$S_1 = 0$

All basic variables will take RHS values
All non-basic variables will take 0

C_j	Basis	X_1	X_2	X_3	S_1	S_2	S_3	RHS
40	X_2	1/3	1	0	1/3	-1/3	0	12
35	X_3	5/6	0	1	-1/6	2/3	0	21
0	S_3	-5/3	0	0	-2/3	-1/3	1	15
	Z_j	42.5	40	35	45/6	10	0	$Z = 1215$
	Δ_j	-12.5	0	0	-7.5	-10	0	

[Shortcut:
$Z = 900$
+

$X_2 = 12$
$X_3 = 21$
$S_3 = 15$
$X_1 = 0$
$S_1 = 0$
$S_2 = 0$
∴ $Z = 1215$

Note: i) Form a 4-element sq. with the pivot row & pivot column

ii) Basic variables will correspond to identity matrix

iii) I'm making some profit. If I start making something else, I've to forego some profit (existing). This is opportunity cost.

iv) The basic variables in the Δj row will be zero (compulsory)

(b) So, product A will not be produced by the firm. If X_1 is being made, every unit of X_1 produced will reduce the profit by 12.5.

(c)

	M/c hrs	Unutilised hrs.	Utilised hrs	% capacity utilisat^n
I	90	0	90	100%
II	54	0	54	100%
III	93	15	78	83.9%

(d)

i) obtaining an order/contract for 4 units of product, will change:

New profit $= 1215 - 4 \times 12.5$

$= 1215 - 50 = 1165$

Current mix		Red. for making 4 units of X_1	New mix
X_2	12	$4 \times \frac{1}{3} = 4/3$	$12 - 4/3 = \frac{32}{3}$
X_3	21	$4 \times \frac{5}{6} = 10/3$	$21 - \frac{10}{3} = \frac{53}{3}$
S_3	15	$4 \times (\frac{-5}{3}) = -\frac{20}{3}$	$15 - (-\frac{20}{3}) = \frac{65}{3}$
X_1	-	-	4

New Profit:

$\frac{32}{3} \times 40 = 426.67$

$\frac{53}{3} \times 35 = 618.33$

$\frac{65}{3} \times 0 = 0$

$4 \times 30 = 120$

1165.0

(column variable),
~~of S_1, 1 unit~~
ic variables)

Replacement
Ratios

$\frac{90}{4} = 22.5$ ← Leaving variable (min +ve replacement ratio)
$\frac{54}{1} = 54$
$\frac{93}{3} = 31$

$\therefore S_1 = 90$
$S_2 = 54$
$S_3 = 93$
& $X_1 = 0$
$X_2 = 0$
$X_3 = 0$
& $Z = 0$

~~X_2 in place of~~

Diving by
← pivot element

Ratios
$= 45$
$\frac{2}{3} = 21$
$= 51$

$\therefore X_2 = 22.5$
$S_2 = 31.5$
$S_3 = 25.5$
$X_1 = 0$
$X_3 = 0$
$S_1 = 0$

basic variables
take RHS values
non-basic
bles will take 0

$X_2 = 12$
$X_3 = 21$
$S_3 = 15$
$X_1 = 0$
$S_1 = 0$
$S_2 = 0$
$\therefore z = 1215$

ii) An increase of 1 hr. of m/c #1 $\overset{(S_1)}{\cancel{(S_3)}}$ will increase the profit by 7.5 & decrease of 1 hr. will decrease the profit by that value.

20% of 90 hrs. = 18 hrs.

∴ 18 more hrs. of m/c 1, will increase the profit to: $1215 + 18 \times 7.5$

$= 1350$

	Current mix	Increase bec. of additional 18 hrs	New mix	New profit
X_2	12	$18 \times \frac{1}{3} = 6$	$12+6=18$	$18 \times 40 = 720$
X_3	21	$18 \times \left(-\frac{1}{6}\right) = -3$	$21-3=18$	$18 \times 35 = 630$
S_3	15	$18 \times \left(-\frac{2}{3}\right) = -12$	$15-12=3$	$3 \times 0 = 0$
				Total profit: 1350

I.) <u>Note</u>: If all replacement ratios are either -ve or infinity, that represents unboundedness in Simplex.

II.) Multiple optima in Simplex means:
a non-basic variable has got 0 value in the Δj row

III.) Infeasible condition in Simplex means:
an artificial ~~value~~ variable is there in the optimal solution

IV.) Degeneracy : A degenerate variable is a condition where a basic variable (on the RHS) takes the value 0. So, replacement ratio & hence entering value will both become 0. So, on encountering degeneracy, new variables are ready to be entered into the system, but profits won't increase & hence iterat^n^s need to keep on going as optimal solution has not been reached.

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ix New profi
18×40 = 72
18×35 = 630
3×0 = 0
Total profit: 1350

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Degeneracy doesn't cause any problem if it appears in the optimal solution (only when it happens on-route to optimal solution, it will cause multiple optimal solut^{n}s, & hence need to be identified)

When there is a tie in replacement ratios, the next table will be degenerate table.

* where y_1, y_2 & y_3 are the opportunity costs & tell what is the min rental value of the resources.
This also tells what is the maximum profit the resources are capable of generating for

Ques 4) Maximise $Z = 30X_1 + 40X_2 + 35X_3$

subject to

$$3X_1 + 4X_2 + 2X_3 \leq 90$$
$$2X_1 + X_2 + 2X_3 \leq 54$$
$$X_1 + 3X_2 + 2X_3 \leq 93$$
$$X_1, X_2, X_3 \geq 0$$

The above is 'Primal problem'.

There is one 'Dual problem' for every Primal problem. If primal problem is maximisat^n problem, the dual is minimisat^n problem.

So, the above problem would become:

Minimise $G = 90y_1 + 54y_2 + 93y_3$ *

subject to

$$\left.\begin{aligned} 3y_1 + 2y_2 + y_3 &\geq 30 \\ 4y_1 + y_2 + 3y_3 &\geq 40 \\ 2y_1 + 2y_2 + 2y_3 &\geq 35 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}\right\} \text{Transpose of the Primal problem}$$

Note: 1.) The value of G & Z would be the same

2.) Whatever are the Δj values in Primal problem would become the solutions in Dual & whatever are the ~~solutions~~ Δj values in Dual will be the solutions in Primal.

Similarly, X_1 & X_2 values will denote S_1 & S_2 variables in Dual (∵ Dual is transpose of Primal inter-replacing resources with constraints & vice-versa).

Note: Just remember to ignore the signs of the values as maximisation in Primal becomes minimisat^n in Dual.

- - - - - - - - - - - - - - - - - - - -

Let's have a look at the optimal matrix of Ques. 3 :

c_j	Basis	X_1	X_2	X_3	S_1	S_2	S_3	b_i	
40	X_2	$1/3$	1	0	$1/3$	$-1/3$	0	12	$\therefore X_2 = 12$ $X_1 = 0$
35	X_3	$5/6$	0	1	$-1/6$	$2/3$	0	21	$X_3 = 21$ $S_1 = 0$
0	S_3	$-5/3$	0	0	$-2/3$	$-1/3$	1	15	$S_3 = 15$ $S_2 = 0$
	Δ_j	$-25/2$	0	0	$-15/2$	-16	0	$z = 1215$	₹ $z = 1215$

↑ If we have 1 more unit of S_1 the profit would increase by $15/2$. On the contorary, if we have 1 less unit of S_1, the profit would reduce by $15/2$. So, this provides marginal change in profit.

Also, Δ_j tells imputed/shadow price for the resources

ie. Each unit of S_1 is worth $15/2$

Each unit of S_2 is worth 10 ₹

Each unit of S_3 is worth 0.

$\therefore$ By increase in 1 unit of X_1, the profit would be affected as follows :

$$3 \times 15/2 = 22 \cdot 5$$
$$2 \times 10 = 20$$
$$1 \times 0 = 0$$
$$\overline{42 \cdot 5 > 30} \quad (\text{by } 12 \cdot 5)$$

Similarly for X_3 :

$$2 \times 15/2 = 15$$
$$2 \times 10 = 20$$
$$2 \times 0 = 0$$
$$\overline{35}$$

If profit of one variable increases or falls, how much would the mix change

SENSITIVITY ANALYSIS

Sensitivity Analysis is done only on optimal tables.

Case 1 : When a variable is not in the basis, till the variable is [Profit + ~~e~~ marginal value], the mix will continue to be optimal. So, the profit will have to be more than the marginal value for us to start making ~~it~~ the variable. So the range for X_1 here is :

$$(-\infty, 30 + 25/2)$$

Let's consider the previous problem again :

	C_j	30	$40+x$	35	0	0	0
C_j	Basic	X_1	X_2	S_3	S_1	S_2	S_3
$40+x$	X_2						
35	X_3						
0	S_3						
	Δj						
	z_j	$\frac{85}{2}+\frac{x}{3}$	$40+x$	35	$\frac{15}{2}+\frac{x}{3}$	$\frac{30}{3}-\frac{x}{3}$	0
	Δj	$-\frac{25}{2}-\frac{x}{3}$	0	0	$-\frac{15}{2}-\frac{x}{3}$	$-10+\frac{x}{3}$	0

Now,

$$-\frac{25}{2}-\frac{x}{3}=0 \Rightarrow x = -\frac{75}{2}$$

$$-\frac{15}{2}-\frac{x}{3}=0 \Rightarrow x = -\frac{45}{2}$$

$$-10+\frac{x}{3}=0 \Rightarrow x = 30$$

Find out the least -ve (closest to zero) & the least +ve (closest to zero on the other side)

So, in this range $[40-22.5, 40+30]$, the profit of the mix would continue to be optimal

Short-cut : The same can be obtained by :

$$\begin{array}{c|cccccc} \frac{\Delta j}{x_2} & -\frac{75}{2} & 0 & - & -\frac{45}{2} & 30 & - \end{array}$$

Similarly,

$$\begin{array}{c|cccccc} \frac{\Delta j}{x_3} & -15 & - & 0 & 45 & -15 & - \end{array}$$

Case 2 : How many units of a resource can we keep on adding/decreasing without affecting its shadow price ?

If the variable is in the basis, continuing to add it won't cause any change.

In the optimal solution for the above quest^n, there are 15 surplus units of S_3 for m/c 3. So, decreasing S_3 resource for m/c 3 won't cause any change ; thereafter it will begin to. In other words, between $[93-15, \infty]$ (ie. between $[78, \infty]$), the shadow price of the resource would continue to be zero.

In general, scarce resource affects shadow price but surplus resource doesn't.

b_i/S_1	b_i/S_2
$12/\tfrac{1}{3} = 36$	$12/-\tfrac{4}{3} = -36$ ↗ least −ve
$21/\tfrac{1}{6} = 126$	$21/\tfrac{2}{3} = 31.5$ → least +ve
$15/-\tfrac{2}{3} = -22.5$	$15/-\tfrac{1}{3} = -45$

Note: Transpose of a Primal matrix is a 'Dual' matrix & the transpose requires a sign change. [Also, the R.H.S. values of a Primal ~~becomes~~ the Δj values in the Transpose]

Therefore, for S_1:

In the range $[90-36,\ 90+22.5]$ (ie $[54, 112.5]$), the shadow price remains the same.

Similarly, for S_2:

In the range $[54-31.5,\ 54+36]$ (ie $[22.5, 90]$), the shadow price remains the same.

b_i/S_2 least -ve

$12/-4/3 = -36$ ↗

$21/2/3 = 31.5$ → lea +ve

$15/-1/3 = -45$

a 'Dual'

gn change. values in the Transpose]

4, 112.5]),

22.5, 90]),

Q1. The Simplex table for a maximisatⁿ problem is given below:

		4	5	0	0	
C_j	Basis	X_1	X_2	S_1	S_2	b_i
5	X_2	1	1	1	0	10
0	S_2	1	0	-1	1	3

Answer the foll. questⁿs giving reasons.

i) Is this sol optimal?

ii) Is there more than one optimal solution?

iii) Is this sol. degenerate?

iv) Is this sol. feasible?

v.) If S_1 is slack in machine 1 in hours/week & S_2 is slack in m/c 2, which of the m/c s is being used in full capacity acc. to this sol?

vi) A customer would like to have 1 unit of X_1 & is willing to pay in excess of the normal price, how much should ~~the~~ he be charged?

vii) How many units of X_1 & X_2 are being produced & what's the total profit?

viii) M/c 1 has to be shut down for 2 hrs next week. What would be the effect on profits & how many units of X_1 & X_2 would be

ix.) ~~viii)~~ How much would you be prepared to pay for another extra hr. on m/c 1 & m/c 2?

x.) ~~ix.)~~ A new product is proposed to be introduced which requires 1/2 hr on m/c 1 & 20 min. on m/c 2. It would give a profit of Rs 3/unit. Should the product be introduced?

Ans) i)

	C_j	4	5	0	0	
C_j	Basic	X_1	X_2	S_1	S_2	b_i
5	X_2	1	1	1	0	10
0	S_2	1	0	-1	1	3
	Z_j	5	5	5	0	0
	Δ_j	-1	0	-5	0	

Since all Δ_j values are 0 or -ve, the sol. is optimal.

ii) Since none of the non-basic variables has got 0 in the Δ_j row, there is only one optimal solution.

iii) Since all basic variables are non zero (i.e. $X_2 = 10$ & $S_2 = 3$), the solution is not degenerate.

iv) Since there is no artificial variable in the optimal solution, the solution is feasible.

v) M/c 1 is fully utilised as $S_1 = 0$
M/c 2 is unutilised for 3 hrs. as $S_2 = 3$.

vi) At least 1 more unit should be charged.

vii) Profit = 50
Every unit of X_2 would generate a profit of 5.

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vii)

New profit $= 50 - 10$
$= 40$

Current mix		Reduction in quantity	New quantity
X_2	10	$2 \times 1 = 2$	8
S_2	3	$2 \times -1 = -2$	5

$\therefore$ New Profit $= 8 \times 5$
$= 40$

ix) ~~viii)~~ For m/c 1, I would be willing to pay a max. of Rs. 5

For m/c 2, I won't be willing to pay anything as they are unutilised hours.

x) ~~ix)~~

Opportunity cost:

$\frac{1}{2} \times 5 = \frac{5}{2}$

$\frac{1}{3} \times 0 = 0$

$\frac{5}{2} = 2.5$

Profit offered = Rs. 3

Hence, the product should be introduced.

Q2. A company manufactures & sells pressure cookers. Supplies of Al are limited to 750 kgs per week & the availability of m/c hrs are limited to 600 hrs/week. The resource usage of the 3 models of cookers are given below:

	M_1	M_2	M_3
Al/unit	6	3	5
M/c hrs	3	4	5
Contribut^n/unit	60	20	80

a.) Formulate the problem as an LPP.

b.) Foll. table was obtained while solving with Simplex

	X_1	X_2	X_3	S_1	S_2
X_1	1	$-1/3$	0	$1/3$	
X_3	0				

Verify the sol. for optimality.

c.) Interpret the sol. Comment on the use of the resources.

d.) Is the sol. feasible ?

e.) Does the sol. have multiple optima ?

f.) Is the sol. degenerate ?

g.) What happens if an additional 150 kg. of Al becomes available ? Give the impact on profit & product mix

h.) If the m/c hrs. available reduce from 600 to 450, what will be the new profit & the product mix ?

i.) There is a reduct^n in the S.P of M_3 whereby its contribut^n margin decreases by 15 Rs. Will the optimal mix change?

j) What min. contribut^n of M_2 would make it feature in the optimal sol.

k.) A new model has been developed requiring 3 kg. of Al & 3 hrs. If it has a contribut^n of 40 Rs., would it be worthwhile to manufacture the product?

Ans.) a) Let the X_1 = Amt. of Al required for M/c 1
X_2 = M/c 2
X_3 = M/c 3
Y_1 = No. of hrs. available on M_1
∴ Maxim Y_2 = M_2
Y_3 = M_3

$$\therefore \text{Maximize } Z = \frac{60X_1}{Y_1} + \frac{20X_2}{Y_2} +$$

X_1, X_2, X_3 = No. of units of M_1, M_2, M_3 to produce

$$\text{Maximise } Z = 60X_1 + 20X_2 + 80X_3$$

Subject to:

$$6X_1 + 3X_2 \leq 750$$
$$\leq 600$$

b.)

		C_j	60	20	80	0	0			
	C_j'	Basis	X_1	X_2	X_3	S_1	S_2	b_i	b_i/S_1	b_i/S_2
	60	X_1	1	$-1/3$	0	$1/3$	$-1/5$	50	$50/1/3 \leftarrow$ least +ve	$-250 \rightarrow$ least -ve
	80	X_3	0	1	1	$-1/5$	$2/5$	90	$90/-1/5 = -450$ ↓ least -ve	225 → least +ve
		z_j	60	60	80	4	20			
		Δ_j	0	-40	0	-4	-20			

∵ all Δj values are 0 or -ve, the solution is optimal.

c.) $X_1 = 50$ $X_2 = 0$

$X_3 = 90$ $S_1 = 0$

$S_1 = 0$

$z = 10,200$

Both S_1 & S_2 are 0, hence both the resources are fully utilized.

d.) No artificial variable in the optimal solution hence the solution is feasible.

e.) No non-basic variable has got 0 in the Δj row, so there is no multiple optimum (ie. unique solution).

f.) Sol. is not degenarate because basic variables are not zero.

g.) Do R.H.S ranging to check for allowable range.

b_i/S_1	b_i/S_2
$50/\frac{1}{3}$ ← least +ve	-250 → least -ve
$90/-\frac{1}{5} = -450$ ↓ least -ve	225 → least +ve

olution

solution

the Δj
(ie.

variables

So, resource can be increased by 450 without effecting the shadow price (least +ve tells so).

Shadow price for 150 kgs added ie. new profit

$= 10,200 + 4 \times 150$

$= 10,800$

	current mix	↑ of 150 kg Al	New Mix	∴ New profit
X_1	50	$\frac{1}{3} \times 150 = 50$	100	$100 \times 60 = 6000$
X_3	90	$-\frac{1}{5} \times 150 = -30$	60	$60 \times 80 = 4800$
				10,800

ii) M/c hrs can be reduced by 225 without affecting shadow price

New profit $= 10200 - 150 \times 20$

$= 10200 - 3000$

$= 7200$

Current mix	↓ of 150 hrs	New mix	New profit
50	$150 \times -\frac{1}{5} = -30$	80	4800
90	$150 \times \frac{2}{5} = 60$	30	2400
			7,200

i)

$\frac{\Delta j}{X_3}$	–	-40	0	20	-50
		↑ least -ve		↑ least +ve	

∴ Profit on M_3 can be reduced by 40 Rs. or increased by 20 Rs. without effecting the optimal mix.

New profit $= 60 \times 50 + 90 \times 65$

$= 3000 + 5850$

$= 8250$

j) To start making M_2, profit on M_2 has to be greater by 40 Rs.

$\therefore \; -20 + 60 = 40$

k) opportunity of making 1 unit of the new product

$3 \times 4 = 12$

$3 \times 20 = \underline{60}$

$72 < 40$

Hence, new product should not be made

has to

product

made

Q3.) An Electronics company produces 3 models of satellite dishes A, B & C which have contributionˢ 400, 200 & 100. The no. of hours in the 2-stage productⁿ process per unit area are as follows:

	A	B	C	Process hrs. available
Process 1	2	3	2·5	1920
Process 2	3	2	2	2200

Sales for model A will not be more than 200 per period. Fixed cost are 40,000 per period.

a) Formulate the data into an LP problem.

b) Interpret the Simplex Table below:

C_j	400	200	100	0	0	0		
Basis	X_1	X_2	X_3	S_1	S_2	S_3	Solutⁿ	Sol/S_1
X_2	0	1	·83	·33	0	-·67	506·7	1535·45
~~S~~ S_2	0	0	·33	-·67	1	-1·67	586·7	-275·67 least -ve
X_1	1	0	0	0	0	1	200	—
Z_j	400	200	166	66	0	266	Z = 181340 - 40000 = 141340	
$(C_j - Z_j)$ Δ_j	0	0	-66	-66	0	-266		

$\therefore X_2 = 506.7 \quad S_1 = 0$

$X_1 = 200 \quad S_2 = 586.7$

$X_3 = 0 \quad S_3 = 0$

c) Is the solution optimal?

d) Is there more than one optimal solutions?

e) Is the solution feasible?

f) Is the solution degenerate?

g) Investigate the effect on the solutⁿ for each of the foll:

i) An increase of 20 hrs. per period in Process 1

ii) An increase of 10 units per period in the o/p of A

iii) Receiving an order which must be met for 10 units of C.

Ans.) (a) Let X_1 = No. of units of A to make

X_2 = No of units of B to make

& X_3 = No. of units of C to make

∴ Maximise $Z = 400X_1 + 200X_2 + 100X_3 - 40,000$

subject to

$$2X_1 + 3X_2 + 2.5X_3 \leq 1920$$

$$3X_1 + 2X_2 + 2X_3 \leq 2000$$

$$X_1 \leq 200$$

$$X_1, X_2, X_3 \geq 0$$

(c) Solution is optimal because all Δ_j values are 0 or -ve

(d) The solution is unique OR there is only one optimal solution as no non-basic valiable has zero Δj value.

(e) The solution is feasible as there is no artificial value in the optimal solution.

(f) The solution is not degenerate because basic variables are not zero in the solution.

(g) i) Least -ve tells how much the resource could be increased without effecting shadow price (i.e. do RHS ranging)

$$20 \times 166 = 1320$$

∴ New Profit = 1,41,340

$$\begin{array}{r} 1,41,340 \\ +\ 1320 \\ \hline 1,42,660 \end{array}$$

$0X_3 - 40,000$

are 0

one optimal

as zero

artificial

sic

could be

∴ do RHS ranging)

current mix	↑ of 20 hrs in Process 1	New mix	New profit	Sol/S_3
$X_2 = 506.7$	$20 \times .33 = 6.6$	513.3	102660	~~756.27~~
$S_2 = 586.7$	$20 \times -.67 = -13.4$	573.3	0	~~350.89~~
$X_1 = 200$	$20 \times 0 = 0$	200	80000	~~200~~
			1,42,660	~~least -ve~~

ii) Since 10 units fall within the 350.89 range, the profit of 266 would be valid

Sol/S_3
756.27
350.89 ← least -ve
200

∴ (increase) ↑ in profit = $10 \times 266 = 2660$
141340
1,44,000

current mix	↑ of 10 units in req. of X_1	New mix	New Profit
$X_2 = 506.7$	$10 \times -0.67 = -6.7$	500	1,00,000
$S_2 = 586.7$	$10 \times -1.67 = -16.7$	570	0
$X_1 = 200$	$10 \times 1 = 10$	210	24,000
			1,84,000
			-40,000
			1,44,000

iii.) ↓ in profit = $66 \times 10 = 660$

New profit = 1,41,340 − 660 = 1,40,680

current mix	10 units of X_3	New mix	New profit
$X_2 = 506.7$	$.83 \times 10 = 8.3$	506.7 − 8.3 = 498.4	99,680
$S_2 = 586.7$	$.33 \times 10 = 3.3$	586.7 − 3.3 = 523.4	0
$X_1 = 200$	$0 \times 10 = 0$	200	80,000
$X_3 = 0$		10	1,000
			1,80,680
			− 40,000
			1,40,680

Q4) A company makes 2 products:

Product	Machining hrs	Fabrication hrs	Assembly hrs	Profit/unit
A	1	5	3	80
B	2	4	1	100
Available	720	1800	900	

(a) Formulate the problem as an LP.

(b) The foll. Simplex table was obtained

	C_j	80	100	0	0	0		
C_j	Basis	X_1	X_2	S_1	S_2	S_3	b_i	b_i/s_1
100	X_2	0	1	5/6	−1/6	0	300	360
80	X_1	1	0	−2/3	−2/3	0	120	−180 → least −ve
0	S_3	0	0	7/6	7/6	1	240	205.7
	Z_j	80	100	30	10	0	$Z = 39,600$	
	$(C_j - Z_j)\ \Delta_j$	0	0	−30	−10	0		

(c) Verify the solutⁿ for optimality.

(d) Identify the values of all the variables & the obj. functⁿ for this solutⁿ. Comment on the utilization of resources.

(e) What are the values of the dual variables?

(f) Suppose the cost of overtime in each of the depts. is 15 Rs./hr. Would it be advisable to work any of the depts. on overtime? What would be the max amt. of overtime authorized if any? How would your answer change if the overtime cost is Rs. 8/hrs instead?

(g) Suppose a price change is under consideratⁿ for "A" raising the profit for this product to 100 Rs. Would this change the optimal Production Plan?

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80
100

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b_i	b_i/S_1
300	360 →least -ve
120	-180
240	205.7

Z = 39,600

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resource → R.H.S.

What is the max amount of change in profit for product that would not cause a change in the optimal Product^n Plan?

(h) How far can the unit profit on B vary without effecting the optimal Product^n Plan?

(i) The company is planning to introduce a new product C with the foll:

Machining	Fabricat^n	Assembling
2 hrs	3 hrs	2 hrs

What profit would be necessary before the company considers product^n of C?

Ans. (a) Let X_1 = No. of units of A to produce

X_2 = No. of units of B to produce

∴ Maximise $Z = 80X_1 + 100X_2$

subject to

$$X_1 + 2X_2 \leq 720$$
$$5X_1 + 4X_2 \leq 1800$$
$$3X_1 + X_2 \leq 900$$

(c) Solution is optimal because all Δj values are either 0 or -ve.

(d) $X_2 = 300 \quad S_1 = 0$
$X_1 = 120 \quad S_2 = 0$
$S_3 = 240$

Machining & Fabricat^n hrs are fully utilised because S_1 & $S_2 = 0$.

Assembly hrs are unutilised to the extent of 240 units as $S_3 = 240$

(e) The values of the dual variables are:

$y_1 = 30$

$y_2 = 10$

& $y_3 = 0$

(f) Overtime in Assembly deptt is not required as 240 hours are already unutilised.

For Fabricatn deptt: Every Fabrication hr. overtime adds Rs. 10 to the profit. So, increasing an hour (overtime) of Fabricatn @ 15 Rs. would not be viable.

For Machinery deptt: Every Machinery hr. overtime adds Rs. 30 to the profit. So, increasing an hour (overtime) of Machinery @ 15 Rs. would be viable, as it is profitable. Hence, machinery deptt. should be worked overtime.

However, if overtime cost is Rs. 8, hrs can be increased in both the departments.

~~(g)~~ Now, b_i / S_2

$-1200 \rightarrow$ least +ve

360

$-288 \rightarrow$ least −ve

So, a maximum of 288 hrs. can be increased in the Fabricatn deptt. ~~without changing the Optimal Productn Plan~~

(g) $\left.\frac{\Delta_j}{X_1}\right| \quad 0 \quad - \quad 45 \quad -30 \quad -$

$\uparrow$ least +ve

Least -ve tells how much the profit could be increased as 240 hours

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∴ $[80 - 30,\ 80 + 45]$ or $[50, 125]$ is the range in which the profit of the mix would not change.

∴ New profit $= 300 \times 100 + 120 \times 100$
$= 42,000$

(h) $$\left.\frac{\Delta_j}{X_2}\right| \quad - \quad 0 \quad \underset{\text{least -ve}}{\underset{\uparrow}{-36}} \quad \underset{\text{least +ve}}{\underset{\uparrow}{60}} \quad -$$

∴ Profit on B can increase by $[100 - 36,\ 100 + 60]$ without effecting optimal mix.

(i) Opportunity cost of making C:

M/c hrs $= 2 \times 30 = 60$
Fab. hrs $= 3 \times 10 = 30$
Assem. hrs $= 2 \times 0 = \underline{0}$
$\underline{90}$

∴ Profit on C should be at least 90 to consider for Product^n

TRANSPORTATION

Transportation is a special case of LP and is a minimisatⁿ problem.

Q1) A firm has 3 manufacturing plants at A, B & C with daily output of 500, 300 & 200 resp. It has warehouses at P, Q, R & S with daily requirements of 180, 150, 350 & 320 resp. Per unit shipping costs are given below :

	Destinations				
Sources	P	Q	R	S	Supply
A	12	10	12	13	500
B	7	11	8	14	300
C	6	16	11	7	200
Demand	180	150	350	320	

How should it route its output (O/P) to minimise overall transportation costs?

Ans.) First verify if the solution is balanced ie. Σ supply = Σ Demand.

(Here both are 1000, hence O.K.)

Let X_{ij} = quantity to be shipped from source to destination

$$\therefore \text{Minimise } Z = 12X_{11} + 10X_{12} + 12X_{13} + 13X_{14} + 7X_{21} + 11X_{22} + 8X_{23} + 14X_{24} + 6X_{31} + 16X_{32} + 11X_{33} + 7X_{34}$$

subject to :

$$X_{11} + X_{12} + X_{13} + X_{14} = 500$$

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500
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(O/P) to

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$x_{13} + 13x_{14}$
$x_{23} + 14x_{24}$
$x_{33} + 7x_{34}$

500

$$X_{21} + X_{22} + X_{23} + X_{24} = 300$$
$$X_{31} + X_{32} + X_{33} + X_{34} = 200$$
$$X_{11} + X_{21} + X_{31} = 180$$
$$X_{12} + X_{22} + X_{32} = 150$$
$$X_{13} + X_{23} + X_{33} = 350$$
$$X_{14} + X_{24} + X_{34} = 320$$
$$\& \; X_{ij} \geq 0$$

a) North West (NW) Corner Rule :

	P	Q	R	S	Supply
A	$\boxed{180}$ 12	$\boxed{150}$ 10	$\boxed{170}$ 12	13	~~500~~ ~~320~~ 170
B	7	11	$\boxed{180}$ 8	$\boxed{120}$ 14	~~300~~ 120
C	6	16	11	$\boxed{200}$ 7	200
D'd	180	150	~~350~~ 180	~~320~~ 200	

$$\therefore \text{Cost} = 12 \times 180 + 10 \times 150 + 12 \times 170 + 8 \times 120 + 14 \times 120 + 7 \times 200$$
$$= 10,220$$

b) Least Cost Method :

	P	Q	R	S	Supply
A	12	$\boxed{150}$ 10	12 $\boxed{50}$	13 $\boxed{300}$	500
B	7	11	8 $\boxed{300}$	14	~~300~~
C	$\boxed{120}$ 6	16	11	7 $\boxed{20}$	~~200~~ 20
	120	150	~~350~~ 50	~~320~~ 300	

$$\therefore \text{Cost} = 6 \times 120 + 10 \times 150 + 12 \times 50 + 13 \times 300 + 8 \times 300 + 7 \times 20$$
$$= 9620$$

So, this method is cheaper than NW, but you may be forced to allocate to a very high cost cell.

c.) Vogel's Approximation Method (VAM):

	P	Q	R	S	Supply	I	II	III
A	12	10 $\boxed{150}$	12 $\boxed{230}$	13 $\boxed{120}$	500	2	2	2
B	7 $\boxed{180}$	11	8 $\boxed{120}$	14	~~300~~ 120	1	1	3
C	6	16	11	7 $\boxed{200}$	200	1	–	–
D'd	180	150	~~350~~ 230	~~320~~ 120	1000			
I	1	1	3	⑥				
II	⑤	1	4	1				
III	–	1	④	1				

$\therefore$ Cost $= 10 \times 150 + 12 \times 230 + 13 \times 120 + 7 \times 180 +$
$8 \times 120 + 7 \times 200$
$= 9440$

$\therefore$ Reaching optimal solutⁿ is cheaper in VAM.

, but you
igh cost

II	III
2	2
1	3
-	-

7 × 180 +

NAM.

Q2. A cement company has 3 factories which manufacture cement which is then transported to 4 distribut? centres. The cost of distribut? per ton & the demand in productⁿ are given below:

Destinations

Factories	W	X	Y	Z	Supply	I	II	III
A	10	8	7000 5	4	7000	1	-	-
B	7	6000 9	1000 15	1000 8	8000	1	1	1
C	6000 6	10	14	4000 8	4000 ~~10000~~	2	②	②
D'd	6000	6000	1000 ~~8000~~	1000 ~~5000~~	25000			

Suggest the optimal transportatⁿ schedule.

Ans:)

I	1	1	⑨	4
II	1	1	1	0
III	-	1	1	0

Now, we need to check the optimality of this sol.

	W	X	Y	Z	Supply ('000)
A	10	8	7 5	4	7
B	7	6 9	1 15	1 8	8
C	6 6	10	14	4 8	10
D'd ('000)					

cost = 5×7 + 9×6 + 15×1 + 8×1 + 6×6 + 8×4
= 1,80,000

Filled cells are like basic-variables. For all un-filled cells (non-basic variables), we need to find out how the cost would be effected if we make allocatⁿ to the un-filled cells. But, for this values in the filled cells will have to be adjusted in a loop, till we reach the starting pt.

So, net impact on the cost if a unit is allocated to the unfilled cell AW.

$$\begin{array}{r} +10 \\ -5 \\ +15 \\ -8 \\ +8 \\ -6 \\ \hline +14 \end{array}$$

Note: If the sol. consists of $\overset{\text{no. of rows}}{m} + \overset{\text{no. of columns}}{n} - 1$ filled cells, there would be one & only one loop possible from every un-filled cell.

Similarly, starting from unfilled cell AX

$$\begin{array}{r} +8 \\ -5 \\ +15 \\ -9 \\ \hline +9 \end{array}$$

This is known as "Stepping Stone" method & is very cumbersome.

A shorter method (& more efficient) is MODI (Modified Distributn) Method h the

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Note: Only one u_i or v_j value can be assumed to be zero

	W	X	Y	Z	Supply	u_i
A	(-14) 10	(-9) 8	$\boxed{7}$ 5	(-6) 4	7	0
B	(-1) 7	$\boxed{6}$ 9	− ~~$\boxed{1}$~~ 15	+ ~~$\boxed{1}$~~ 8 $\boxed{2}$	8	10
C	$\boxed{6}$ 6	(-1) 10	(1) + 14 $\boxed{1}$	$\boxed{4}$ − 8 $\boxed{3}$	10	10
D'd	6	6	8	5	25	
v_j	-4	-1	5	-2		

For Filled cells : $u_i + v_j = c_{ij}$

For un-filled cells : $c_{ij} - (u_i + v_j)$

So for cell AW : $c_{ij} - (u_i + v_j) = 10 - (0 + -4)$
$= 14$

(OR)

$(u_i + v_j) - c_{ij}$ (Acc. to convention used in book)

Note: +ve value tells there would be a saving if allocatn is made to the cell.
-ve value tells the cost would increase

Hint: Among the subtractn columns, choose the smallest.

So, the new table becomes :

	W	X	Y	Z	Supply	u_i
A	10	8 (-8)	5 $\boxed{7}$	4 (-5)	7	0
B	(-1) 7	9 $\boxed{6}$	15 (-1)	8 $\boxed{2}$	8	9
C	$\boxed{6}$ 6	(-1) 10	$\boxed{1}$ 14	$\boxed{3}$ 8	10	9
D'd	6	6	8	5	25	
v_i	-3	0	5	-1		

So, the new cost becomes $35 + 16 + 54 + 36 + 14 + 24$
$= 179$

So, introductn of 1 unit reduces the cost by 1 unit (from 180 to 179) So, it acts like Δj.

Now, we will again check test if this solut^n is optimal or not.

Again choose u_i & v_j & solve.

Since this time all ~~u_i~~ Δ_j values are -ve, we have reached an optimal solution.

solutⁿ

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n.

Q3. Consider the following transportatⁿ problem :

Supply

D'd	1	2	3	Supply	I	II	III	IV
A	8	4 $\boxed{10}$	10	~~10~~	4	④	–	–
B	9 $\boxed{60}$	7 $\boxed{10}$	9 $\boxed{10}$	~~80~~ 70	2	2	2	②
C	6 $\boxed{15}$	5	8	~~15~~	1	1	1	–
Dummy row	0	0	0 $\boxed{40}$	~~40~~	0	–	–	–
D'd	~~75~~ 60	~~20~~ 10	~~50~~ 10	145				

Since initially $\sum$ Demand > $\sum$ Supply, we had to introduce a dummy row (with supply 40), so that the $\sum$ Demand = $\sum$ Supply

I	6	4	⑧
II	2	1	1
III	③	2	1

Now, check for optimality (ie u_i & v_j)

	1	2	3	Supply	u_i
A	8 (-2)	4 $\boxed{10}$	10 (-4)	10	0
B	$\boxed{20}$ ~~$\boxed{60}$~~ 9	7 $\boxed{10}$	+ $\boxed{50}$ ~~$\boxed{10}$~~ 9	80	3
C	6 $\boxed{15}$	5 (-1)	8 (-2)	15	0
Dummy	$\boxed{40}$ + 0 ⓪	0 (-2)	0 ~~$\boxed{40}$~~	40	-6
D'd	75	20	50	145	
v_j	6	4	6		

← start with assuming this u_i as 0

∴ cost = 540 + 70 + 40 + 90 = 830

Now calculate Δj value for non-filled cells.

∵ All values are either 0 or -ve, sol is optimal

However since Dummy 1 cell is ⓪ this means multiple optima is present (ie Δj value of an un-filled cell is 0). So, to find this another optima, construct a loop from this cell with +ve & -ve corners & calculate new cost which should be equal to the cost before

ie. cost = $180 + 70 + 40 +$

$= 830$

Note: In problems where the route is prohibited or blocked, we assign a very high cost to the cell in the route.

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Q4 consider the foll. transportation problem

	A	B	C	Supply	I	II	III	IV	V
1	~~10~~ M	12	7 $\boxed{180}$	180	~~3~~5	⑤	-	-	-
2	14 $\boxed{60}$	11	6 $\boxed{140}$	~~100~~ 60	5	5	⑤	3	3
3	9	5 $\boxed{160}$	~~13~~ M	160	4	4	4	④	-
4	11 $\boxed{80}$	7 $\boxed{40}$	9	~~120~~ 80	2	2	2	4	④
Dummy row	0 $\boxed{100}$	0	0	100	0	-	-	-	-
	~~240~~ 140	~~200~~ 40	~~220~~ 40	560					
I	⑨	5	6						
II	2	2	1						
III	2	2	3						
IV	2	2	-						
V	3	4	-						

Condil Nothing can be sent from WH 1 to market A & 3 to C.

So, cost = 4300

Now, check for optimality

	A	B	C	Supply	u_i
1	M (−M)	12 (−12)	7 $\boxed{180}$		0
2	14 $\boxed{60}$	11 (−1)	6 $\boxed{40}$		−1
3	+ 9 ⓪ 80	− 5 $\boxed{\cancel{160}}$ 80	M (−M)		−~~4~~6
4	− 11 $\boxed{\cancel{80}}$	7 + $\boxed{\cancel{40}}$ 120	9 (−6)		−~~15~~ 4
Dummy	0 $\boxed{100}$	0 (−4)	0 (−8)		−15
D'd	240	200	220	660	
v_j	15	11	7		

So, there is multiple optima.

Q5. Consider the foll. transportatn problem :

	1	2	3	4	Supply	I	II	III	ui
A	7	$\boxed{20}$ 3	8	$\boxed{40}$ 6	~~60~~ 20	3	3	④	0
B	$\boxed{20}$ 4	$\boxed{30}$ 2	$\boxed{50}$ 5	10	100	2	2	2	-1
C	2	6	5	$\boxed{40}$ 1	40	1	–		-5
D'd	20	~~50~~ 30	50	~~80~~ 40	200				
I	2	1	0	⑤					
II	3	1	3	④					
III	3	1	3	—					
v_j		3		6					

∴ cost = 730