CHAPTER 2

1. Let x_1 and x_2 be the output of P and V respectively.
The LPP is:

Maximise	$Z = 40x_1 + 30x_2$	Profit
Subject to		
	$400x_1 + 350x_2 \leq 250{,}000$	Steel
	$85x_1 + 50x_2 \leq 26{,}100$	Lathe
	$55x_1 + 30x_2 \leq 43{,}500$	Grinder
	$20x_2 \leq 17{,}400$	Polishing
	$x_1 \leq x_2$	Sales
	$x_1, x_2 \geq 0$	

2. Let the daily output of products A, B and C be x_1, x_2 and x_3 respectively. We have

Maximise	$Z = 500x_1 + 600x_2 + 1200x_3$	Profit
Subject to		
	$2x_1 + 4x_2 + 6x_3 \leq 160$	Platinum
	$3x_1 + 2x_2 + 4x_3 \leq 120$	Gold
	$x_1, x_2, x_3 \geq 0$	

3. Let x_1, x_2, x_3, and x_4 represent the number of three-compartment bags, shoulder—strap bags, tote bags, and pocket purses, respectively, to be produced per day. With the given data, the LPP is given below:

Maximise	$Z = 16x_1 + 25x_2 + 12x_3 + 12x_4$	
Subject to		
	$45x_1 + 60x_2 + 45x_3 + 30x_4 \leq 1920$	Assembly line
	$x_4 \leq 30$	Pins 1
	$x_1 + x_2 + x_3 \leq 70$	Pins 2
	$x_3 + x_4 \leq 60$	Raw material
	$x_4 \geq 6$	Minimum
	$x_2 \geq 10$	demand
	$x_1, x_2, x_3, x_4 \geq 0$	

4 Profit per unit of C_1 = Rs 30 – (5 + 5) = Rs 20, and
Profit per unit of C_2 = Rs 70 – (25 + 15) = Rs 30
Let x_1 and x_2 be the number of units of C_1 and C_2 respectively, produced and sold. The LPP is:

Maximise	$Z = 20x_1 + 30x_2$	Profit
Subject to		
	$10x_1 + 40x_2 \leq 4{,}000$	Cash
	$3x_1 + 2x_2 \leq 2{,}000$	Machine time
	$2x_1 + 3x_2 \leq 1{,}000$	Assembly time
	$x_1, x_2 \geq 0$	

5. Let x_{ij} be the amount of money invested at the beginning of month i for a period of j months. For every month, we have: money invested plus bills paid = money available. Accordingly, the LPP is:

Maximise	$Z = 1.72x_{14} + 1.45x_{23} + 1.02x_{32} + 1.005x_{41}$	
Subject to		
	$x_{11} + x_{12} + x_{13} + x_{14} + 36{,}000 = 30{,}000 + 28{,}000$	Month 1
	$x_{21} + x_{22} + x_{33} + 31{,}000 = 1.005x_{11} + 52{,}000$	Month 2
	$x_{31} + x_{32} + 40{,}000 = 1.02x_{12} + 1.005x_{21} + 3{,}400$	Month 3
	$x_{41} + 20{,}000 = 1.45x_{13} + 1.02x_{22} + 1.005x_{31} + 22{,}000$	Month 4

all variables ≥ 0

6. Let x_1, x_2 be the number of issues of Daily Life, Agriculture Today and Surf's Up, respectively, published every week.

Maximise $Z = 22.50x_1 + 40x_2 + 15x_3$

Subject to

$$0.01x_1 + 0.03x_2 + 0.02x_3 \le 120$$
$$0.20x_1 + 0.50x_2 + 0.30x_3 \le 3{,}000$$
$$x_1 + x_2 + x_3 \ge 5{,}000$$
$$x_1 \le 3{,}000$$
$$x_2 \le 2{,}000$$
$$x_3 \le 6{,}000$$
$$x_1, x_2, x_3 \ge 0$$

7. Let x_1, x_2, x_3, and x_4 be the amount invested (in lakh) in government bonds, blue chip stocks, speculative stocks, and short-term deposits respectively. We may state the LPP as follows:

Maximise $Z = 0.14\,x_1 + 0.19\,x_2 + 0.23\,x_3 + 0.12\,x_4$ Return

Subject to $x_1 + x_2 + x_3 + x_4 \le 20$ Budget

$$\frac{12x_1 + 24x_2 + 48x_3 + 64x_4}{x_1 + x_2 + x_3 + x_4} \le 42 \quad \text{Average risk}$$

$$x_4 \ge 2 \quad \text{Short-term deposits}$$
$$x_3 \le 0.20\,(x_1 + x_2 + x_3 + x_4) \quad \text{Speculative stocks}$$
$$x_1, x_2, x_3, x_4 \ge 0$$

8. Let x_1, x_2, and x_3 respectively represent the number of units of the parts A, B, and C produced per hour. With the given information, the hourly profit would be:

$$\text{Profit} = (8-5)x_1 - \left[\frac{20}{25} + \frac{30}{25} + \frac{30}{40}\right]x_1 + (10-6)x_2 - \left[\frac{20}{40} + \frac{30}{20} + \frac{30}{30}\right]x_2$$
$$+ (14-10)x_3 - \left[\frac{20}{25} + \frac{30}{20} + \frac{30}{40}\right]x_3$$
$$= 0.25x_1 + x_2 + 0.95x_3$$

Thus, the LPP is:

Maximise $Z = 0.25x_1 + x_2 + 0.95x_3$

Subject to

$$\frac{x_1}{25} + \frac{x_2}{40} + \frac{x_3}{25} \le 1$$

$$\frac{x_1}{25} + \frac{x_2}{20} + \frac{x_3}{20} \le 1$$

$$\frac{x_1}{40} + \frac{x_2}{30} + \frac{x_3}{40} \le 1$$

$$x_1, x_2, x_3 \ge 0$$

9. Let x_1, x_2, ... x_{12} be the number of nurses joining at 12 midnight, 2 a.m., 4 a.m., ... and 10 p.m. respectively.

Minimise $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$

Subject to

$$x_1 + x_2 + x_3 + x_4 \ge 50$$
$$x_2 + x_3 + x_4 + x_5 \ge 60$$
$$x_3 + x_4 + x_5 + x_6 \ge 80$$
$$x_4 + x_5 + x_6 + x_7 \ge 80$$
$$x_5 + x_6 + x_7 + x_8 \ge 70$$

$$x_6 + x_7 + x_8 + x_9 \geq 70$$
$$x_7 + x_8 + x_9 + x_{10} \geq 60$$
$$x_8 + x_9 + x_{10} + x_{11} \geq 50$$
$$x_9 + x_{10} + x_{11} + x_{12} \geq 50$$
$$x_{10} + x_{11} + x_{12} + x_1 \geq 30$$
$$x_{11} + x_{12} + x_1 + x_2 \geq 20$$
$$x_{12} + x_1 + x_2 + x_3 \geq 40$$
$$x_i \geq 0,\ i = 1 \text{ to } 12$$

10. Let $x_1, x_2, \ldots, x_7$ be the number of doctors starting on day 1, 2, …, 7, respectively, beginning with Sunday. The LPP may be stated as given below:

Minimise $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

Subject to

$$x_1 + x_4 + x_5 + x_6 + x_7 \geq 35$$
$$x_1 + x_2 + x_5 + x_6 + x_7 \geq 55$$
$$x_1 + x_2 + x_3 + x_6 + x_7 \geq 60$$
$$x_1 + x_2 + x_3 + x_4 + x_7 \geq 50$$
$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 60$$
$$x_2 + x_3 + x_4 + x_5 + x_6 \geq 50$$
$$x_3 + x_4 + x_5 + x_6 + x_7 \geq 45$$
$$0 \leq x_i \leq 40,\ i = 1, 2, \ldots, 7$$

11. Let production lines 1, 2, and 3 are run for x_1 x_2 and x_3 days respectively.

Minimise $Z = 600x_1 + 500x_2 + 400x_3$

Subject to

$$\left.\begin{aligned} 150x_1 + 200x_2 + 160x_3 &\geq 2{,}000 \\ 100x_1 + 100x_2 + 80x_3 &\geq 3{,}000 \\ 500x_1 + 760x_2 + 890x_3 &\geq 3{,}000 \\ 400x_1 + 400x_2 + 600x_3 &\geq 6{,}000 \end{aligned}\right\} \text{Output}$$

$$\left.\begin{aligned} x_1 &\leq 20 \\ x_2 &\leq 20 \\ x_3 &\leq 18 \end{aligned}\right\} \text{Capacity}$$

$$x_1, x_2, x_3 \geq 0$$

12. Let x_{ij} be the number of units produced in plant i and sent to customer j; $i = 1, 2$ and $j = 1, 2, 3, 4$.

Minimise $Z = 25x_{11} + 30x_{12} + 40x_{13} + 45x_{14} + 51x_{21} + 41x_{22} + 36x_{23} + 31x_{24}$

Subject to

$$0.10x_{11} + 0.10x_{12} + 0.10x_{13} + 0.10x_{14} \leq 120$$
$$0.20x_{11} + 0.20x_{12} + 0.20x_{13} + 0.20x_{14} \leq 260$$
$$0.11x_{21} + 0.11x_{22} + 0.11x_{23} + 0.11x_{24} \leq 140$$
$$0.22x_{21} + 0.22x_{22} + 0.22x_{23} + 0.22x_{24} \leq 250$$
$$x_{11} + x_{21} = 500$$
$$x_{12} + x_{22} = 300$$
$$x_{13} + x_{23} = 1{,}000$$
$$x_{14} + x_{24} = 200$$
$$x_{ij} \geq 0$$

13. Let x_1, x_2 and x_3, respectively, be the number of vehicles A, B and C purchased.

Maximise $Z = 6{,}300x_1 + 10{,}800x_2 + 11{,}340x_3$

Subject to

$$80{,}000x_1 + 130{,}000x_2 + 150{,}000x_3 \leq 40{,}00{,}000$$
$$x_1 + x_2 + x_3 \leq 30$$
$$3x_1 + 6x_2 + 6x_3 \leq 150$$

$x_1, x_2, x_3 \leq 0$

14. Let x_{ij} be the quantity of ith crude mixed in jth grade petrol.
Maximise $Z = -1.5x_{11} + 3.5x_{21} + 2.9x_{31} - 3.0x_{21} + 2.0x_{22} + 1.4x_{32} - 4.0x_{13} + x_{23} + 0.4x_{33}$
Subject to

$$x_{11} + x_{12} + x_{13} \leq 500{,}000$$
$$x_{21} + x_{22} + x_{23} \leq 500{,}000$$
$$x_{31} + x_{32} + x_{33} \leq 360{,}000$$
$$x_{11} \geq 0.50\ (x_{11} + x_{21} + x_{31})$$
$$x_{21} \leq 0.25\ (x_{11} + x_{21} + x_{31})$$
$$x_{12} \geq 0.25\ (x_{12} + x_{22} + x_{32})$$
$$x_{22} \leq 0.50\ (x_{12} + x_{22} + x_{32})$$
$$x_{ij} \geq 0\ \ i = 1, 2, 3;\ j = 1, 2, 3$$

15. Let x_{ijk} be the quantity produced in quarter $i(i = 1, 2, 3, 4)$, in time $j(j = 1$ as regular time and $j = 2$ as overtime) and supplied in quarter $k(k = 1, 2, 3, 4)$. The total cost that is sought to be minimised comprises the production and storage costs. The problem may be stated as follows:
Minimise
$$Z = 16x_{111} + 20x_{121} + 18x_{112} + 22x_{122} + 20x_{113} + 24x_{123} + 22x_{114} + 26x_{124} + 16x_{212} + 20x_{222} + 18x_{213} + 22x_{223} + 20x_{214} + 24x_{224} + 16x_{313} + 20x_{323} + 18x_{314} + 22x_{324} + 16x_{414} + 18x_{424}$$
Subject to

$x_{111} + x_{112} + x_{113} + x_{114}$	≤ 80	Regular
$x_{212} + x_{213} + x_{214}$	≤ 90	time
$x_{313} + x_{314}$	≤ 95	constraints
x_{414}	≤ 70	
$x_{121} + x_{122} + x_{123} + x_{124}$	≤ 10	
$x_{222} + x_{223} + x_{224}$	≤ 10	Overtime
$x_{323} + x_{324}$	≤ 20	constraints
x_{424}	≤ 10	
$x_{111} + x_{121}$	$= 65$	
$x_{112} + x_{122} + x_{212} + x_{222}$	$= 80$	Demand
$x_{113} + x_{123} + x_{213} + x_{223} + x_{313} + x_{323}$	$= 135$	constraints
$x_{114} + x_{124} + x_{214} + x_{224} + x_{314} + x_{324} + x_{414} + x_{424}$	$= 75$	

$x_{ijk} \geq 0$, for $i = 1, 2, 3, 4$
$j = 1, 2$
$k = 1, 2, 3, 4$

16. The problem here is to maximise total effective exposures. The coefficients of the objective function are obtained by the product of audience size multiplied by the 'effectiveness coefficient' of each magazine which, in turn, is calculated on the basis of audience characteristics, their relative importance, and efficiency indices of the colour, and black and white advertisements. To illustrate, for magazine M_1,
Effectiveness coefficient $= [0.70(0.3) + 0.50(0.5) + 0.80(0.2)][0.3x_{11} + 0.2x_{12}]$
$= 0.186x_{11} + 0.124x_{12}$
where x_{11}: No. of colour advertisements in magazine M_1
x_{12}: No. of black and white advertisements in magazine M_1
Similarly, for magazine M_2, if x_{21}, and x_{22} represent the number of colour, and black and white advertisements in M_2,
We have
Effectiveness coefficient $= [0.60(0.3) + 0.40(0.5) + 0.70(0.2)][0.3x_{21} + 0.2x_{22}]$
$= 0.156x_{21} + 0.104x_{22}$
For magazine M_3,
Effectiveness coefficient $= [0.90(0.3) + 0.75(0.5) + 0.80(0.2)][0.3x_{31} + 0.2x_{32}]$

$= 0.2415x_{31} + 0.161x_{32}$

Now, objective function coefficients are:

For M_1: $(0.186x_{11} + 0.124x_{12})(400{,}000) = 74{,}400x_{11} + 49{,}600x_{12}$

For M_2: $(0.156x_{21} + 0.104x_{22})(300{,}000) = 46{,}800x_{21} + 31{,}200x_{22}$

For M_3: $(0.2415x_{31} + 0.161x_{32})(200{,}000) = 48{,}300x_{31} + 32{,}200x_{32}$

The LPP is:

Maximise $Z = 74{,}400x_{11} + 49{,}600x_{12} + 46{,}800x_{21} + 31{,}200x_{22} + 48{,}300x_{31} + 32{,}200x_{32}$

Total exposure

Subject to

$18{,}000x_{11} + 12{,}000x_{12} + 16{,}000x_{21} + 10{,}000x_{22} + 19{,}000x_{31} + 15{,}000x_{32} \le 500{,}000$

Budget

$$\left.\begin{array}{l} x_{11} + x_{12} \le 12 \\ x_{21} + x_{22} \le 24 \\ x_{31} + x_{32} \le 12 \end{array}\right\} \text{Maximum number of advertisements}$$

$$\left.\begin{array}{l} x_{11} + x_{12} \ge 5 \\ x_{21} + x_{22} \ge 4 \\ x_{31} + x_{32} \ge 5 \end{array}\right\} \text{Minimum number of advertisements}$$

$x_{ij} \ge 0;\ i = 1, 2, 3;\ j = 1, 2$

17. Let x_{pmsd} be the quantity of product p produced in month m, in shift s, and delivered in month d.

Minimise $Z = 400x_{1111} + 440x_{1112} + 480x_{1113} + 480x_{1121} + 520x_{1122} + 560x_{1123} + 400x_{1212} + 440x_{1213} + 480x_{1222} + 520x_{1223} + 400x_{1313} + 480x_{1323} + 500x_{2111} + 540x_{2112} + 580x_{2113} + 600x_{2121} + 640x_{2122} + 680x_{2123} + 500x_{2212} + 540x_{2213} + 600x_{2222} + 640x_{2223} + 500x_{2313} + 600x_{2323} + 500x_{3111} + 540x_{3112} + 580x_{3113} + 600x_{3121} + 640x_{3122} + 680x_{3123} + 500x_{3212} + 540x_{3213} + 600x_{3222} + 640x_{3223} + 500x_{3313} + 600x_{3323} + 700x_{4111} + 740x_{4112} + 780x_{4113} + 840x_{4121} + 880x_{4122} + 920x_{4123} + 700x_{4212} + 740x_{4213} + 840x_{4222} + 880x_{4223} + 700x_{4313} + 840x_{4323}$

Subject to

$4x_{1111} + 4x_{1112} + 4x_{1113} + 5x_{2111} + 5x_{2112} + 5x_{2113} + 5x_{3111} + 5x_{3112} + 5x_{3113} + 7x_{4111} + 7x_{4112} + 7x_{4113} \le 1{,}10{,}000$

$4x_{1121} + 4x_{1122} + 4x_{1123} + 5x_{2121} + 5x_{2122} + 5x_{2123} + 5x_{3121} + 5x_{3122} + 5x_{3123} + 7x_{4121} + 7x_{4122} + 7x_{4123} \le 1{,}00{,}000$

$4x_{1212} + 4x_{1213} + 5x_{2212} + 5x_{2213} + 5x_{3212} + 5x_{3213} + 7x_{4212} + 7x_{4213} \le 1{,}30{,}000$

$4x_{1222} + 4x_{1223} + 5x_{2222} + 5x_{2223} + 5x_{3222} + 5x_{3223} + 7x_{4222} + 7x_{4223} \le 1{,}20{,}000$

$4x_{1313} + 5x_{2313} + 5x_{3313} + 7x_{4313} \le 1{,}15{,}000$

$4x_{1323} + 5x_{2323} + 5x_{3323} + 7x_{4323} \le 1{,}16{,}000$

$x_{1111} + x_{1121} = 8{,}000$

$x_{2111} + x_{2121} = 19{,}000$

$x_{3111} + x_{3121} = 4{,}000$

$x_{4111} + x_{4121} = 7{,}000$

$x_{1112} + x_{1122} + x_{1212} + x_{1222} = 7{,}000$

$x_{2112} + x_{2122} + x_{2212} + x_{2222} = 19{,}000$

$x_{3112} + x_{3122} + x_{3212} + x_{3222} = 15{,}000$

$x_{4112} + x_{4122} + x_{4212} + x_{4222} = 7{,}000$

$x_{1113} + x_{1123} + x_{1213} + x_{1223} + x_{1313} + x_{1323} = 6{,}000$

$x_{2113} + x_{2123} + x_{2213} + x_{2223} + x_{2313} + x_{2323} = 18{,}000$

$x_{3113} + x_{3123} + x_{3213} + x_{3223} + x_{3313} + x_{3323} = 17{,}000$

$x_{4113} + x_{4123} + x_{4213} + x_{4223} + x_{4313} + x_{4323} = 7{,}000$

all variables ≥ 0

18. Minimise $Z = 350{,}000x_{111} + 353{,}000x_{112} + 356{,}000x_{113} + 390{,}000x_{121} + 393{,}000x_{122} + 396{,}000x_{123} +$

$430{,}000x_{212} + 433{,}000x_{213} + 470{,}000x_{222} + 473{,}000x_{223} + 400{,}000x_{313} + 450{,}000x_{323}$

Subject to

$$x_{111} + x_{121} = 2$$
$$x_{112} + x_{122} + x_{212} + x_{222} = 2$$
$$x_{113} + x_{123} + x_{213} + x_{223} + x_{313} + x_{323} = 2$$
$$x_{111} + x_{112} + x_{113} \leq 1$$
$$x_{121} + x_{122} + x_{123} \leq 2$$
$$x_{212} + x_{213} \leq 2$$
$$x_{222} + x_{223} \leq 2$$
$$x_{313} \leq 3$$
$$x_{323} \leq 2$$

all variables ≥ 0

19. Let x_1, x_2, and x_3 be the number of Manual, Electronic and Deluxe electronic typewriters respectively.

With selling prices and variable costs given, the profit contribution per unit for the three typewriters is Rs 1,600, Rs 3,000, and Rs 5,600 respectively.

The LPP may be stated as follows:

Maximise $Z = 1{,}600x_1 + 3{,}000x_2 + 5{,}600x_3$ Profit

Subject to

$15x_1 + 12x_2 + 14x_3 \leq 3{,}000$	Machine time
$4x_1 + 3x_2 + 5x_3 \leq 1{,}200$	Assembly time
$x_1 \geq 2$, $x_3 \geq 8$	Committed supply
$2{,}500x_1 + 4{,}500x_2 + 9{,}000x_3 \leq 136{,}800$	Cash
$x_2 \geq 0$	

Note: The cash requirement is $2{,}500x_1 + 4{,}500x_2 + 9{,}000x_3$, while the cash availability is Rs 136,800, worked out as below:

Cash availability = Cash balance + Receivables – Loan to repay to cooperative bank – Interest on loan from TNC bank and cooperative bank – Interest on long-term loans – Top management salary and other fixed overhead

= Rs 140,000 + Rs 50,000 – Rs 40,000 – Rs 1,200 – Rs 2,000 – Rs 10,000

= Rs 136,800

20. Here $A(2, 0) = 2$

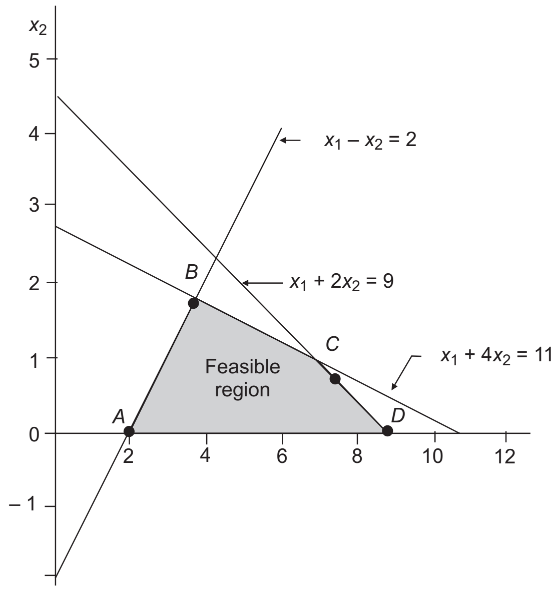
$B(3.8, 1.8) = 9.2$

$C(7, 1) = 10$

$D(9, 0) = 9$

Optimal solution is:

$x_1 = 7$, $x_2 = 1$ for $Z = 10$



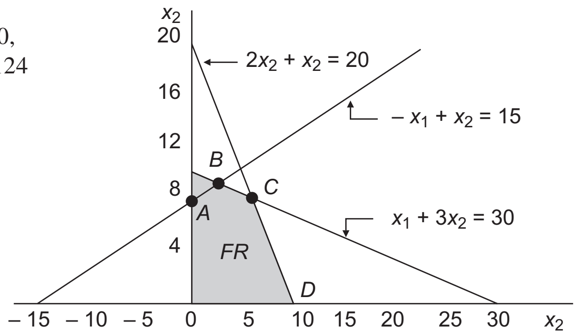
21. From the graph,

$$Z(0 : 0, 0) = 0,\ Z(A : 0, 7.5) = 60,$$
$$Z(B : 3, 9) = 102,\ Z(\text{C} : 6, 8) = 124$$

and $$Z(D : 10, 0) = 100.$$

Optimal solution:

$x_1 = 6$ and $x_2 = 8$, for $Z = 124$.

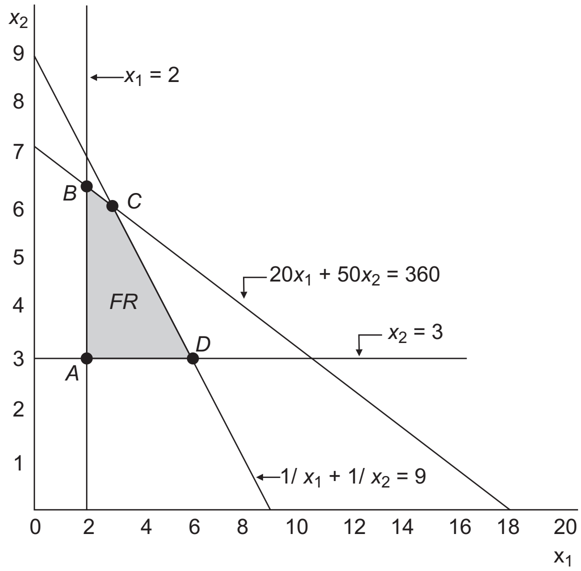


22. Here,

$$Z(A : 2, 3) = 520,$$
$$Z(B : 2, 6.4) = 928$$
$$Z(C : 3, 6) = 960$$
$$Z(D : 6, 3) = 840$$

Optimal solution is

$x_1 = 3$, and $x_2 = 6$, for $Z = 960$.



23. Let x_1 and x_2 be the number of packages of economy and special type, respectively.

LPP is:

Maximise $Z = 5x_1 + 8x_2$

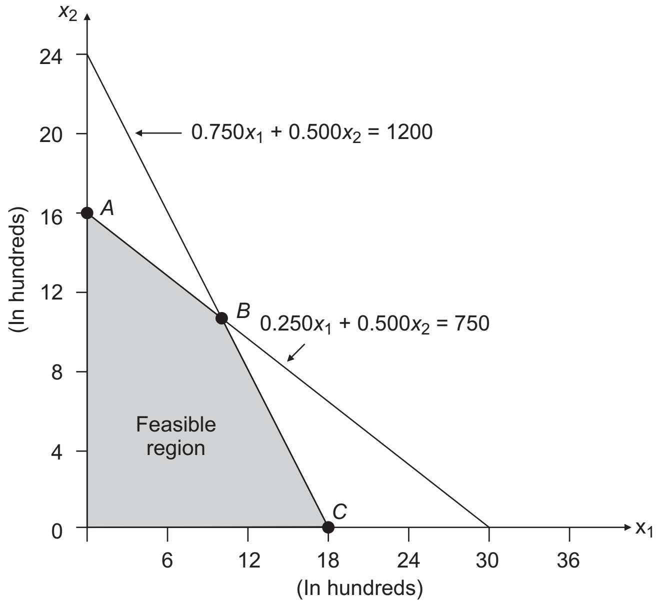
Subject to

$0.250x_1 + 0.500x_2 \le 750$ Grade I

$0.750x_1 + 0.500x_2 \le 1,200$ Grade II

$x_1, x_2 \ge 0$

From the graph, $Z(0 : 0, 0) = 0$, $Z(A : 0, 1500) = 12,000$, $Z(B : 900, 1050) = 12,900$ and $Z(C : 1600,0) = 8000$. Thus, $Z(B)$ gives optimal solution. If the profit margin is Rs 10 on special pack, we have $Z(0) = 0$, $Z(A) = 15,000$, $Z(B) = 15,000$, and $Z(C) = 8,000$. As such, the company can have either $x_1 = 0$ and $x_2 = 1500$, or $x_1 = 900$ and $x_2 = 1050$.



24. Let x_1 : daily production of pencil *A*, and
x_2 : daily production of pencil *B*

The LPP is:

Maximise $Z = 5x_1 + 3x_2$ Profit

Subject to

$2x_1 + x_2 \le 1000$ Raw material

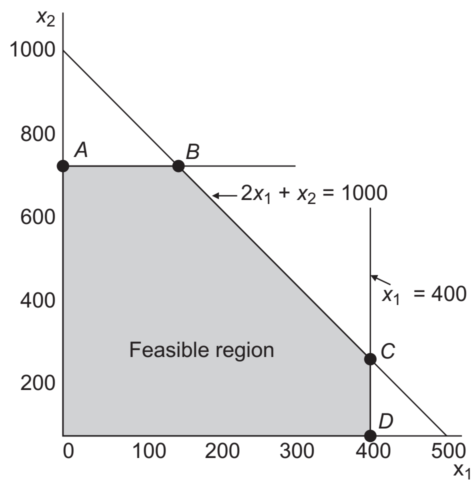
$x_1 \le 400$ Clips for *A*

$x_2 \le 700$ Clips for *B*

$x_1, x_2 \ge 0$

The constraints are shown plotted in the figure.

Point	x_1	x_2	*Z*	
0	0	0	0	
A	0	700	2100	
B	150	700	<u>2850</u>	Optimal product
C	400	200	2600	mix
D	400	0	2000	



25. Let x_1 and x_2 be the number of inspectors employed daily of grade 1 and grade 2 respectively.

Total cost = Inspection charges + Cost of errors

Inspection charges = $5 \times 8 \times x_1 + 4 \times 8 \times x_2 = 40x_1 + 32x_2$

Cost of errors = $3 \times 0.03 \times 40 \times 8 \times x_1 + 3 \times 0.05 \times 30 \times 8 \times x_2$

$= 28.80x_1 + 36x_2$

The LPP is:

Minimise $Z = 68.80x_1 + 68.00x_2$

Subject to

$$320x_1 + 240x_2 \ge 2000$$
$$x_1 \le 9$$
$$x_2 \le 11$$
$$x_1, x_2 \ge 0$$

The constraints are plotted on the graph and feasible region is marked *ABCDE*. We have

Cost

$A(0, \left(0, 8\frac{1}{3}\right) : 566\frac{2}{3}$

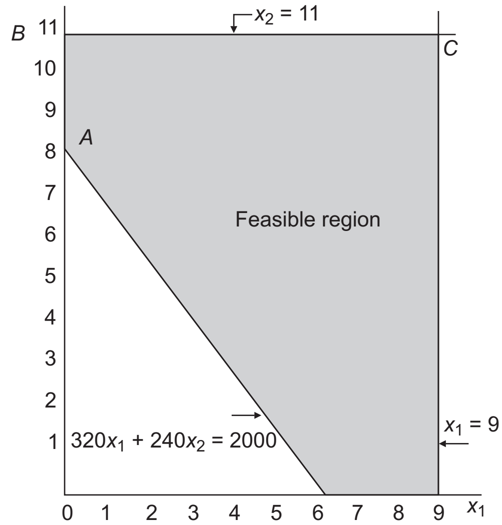
$B(0,11) : 748$

$C(9, 11) : 1367\frac{1}{5}$

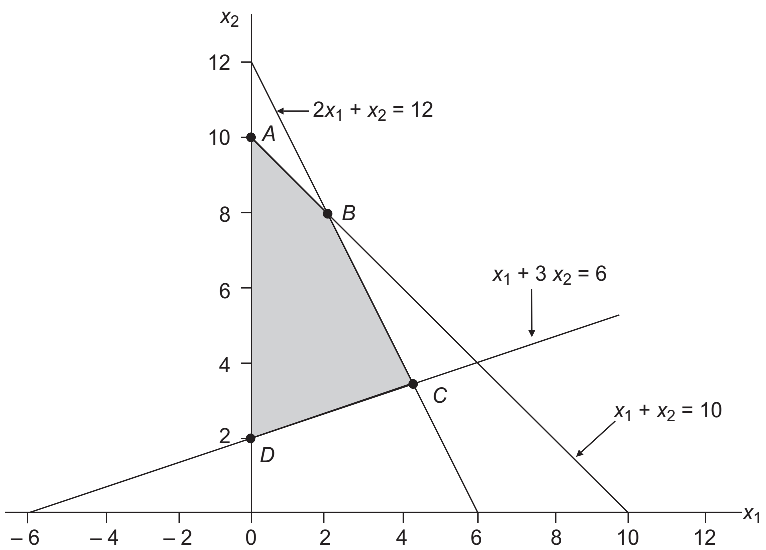
$D(9, 0) : 619\frac{1}{5}$

$E\left(6\frac{1}{4}, 0\right) : 430$

Point *E* represents optimal solution.



26. Here, $Z(A : 0, 10) = 20$, $Z(B : 2, 8) = 22$, $Z(C : 30/7, 24/7) = 138/7$ and $Z(D : 0, 2) = 4$. Accordingly, optimal solution is: $x_1 = 2$, $x_2 = 8$, and $Z = 22$.

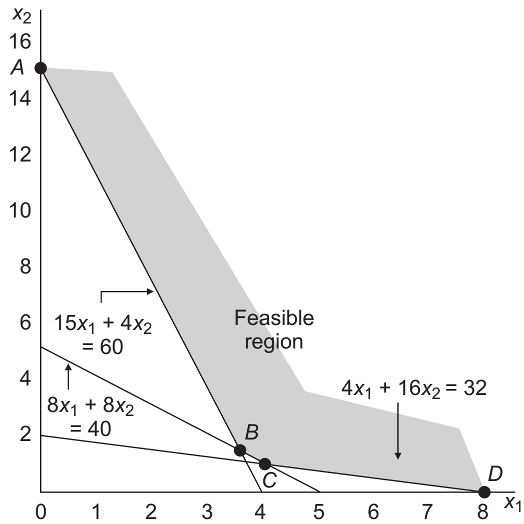


27. We have:

$Z(A : 0, 15) = 150$
$Z(B : 40/11, 15/11) = 270/11$
$Z(C : 4, 1) = 22$
$Z(D : 8, 0) = 24.$

Optimal solution:

$x_1 = 4$ and $x_2 = 1$; and $Z = 22$.



28. Here,

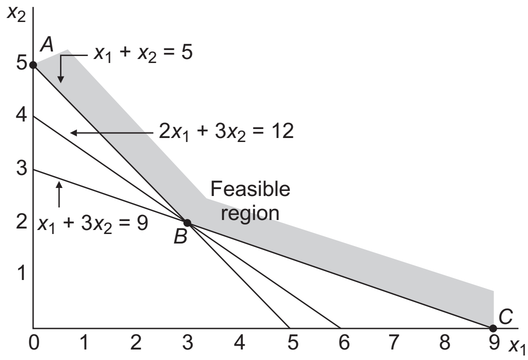
$Z(A : 0, 5) = 15,$

$Z(B : 3, 2) = 18,$ and

$Z(C : 9, 0) = 36$

Hence, optimal solution is:

$x_1 = 0$ and $x_2 = 5$; for $Z = 15$.



29. Let x_1 and x_2 be the number of units of deluxe and standard machines to be produced. From the given information, the LPP may be stated as follows:

Maximise	$Z = 400x_1 + 200x_2$	Total profit
Subject to		
	$18x_1 + 3x_2 \leq 800$	Labour time
	$9x_1 + 4x_2 \leq 600$	Testing time
	$x_2 \leq 150$	Demand
	$x_1, x_2 \geq 0$	

$Z(0 : 0, 0) = 0$

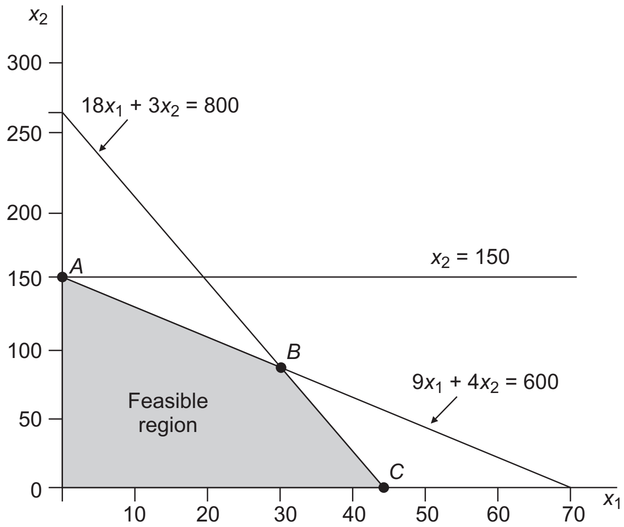
$Z(A : 0, 150) = 30{,}000$

$Z(B : 280/9, 80) = 28444.4$

$Z(C : 400/9, 0) = 17777.8$

Optimal solution:

Produce 150 units of standard machines and none of the deluxe machines.



30. Let x_1 and x_2 be the number of units of products *A* and *B*, respectively, to be purchased. The LPP may be stated as follows:

Minimise $Z = 20x_1 + 40x_2$ Total cost

Subject to

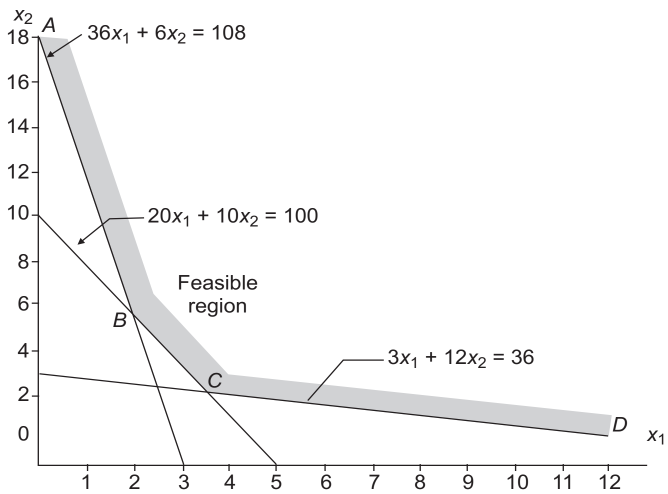
$36x_1 + 6x_2 \geq 108$ Nutrient 1

$3x_1 + 12x_2 \geq 36$ Nutrient 2

$20x_1 + 10x_2 \geq 100$ Nutrient 3

$x_1, x_2 \geq 0$

The feasible area has extremes *A*(0, 18), *B*(2, 6), *C*(4, 2), and *D*(12, 0). Accordingly, *Z*(*A*) = 720, *Z*(*B*) = 280, *Z*(*C*) = 160, and *Z*(*D*) = 240. Thus, optimal solution is $x_1 = 4$ and $x_2 = 2$.

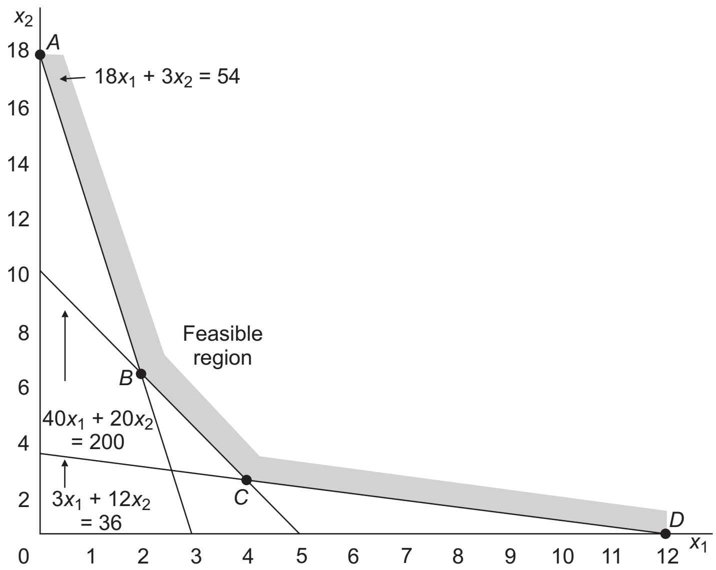


31. We have:

$$Z(A : 0, 18) = 720,$$
$$Z(B : 2, 6) = 280,$$
$$Z(C : 4, 2) = 160,$$
$$Z(D : 12, 0) = 240$$

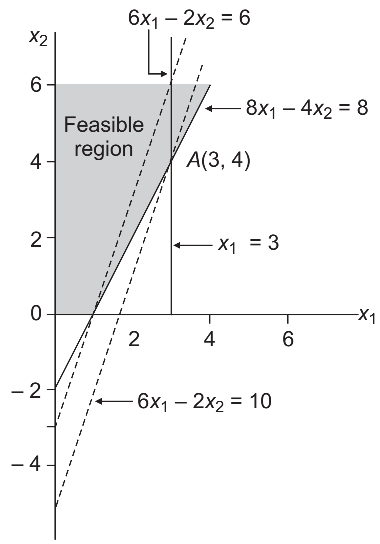
Optimal solution is:

$$x_1 = 4 \text{ and } x_2 = 2, \text{ for } Z = 160$$



32. (a) It is not necessary that the feasible region for a maximisation problem of linear programming be always a bounded one. When the feasible region is bounded in the direction in which iso-profit lines with higher profit values are obtained, the unboundedness nature of the feasible region (in the other direction) would not hinder the obtaining of the optimal solution.

(b) The constraints are plotted in figure. The feasible region, shown shaded, is evidently unbounded. The iso-profit lines are shown. The maximum profit obtainable is 10, which corresponds to $x_1 = 3$ and $x_2 = 4$ as shown by point A. This is the optimal solution to the problem.



33. Let x_1 be the number of bottles of Tonus-2000, and x_2 be the number of bottles of Health-Wealth produced per week. With profit rates as Rs 2.80 and Rs 2.20 per bottle of Tonus-2000 and Health-Wealth respectively, the total profit would be $2.80x_1 + 2.20x_2$. The problem, then, is:

Maximise $Z = 2.80x_1 + 2.20x_2$ Total profit

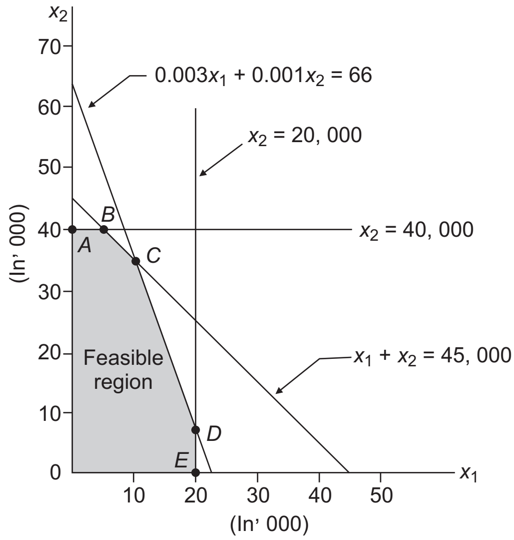
Subject to

$$\left.\begin{array}{l} x_1 \leq 20{,}000 \\ x_2 \leq 40{,}000 \end{array}\right\} \quad \text{Raw material}$$

$0.003x_1 + 0.001x_2 \leq 66$ Filling time

$x_1 + x_2 \leq 45{,}000$ Bottles availability

$x_1, x_2 \leq 0$



We have,

A : 0, 40,000
B : 5000, 40000
C : 10,500, 34500
D : 20,000, 6000
E : 20,000, 0

Further,

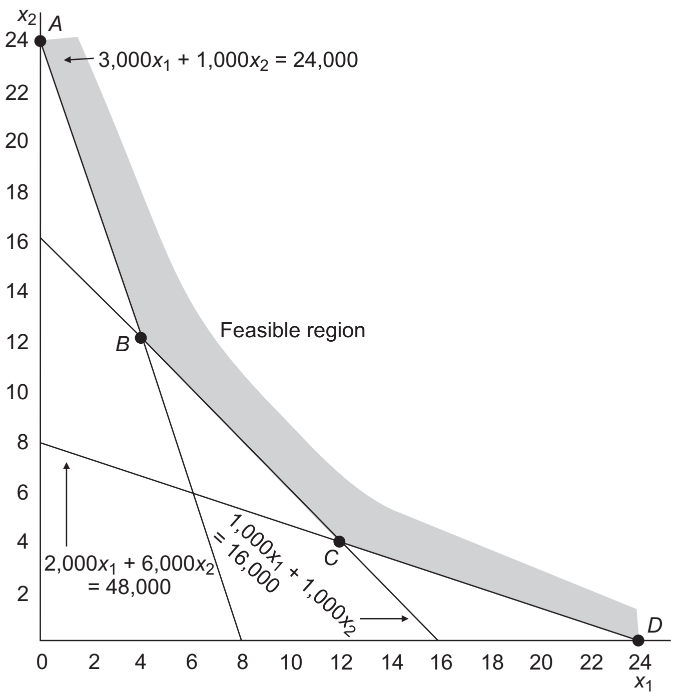
$Z(A) = 88{,}000$
$Z(B) = 102{,}000$
$Z(C) = 105{,}300$
$Z(D) = 69{,}200$
$Z(E) = 56{,}000$

Optimal solution is given by point C.

34. For this,

$Z(A : 0, 24) = 96{,}000$
$Z(B : 4, 12) = 72{,}000$
$Z(C : 12, 4) = 88{,}000$
$Z(D : 24, 0) = 144{,}000$

Thus, the optimal solution is to run plant I for 4 days and plant II for 12 days. TC = Rs 72,000



35. (a) Total hours available:

Department A : $20 \times 40 \times 50 = 40{,}000$
Department B : $15 \times 40 \times 50 = 30{,}000$
Department C : $18 \times 40 \times 50 = 36{,}000$

Contribution margin per unit:

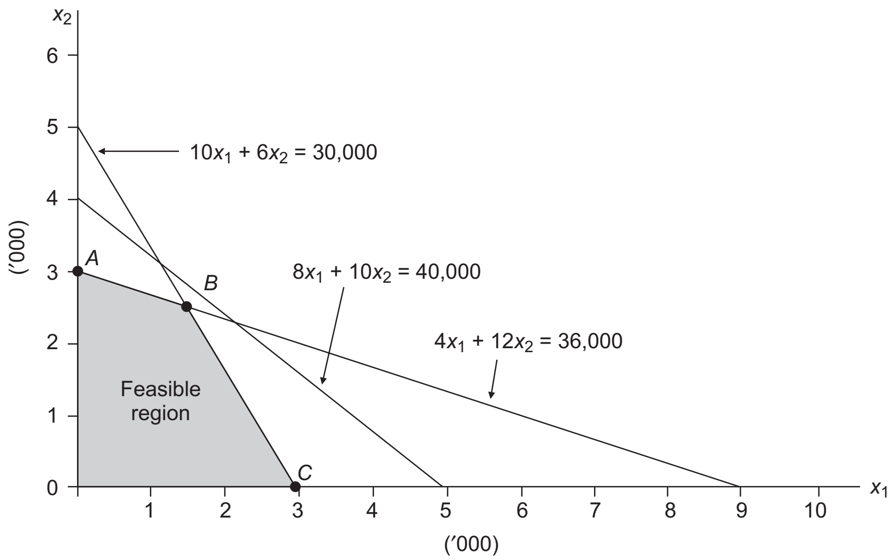
P_1 : $200 - (45 + 8 \times 2 + 10 \times 2.25 + 4 \times 2.50 + 6.50) =$ Rs 100
P_2 : $240 - (50 + 10 \times 2 + 6 \times 2.25 + 12 \times 2.50 + 11.50) =$ Rs 115

Let x_1 and x_2 be the number of units of the products P_1 and P_2 respectively. The problem is:

Maximise $Z = 100x_1 + 115x_2$

Subject to

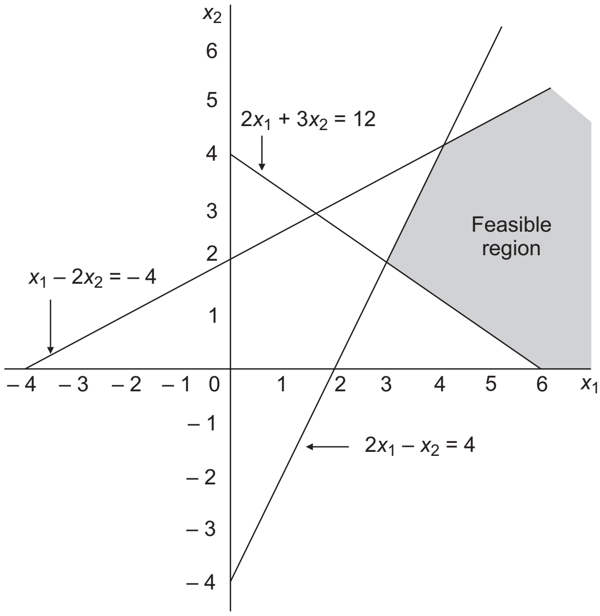
$$8x_1 + 10x_2 \le 40{,}000$$
$$10x_1 + 6x_2 \le 30{,}000$$
$$4x_1 + 12x_2 \le 36{,}000$$
$$x_1,\ x_2 \ge 0$$



The feasible region is given by the polygon $OABC$. Evaluating the objective function at each of these, we get $Z(0) = 0$, $Z(A) = 0 \times 100 + 3{,}000 \times 115 = 345{,}000$, $Z(B) = 1{,}500 \times 100 + 2{,}500 \times 115 = 437{,}500$, and $Z(C) = 3{,}000 \times 100 + 0 \times 115 = 300{,}000$. The optimal solution, therefore, is to produce 1,500 units of P_1 and 2,500 units of P_2. Total Profit = Contribution – Fixed cost = Rs 437,500 – Rs 285,000 = Rs 152,500 p.a.

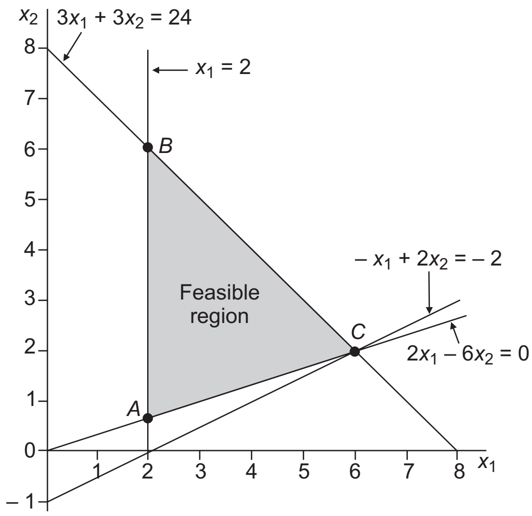
(b) It may be observed from the graph that the constraint representing labour hours in Department A is redundant because its exclusion does not affect the feasible region of the problem.

36. From the feasible region, it is evident that the problem has unbounded solution.



37. The different points are evaluated below:

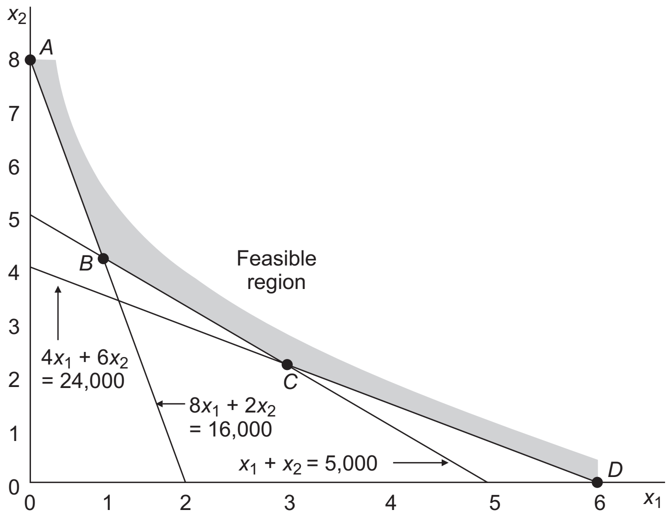
Point	x_1	x_2	$Z = 10x_1 - 4x_2$		$Z = 4x_1 - 10x_2$	
A	2	2/3	17 1/3		14 2/3	Min
B	2	6	–4	Min	68	Max
C	6	2	52	Max	44	



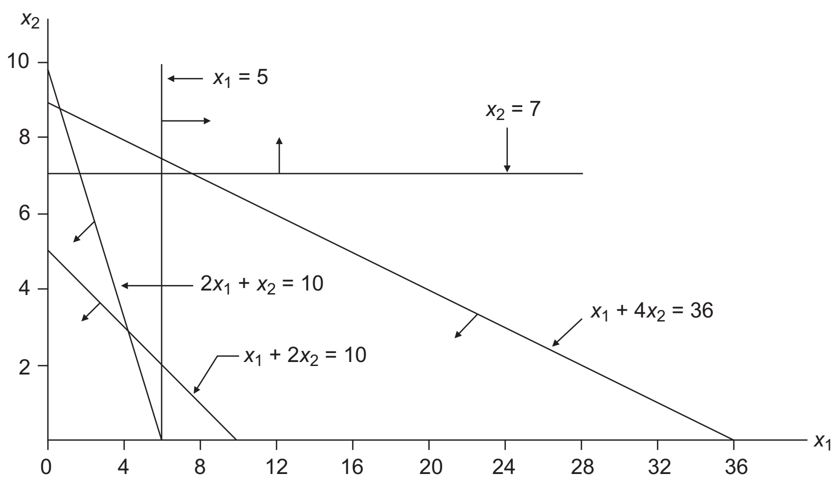
Thus, we have:

(a) Minimise $Z = 10x_1 - 4x_2 = -4$ when $x_1 = 2$, $x_2 = 6$
(b) Maximise $Z = 10x_1 - 4x_2 = 52$ when $x_1 = 6$, $x_2 = 2$
(c) Maximise $Z = 4x_1 + 10x_2 = 68$ when $x_1 = 2$, $x_2 = 6$
(d) Minimise $Z = 4x_1 + 10x_2 = 14\ 2/3$ when $x_1 = 2$, $x_2 = 2/3$

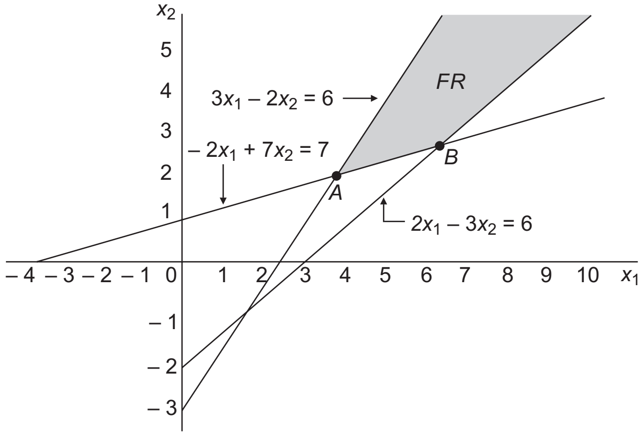
38. Z is minimum at either $x_1 = 0$ and $x_2 = 8$, or $x_1 = 1$ and $x_2 = 4$ since $Z(A : 0, 8) = 24$, $Z(B : 1, 4) = 24$, $Z(C : 3, 2) = 42$ and $Z(D : 6, 0) = 72$.



39. The constraints are plotted graphically. It is evident from it that there is no common point between the feasible regions of all constraints. Thus, the problem has no feasible solution.

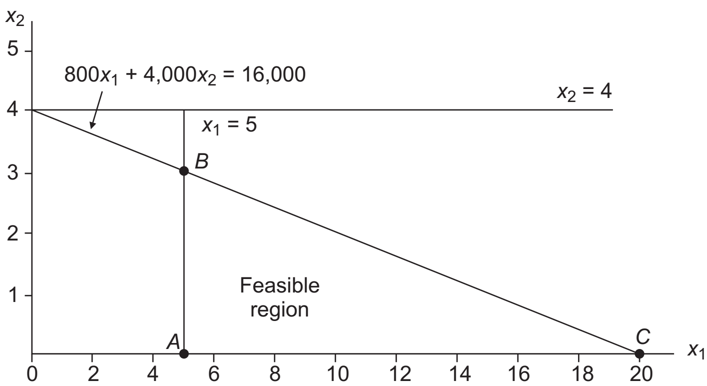


40. Maximise $Z = 8x_1 + 5x_2$: Unbounded solution
Minimise $Z = 8x_1 + 5x_2$
$Z(A : 56/17, 33/17) = 36\ 1/17$, $Z(B : 63/8, 13/4) = 79\ 1/4$.
Thus Z is minimum at $x_1 = 56/17$ and $x_2 = 33/17$.



41. Let x_1 and x_2 be the number of spots on Radio and TV respectively. From the given information, we have
Maximise $Z = x_1 + 6x_2$ Total coverage
Subject to

$800x_1 + 4{,}000x_2 \leq 16{,}000$ Budget
$x_1 \geq 5$, $x_2 \leq 4$ } Availability
$x_1, x_2 \geq 0$



The feasible area has three extreme points: $A(5, 0)$, $B(5, 3)$ and $C(20, 0)$. For these, we have $Z(A) = 5$, $Z(B) = 23$, and $Z(C) = 20$. Thus, the optimal solution is to have 3 spots on TV and 5 spots on radio.

Evidently, if the present restriction on TV spots is not there, it would not affect the optimal solution. It is redundant, in other words.

42. Let x_1 and x_2 be the number of units produced of products *A* and *B* respectively.

Maximise $Z = 500x_1 + 125x_2$

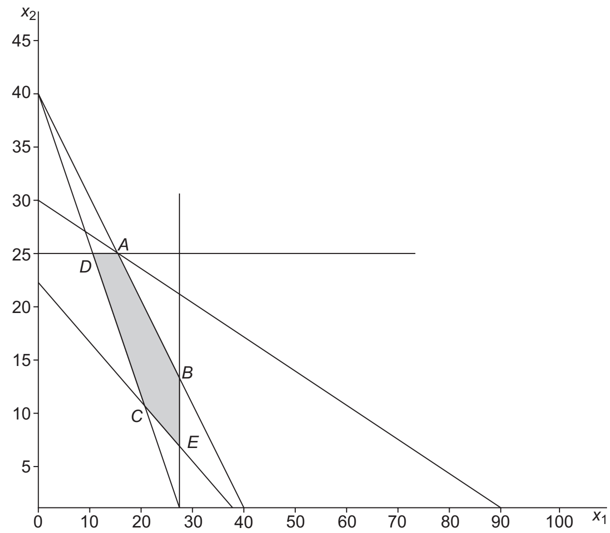
Subject to

$$3x_1 + 3x_2 \le 120$$
$$3x_1 + 9x_2 \le 270$$
$$13x_1 + 8x_2 \ge 330$$
$$4x_1 + 7x_2 \ge 156$$
$$x_1 \le 25$$
$$x_2 \le 25$$
$$x_1, x_2 \ge 0$$

The problem is shown graphically in the figure. The feasible area is shown marked *A*, *B*, *C*, *D*, *E*.

Point	x_1	x_2	*Z*
A	15	25	10,625
B	25	15	14,375
C	18	12	10,500
D	10	25	8,125
E	25	8	13,500

Optimal solution is : $x_1 = 25$, $x_2 = 15$ for Z = Rs 14,375

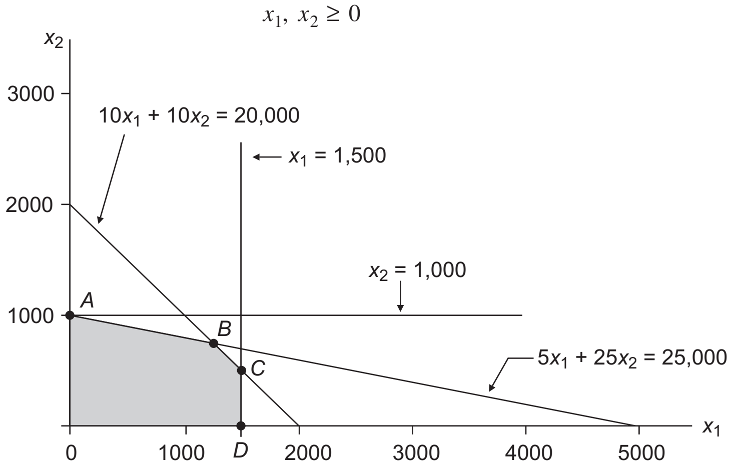


43. Contribution margin calculation:

Product	*Selling Price (Rs)*	*Variable Cost (Rs)*	*Contribution Margin CM (in Rs)*
Pixie	111	25 + 17 + 40 = 82	29
Elf	98	35 + 18 + 30 = 83	15
Queen	122	22 + 15 +75 = 112	10
King	326	25 + 16 + 175 = 216	110

(a) This problem may be solved in two parts. Since Pixie and Elf need only Type I labour and this resource is nor used by Queen and King, we calculate contribution margin per hour for each of these products to decide which one to produce.
For Pixie : Rs 29/8 = Rs 3.63 and for Elf : Rs 15/6 = Rs 2.50
∴ Produce only Pixie. Output = 8,000/8 = 1,000 units.
To determine optimal mix of Queen and King, we have to
Maximise $Z = 10x_1 + 110x_2$
Subject to

$$10x_1 + 10x_2 \leq 20{,}000 \quad \text{Type 2 labour}$$
$$5x_1 + 25x_2 \leq 25{,}000 \quad \text{Type 3 labour}$$
$$x_1 \leq 1{,}500 \quad \text{Demand}$$
$$x_2 \leq 1{,}000 \quad \text{Demand}$$
$$x_1,\ x_2 \geq 0$$



From the graph, the extreme points of feasible region are evaluated now: $Z(0 : 0, 0) = 0$, $Z(A : 0, 1{,}000) = 110{,}000$, $Z(B : 1{,}250, 750) = 95{,}000$, $Z(C) = (1{,}500, 500) = 26{,}000$ and $Z(D : 1{,}500, 0) = 15{,}000$. Optimal solution: 1,000 units of King. The overall solution is:
Pixie: 1,000 units, King: 1,000 units, Contribution = Rs 139,000

(b) If labour Type 1 is paid 1.5 times,
Contribution margin for Pixie = 111 – (25 + 17 + 60) = Rs 9, and
Contribution margin for Elf = 98 – (35 + 18 + 45) = Rs 0
∴ It is worthwhile to pay labour Type 1 time-and-a-half for overtime working to make Pixie, provided fixed costs do not increase.
Extra profit for every 1,000 hours overtime = 1,000 × 9/8 = Rs 1,125

(c) The basic principles used for the solution are:
- The objective is to maximise contribution, no matter only if two of the four products are produced.
- There is no substitution of labour between the two types.
- The objective functions and constraints are both linear in nature.
- The demand limits are fixed and known and there is no probability distribution of demand. It may be difficult to find all the conditions satisfied in a real life situation, yet they represent satisfactory set to investigate solutions to the problem.

(d) A computer can be used for solving linear programming problems using simplex algorithm (discussed in the next chapter). "Canned" programmes are available for handling such problems where a host of information, in addition to the optimal solution, is provided.

44. Let x_1: The output of product *A*

x_2: The output of product *B*

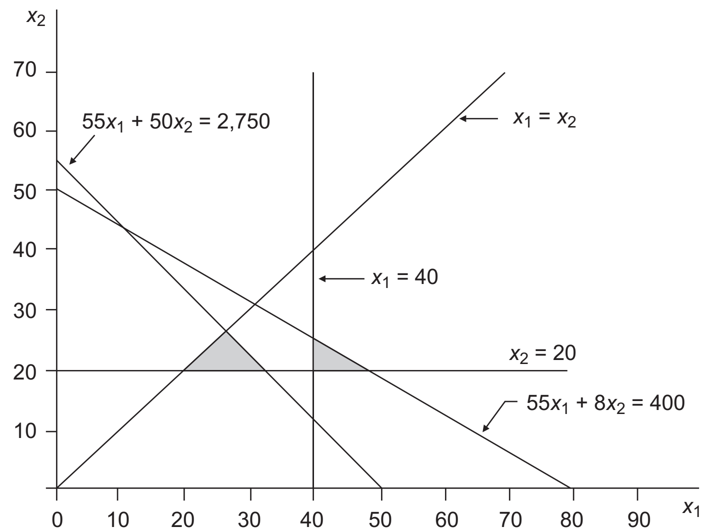
Since the profit rate is the same for both the products, the LPP may be stated as:

Maximise $Z = x_1 + x_2$

Subject to

$$\begin{aligned} 5x_1 + 8x_2 &\leq 400 \\ 55x_1 + 50x_2 &\leq 2{,}750 \\ x_1 &\geq 40 \\ x_2 &\geq 20 \\ x_1 &\geq x_2 \end{aligned}$$

The constraints are plotted on graph. It may be observed that no feasible solution to the problem exists because there is no common point between the feasible regions relating to all constraints.



CHAPTER 3

1. Maximise $Z = 8x_1 - 6x_2 + 7x_3 + 2x_4 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2$

   Subject to

$$4x_1 - 3x_2 + 6x_3 + x_4 + S_1 + 0S_2 + 0S_3 - 0A_1 + 0A_2 = 40$$
$$-x_1 + 2x_2 + 3x_3 + x_4 + 0S_1 + S_2 + 0S_3 + 0A_1 + 0A_2 = 5$$
$$9x_1 - 5x_2 + 7x_3 - x_4 + 0S_1 + 0S_2 - S_3 - A_1 + 0A_2 = 60$$
$$0x_1 + 6x_2 + 2x_3 + 4x_4 + 0S_1 + 0S_2 + 0S_3 + 0A_1 + A_2 = 47$$
$$x_1, x_2, x_3, x_4, S_1, S_2, S_3, A_1, A_2 \geq 0$$

2. **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	3	2	1	0	36	18
S_2	0	1	4*	0	1	10	5/2 ←
C_j		7	14	0	0		
Solution		0	0	36	10	$Z = 0$	
Δ_j		7	14 ↑	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	5/2	0	1	–1/2	31	62/5
x_2	14	1/4*	1	0	1/4	5/2	10 ←
C_j		7	14	0	0		
Solution		0	5/2	31	0	$Z = 35$	
Δ_j		7/2 ↑	0	0	–7/2		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
S_1	0	0	–10	1	–3	6
x_1	7	1	4	0	1	10
C_j		7	14	0	0	
Solution		10	0	6	0	$Z = 70$
Δ_j		0	–14	0	–7	

3. For solving the problem, we need to multiply the first constraint by –1 to have a non-negative b_i value. With slack variables S_1 and S_2, the solution follows.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	1	2	1	0	6	6
S_2	0	4*	3	0	1	12	3 ←
C_j		21	15	0	0		
Solution		0	0	6	12	$Z = 0$	
Δ_j		21	15	0	0		
		↑					

Simplex Tableau 2: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
S_1	0	0	5/4	1	–1/4	3
x_1	21	1	3/4	0	1/4	3
C_j		21	15	0	0	
Solution		3	0	3	0	$Z = 63$
Δ_j		0	–3/4	0	–21/4	

4. **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	4	3	1	1	0	0	40	40/3
S_2	0	2	5*	0	0	1	0	28	28/5 ←
S_3	0	8	2	0	0	0	1	16	8
C_j		20	30	5	0	0	0		
Solution		0	0	0	40	28	16	$Z = 0$	
Δ_j		20	30	5	0	0	0		
			↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	14/5	0	1	1	–3/5	0	116/5	58/7
S_2	30	2/5	1	0	0	1/5	0	28/5	14
S_3	0	36/5*	0	0	0	–2/5	1	24/5	2/3 ←
C_j		20	30	5	0	0	0		
Solution		0	28/5	0	116/5	0	24/5	$Z = 168$	
Δ_j		8	0	5	0	– 6	0		
		↑							

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	0	1*	1	−4/9	−7/18	64/3	64/3
x_2	30	0	1	0	0	2/9	−1/18	16/3	—
x_1	20	1	0	0	0	−1/18	5/36	2/3	—
C_j		20	30	5	0	0	0		
Solution		2/3	16/3	0	64/3	0	0	Z = 520/3	
Δ_j		0 ↑	0	5	0	−50/9	−10/9		

Simplex Tableau 4: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3	5	0	0	1	1	−4/9	−7/18	64/3	—
x_2	30	0	1	0	0	2/9	−1/18	16/3	—
x_3	20	1	0	0	0	−1/18	5/36*	2/3	24/5 ←
C_j		20	30	5	0	0	0		
Solution		2/3	16/3	64/3	0	0	0	Z = 280	
Δ_j		0	0	0	−5	−10/3	5/6 ↑		

Simplex Tableau 5: Optional Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_3	5	14/5	0	1	1	−3/5	0	116/5
x_2	30	2/5	1	0	0	1/5	0	28/5
S_3	0	36/5	0	0	0	−2/5	1	24/5
C_j		20	30	5	0	0	0	
Solution		0	28/5	116/5	0	0	24/5	Z = 284
C_j		−6	0	0	−5	−3	0	

5. Setting $x_2 = x_3 - x_4$, and multiplying constraint involving negative b_i, by −1 the LPP is:

Maximise $Z = 8x_1 - 4x_3 + 4x_4$

Subject to
$$4x_1 + 5x_3 - 5x_4 \le 20$$
$$x_1 - 3x_3 + 3x_4 \le 23$$
$$x_1, x_3, x_4 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_3	x_4	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	4*	5	−5	1	0	20	5 ←
S_2	0	1	−3	3	0	1	23	23
C_j		8	−4	4	0	0		
Solution		0	0	0	20	23	Z = 0	
Δ_j		8 ↑	−4	4	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_3	x_4	S_1	S_2	b_i	b_i/a_{ij}
x_1	8	1	5/4	–5/4	1/4	0	5	—
S_2	0	0	–17/4	17/4*	–1/4	1	18	72/17 ←
C_j		8	–4	4	0	0		
Solution		5	0	0	0	18	$Z = 40$	
Δ_j		0	–14	14 ↑	–2	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_3	x_4	S_1	S_2	b_i
x_1	8	1	0	0	3/17	5/17	175/17
x_2	4	0	–1	1	–1/17	4/17	72/17
C_j		8	–4	4	0	0	
Solution		175/17	0	72/17	0	0	$Z = \frac{1688}{17}$
Δ_j		0	0	0	–20/17	–56/17	

From Table 3, the optimal solution is:

$$x_1 = 175/17,\ x_2 = 0, \text{ and } x_3 = 72/17$$

Accordingly, the solution to the original problem is:

$$x_1 = 175/17 \text{ and } x_2 = x_3 - x_4 = 0 - 72/17 = -72/17 \text{ and}$$
$$Z = 8 \times 175/17 - 4(-72/17) = 1688/17$$

6. From the given information

Profit per unit of A = Rs 9.60 – (0.5 × 8 + 0.3 × 6 + 0.2 × 4) = Rs 3

Profit per unit of B = Rs 7.80 – (0.3 × 8 + 0.3 × 6 + 0.4 × 4) = Rs 2

Now, if x_1 and x_2 be the output and sales of drugs A and B respectively, the LPP may be stated as follows:

Maximise $Z = 3x_1 + 2x_2$

Subject to
$$0.5x_1 + 0.3x_2 \le 1{,}600$$
$$0.3x_1 + 0.3x_2 \le 1{,}400$$
$$0.2x_1 + 0.4x_2 \le 1{,}200$$
$$x_1, x_2 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0.5*	0.3	1	0	0	1,600	3,200 ←
S_2	0	0.3	0.3	0	1	0	1,400	4,667
S_3	0	0.2	0.4	0	0	1	1,200	6,000
C_j		3	2	0	0	0		
Solution		0	0	1,600	1,400	1,200	$Z = 0$	
Δ_j		3 ↑	2	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_1	3	1	0.60	2	0	0	3,200	5,333
S_2	0	0	0.12	−0.6	1	0	440	3,667
S_3	0	0	0.28*	−0.4	0	1	560	2,000 ←
C_j		3	2	0	0	0		
Solution		3,200	0	0	440	560	$Z = 9,600$	
Δ_j		0	0.2 ↑	−6	0	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_1	3	1	0	2.86	0	–2.14	2,000
S_2	0	0	0	−0.43	1	−0.43	200
x_2	2	0	1	–1.43	0	3.57	2,000
C_j		3	2	0	0	0	
Solution		2,000	2,000	0	0	0	$Z = 10,000$
Δ_j		0	0	–5.72	0	−0.72	

From Table 3 it is evident that the optimal product is: drug *A*, 2,000 units; drug *B*, 2,000 units for a total profit of Rs 10,000.

7. Let the output of belts type *A* and type *B* be x_1 and x_2 respectively. The LPP is:

Maximise $Z = 20x_1 + 15x_2$ Total profit

Subject to

$2x_1 + x_2 \le 1,000$ Time availability

$x_1 + x_2 \le 800$ Leather availability

$\left.\begin{array}{l} x_1 \le 400 \\ x_2 \le 700 \end{array}\right\}$ Buckle availability

$x_1, x_2 \ge 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	2	1	1	0	0	0	1,000	500
S_2	0	1	1	0	1	0	0	800	800
S_3	0	1*	0	0	0	1	0	400	400 ←
S_4	0	0	1	0	0	0	1	700	—
C_j		20	15	0	0	0	0		
Solution		0	0	1,000	800	400	700	$Z = 0$	
Δ_j		20 ↑	15	0	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	0	1*	1	0	–2	0	200	200 ←
S_2	0	0	1	0	1	–1	0	400	400
x_1	20	1	0	0	0	1	0	400	—
S_4	0	0	1	0	0	0	1	700	700
C_j		20	15	0	0	0	0		
Solution		400	0	200	400	0	700	Z = 8,000	
Δ_j		0	15	0	0	–20	0		
			↑						

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
x_2	15	0	1	1	0	–2	0	200	—
S_2	0	0	0	–1	1	1*	0	200	200 ←
x_1	20	1	0	0	0	1	0	400	400
S_4	0	0	0	– 0	0	–2	1	500	250
C_j		20	15	0	0	0	0		
Solution		400	200	0	200	0	500	Z = 11,000	
Δ_j		0	0	–15	0	10	0		
						↑			

Simplex Tableau 4: Optional Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i
x_2	15	0	1	–1	2	0	0	600
S_3	0	0	0	–1	1	1	0	200
x_1	20	1	0	1	–1	0	0	200
S_4	0	0	0	1	–2	0	1	100
C_j		20	15	0	0	0	0	
Solution		200	600	0	0	200	100	Z = 13,000
Δ_j		0	0	–5	–10	0	0	

8. **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	4	3	1	0	0	12	3
S_2	0	4*	1	0	1	0	8	2 ←
S_3	0	4	–9	0	0	1	8	2
C_j		3	2	0	0	0		
Solution		0	0	12	8	8		
Δ_j		3	2	0	0	0		
		↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	2*	1	–1	0	4	2 ←
x_1	3	1	1/4	0	1/4	0	2	8
S_3	0	0	–10	0	–1	1	0	—
C_j		3	2	0	0	0		
Solution		2	0	4	0	0	$Z = 6$	
Δ_j		0	5/4 ↑	0	–3/4	0		

Simplex Tableau 3: Optional Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_2	2	0	1	1/2	–1/2	0	2
x_1	3	1	0	–1/8	3/8	0	3/2
S_3	0	0	0	5	– 6	1	20
C_j		3	2	0	0	0	0
Solution		3/2	2	0	0	20	$Z = 8.5$
Δ_j		0	0	–5/8	–1/8	0	

It is evident that the optimal solution contained in Tableau 3 is not degenerate (as none of the basic variables assumes a solution value equal to zero). However, the solution given in Tableau 2 is a degenerate one. The improvement of this solution does not lead to another degenerate solution since the outgoing variable (S_1) is not a degenerate variable. The solution is temporarily degenerate, therefore.

9. After introducing necessary variables, the problem is:

Maximise $Z = 3x_1 + 2x_2 + 3x_3 + 0S_1 + 0S_2 - MA_1$

Subject to

$$2x_1 + x_2 + x_3 + S_1 = 2$$
$$3x_1 + 4x_2 + 2x_3 - S_2 + A_1 = 8$$
$$x_1, x_2, x_3, S_1, S_2, A_1 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	A_1	b_i	b_i/a_{ij}
S_1	0	2	1	1	1	0	0	2	2
A_1	$-M$	3	4	2	0	–1	1	8	2 ←
C_j		3	2	3	0	0	$-M$		
Solution		0	0	0	2	0	8		
Δ_j		$3 + 3M$	$2 + 4M$	$3 + 2M$	0	$-M$	0		

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	A_1	b_i	b_i/a_{ij}
S_1 0	5/4	0	1/2*	1	1/4	–1/4	0	0 ←
x_2 2	3/4	1	1/2	0	–1/4	1/4	2	4
C_j	3	2	3	0	0	$-M$		
Solution	0	2	0	0	0	0	$Z = 4$	
Δ_j	3/2	0	2	0	1/2	$-M-1/2$		

Simplex Tableau 3: Optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	A_1	b_i
x_3 3	5/2	0	1	2	1/2	–1/2	0
x_2 2	–1/2	1	0	–1	–1/2	1/2	2
C_j	3	2	3	0	0	$-M$	
Solution	0	2	0	0	0	0	$Z = 4$
Δ_j	–7/2	0	0	–4	–1/2	$-M-1/2$	

The solution in Simplex Tableau 3 is optimal. It is unique. The solution is degenerate, however.

10. From the given information,
No. of working hours available per machine per month
= No. of hours per day × No. of days × Percentage of effective working. Accordingly, the monthly capacity for the three operations is as follows:

$$X: 3 \times 320 = 960 \text{ hours}$$
$$Y: 2 \times 320 = 640 \text{ hours}$$
$$Z: 1 \times 320 = 320 \text{ hours}$$

The LPP with x_1, x_2, and x_3 representing the output of products *A*, *B*, and *C* respectively, may be stated as under:

Maximise $P = 3x_1 + 4x_2 + 6x_3$

Subject to

$$4x_1 + x_2 + 6x_3 \le 960$$
$$5x_1 + 3x_2 + x_3 \le 640$$
$$x_1 + 2x_2 + 3x_3 \le 320$$
$$x_1, x_2, x_3 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1 0	4	1	6	1	0	0	960	160
S_2 0	5	3	1	0	1	0	640	640
S_3 0	1	2	3*	0	0	1	320	320/3←
C_j	3	4	6	0	0	0		
Solution	0	0	0	960	640	320	$Z = 0$	
Δ_j	3	4	6	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	–3	0	1	0	–2	320	160
S_2	0	14/3*	7/3	0	0	1	–1/3	1,600/3	800/7 ←
x_3	6	1/3	2/3	1	0	0	1/3	320/3	320
C_j		3	4	6	0	0	0		
Solution		0	0	320/3	320	1,600/3	0	$Z = 640$	
Δ_j		1 ↑	0	0	0	0	–2		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
S_1	0	0	– 4	0	1	–3/7	–13/7	640/7
x_1	3	1	1/2	0	0	3/14	–1/14	800/7
x_3	6	0	1/2	1	0	–1/14	5/14	480/7
C_j		3	4	6	0	0	0	
Solution		800/7	0	480/7	640/7	0	0	$Z = 5{,}280/7$
Δ_j		0	–1/2	0	0	–3/14	–27/14	

Thus, optimal solution is: product *A*: 800/7 units, product *B*: nil, product $C = 480/7$ units. Total profit = Rs 5,280/7 or Rs 754.29.

11. Let x_1, x_2, and x_3 represent the daily production of dolls *A*, *B*, and *C* respectively. Using the given information, we may state the LPP as follows:

Maximise $Z = 3x_1 + 5x_2 + 4x_3$ Total Profit

Subject to

$2x_1 + 3x_2 \leq 8$ Machine M_1 time

$2x_2 + 5x_3 \leq 10$ Machine M_2 time

$3x_1 + 2x_2 + 4x_3 \leq 15$ Machine M_3 time

$x_1, x_2, x_3 \geq 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	3*	0	1	0	0	8	8/3 ←
S_2	0	0	2	5	0	1	0	10	5
S_3	0	3	2	4	0	0	1	15	15/2
C_j		3	5	4	0	0	0		
Solution		0	0	0	8	10	15		
Δ_j		3	5 ↑	4	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	5	2/3	1	0	1/3	0	0	8/3	—
S_2	0	–4/3	0	5*	–2/3	1	0	14/3	14/15 ←
S_3	0	5/3	0	4	–2/3	0	1	29/3	29/12
C_j		3	5	4	0	0	0		
Solution		0	8/3	0	0	14/3	29/3		
Δ_j		–1/3	0	4	–5/3	0	0		
				↑					

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	5	2/3	1	0	1/3	0	0	8/3	4
x_3	4	–4/15	0	1	–2/15	1/5	0	14/15	—
S_3	0	41/15*	0	0	–2/15	–4/5	1	89/15	89/41 ←
C_j		3	5	4	0	0	0		
Solution		0	8/3	14/5	0	0	89/15		
Δ_j		11/15	0	0	–17/15	–4/5	0		
		↑							

Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_2	5	0	1	0	15/41	8/41	–10/41	50/41
x_3	4	0	0	1	–6/41	5/41	4/41	62/41
x_1	3	1	0	0	–2/41	–12/41	15/41	89/41
C_j		3	5	4	0	0	0	
Solution		89/41	50/41	62/41	0	0	0	
Δ_j		0	0	0	–45/41	–24/41	–11/41	

From Tableau 4, it is evident that optimal daily output of the three type of dolls is:
Doll *A*: 89/41, Doll *B*: 50/41, Doll *C*: 62/41
The total profit works out to be Rs 765/41 or Rs 18.66. Also, none of the machines would remain idle.

12. Let x_1, x_2, and x_3 be the output of pistons, rings, and valves respectively. Using the given information, we may state the LPP as follows:

Maximise $\quad Z = 10x_1 + 6x_2 + 4x_3 \quad$ Profit

Subject to

$x_1 + x_2 + x_3 \leq 100 \quad$ Preparatory work

$10x_1 + 4x_2 + 5x_3 \leq 600 \quad$ Machinng

$2x_1 + 2x_2 + 6x_3 \leq 300 \quad$ Allied

$x_1, x_2, x_3 \geq 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	1	1	1	1	0	0	100	100
S_2	0	10*	4	5	0	1	0	600	60 ←
S_3	0	2	2	6	0	0	1	300	150
C_j		10	6	4	0	0	0		
Solution		0	0	0	100	600	300		
Δ_j		10 ↑	6	4	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	3/5*	1/2	1	–1/10	0	40	200/3 ←
x_1	10	1	2/5	1/2	0	1/10	0	60	150
S_3	0	0	6/5	5	0	–1/5	1	180	150
C_j		10	6	4	0	0	0		
Solution		60	0	0	40	0	180		
Δ_j		0	2 ↑	–1	0	–1	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_2	6	0	1	5/6	5/3	–1/6	0	200/3
x_1	10	1	0	1/6	–2/3	1/6	0	100/3
S_3	0	0	0	4	–2	0	1	100
C_j		10	6	4	0	0	0	
Solution		100/3	200/3	0	0	0	100	
Δ_j		0	0	–8/3	–10/3	–2/3	0	

The most profitable mix, therefore, is: Pistons = 100/3, Rings = 200/3 and Valves = 0. The corresponding profit = 10 × 100/3 + 6 × 200/3 = Rs 733.33.

13. (a) Let x_1, x_2, and x_3 represent, respectively, the number of units of *A*, *B* and *C*. The linear programming formulation is given here:

Maximise $Z = 12x_1 + 3x_2 + x_3$

Subject to

$$10x_1 + 2x_2 + x_3 \le 100$$
$$7x_1 + 3x_2 + 2x_3 \le 77$$
$$2x_1 + 4x_2 + x_3 \le 80$$
$$x_1, x_2, x_3 \ge 0$$

(b) **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	10*	2	1	1	0	0	100	10 ←
S_2	0	7	3	2	0	1	0	77	11
S_3	0	2	4	1	0	0	1	80	40
C_j		12	3	1	0	0	0		
Solution		0	0	0	100	77	80		
Δ_j		12 ↑	3	1	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_1	12	1	1/5	1/10	1/10	0	0	10	50
S_2	0	0	8/5*	13/10	–7/10	1	0	7	35/8 ←
S_3	0	0	18/5	4/5	–1/5	0	1	60	50/3
C_j		12	3	1	0	0	0		
Solution		0	0	0	0	7	60		
Δ_j		0 ↑	3/5	–1/5	– 6/5	0	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_1	12	1	0	–1/16	3/16	–1/8	0	73/8
x_2	3	0	1	13/16	–7/16	5/8	0	35/8
S_3	0	0	0	–17/16	11/8	–9/4	1	177/4
C_j		12	3	1	0	0	0	
Solution		73/8	35/8	0	0	0	177/4	
Δ_j		0	0	–11/16	–15/16	–3/8	0	

The product mix so as to maximise profit is: product *A*: 73/8 units, product *B*: 35/8 units and product *C*: nil. Total profit = Rs 12 × 73/8 + 3 × 35/8 = Rs 981/8.

(c) From Tableau 3 it is clear that $S_1 = S_2 = 0$, while $S_3 = 177/4$. Thus, there is no unused capacity in machine centres *X* and *Y*, while in machine centre *Z* a total of 177/4 hours would be unused.

14. Let the monthly production of the products 5-10-5, 5-5-10, and 20-5-10 be x_1, x_2 and x_3 kg respectively. The LPP is:

Maximise $Z = 16x_1 + 17x_2 + 10x_3$ Total profit

Subject to

$$\frac{1}{20}x_1 + \frac{1}{20}x_2 + \frac{1}{5}x_3 \le 100 \quad \text{Material } A$$

$$\frac{1}{10}x_1 + \frac{1}{20}x_2 + \frac{1}{20}x_3 \le 180 \quad \text{Material } B$$

$$\frac{1}{20}x_1 + \frac{1}{10}x_2 + \frac{1}{10}x_3 \le 120 \quad \text{Material } C$$

$$x_1 \le 30 \quad \text{Capacity}$$

$$x_1, x_2, x_3 \ge 0$$

Working notes:
Profit per unit is worked out as follows:
5-10-5: 40.50 – (0.05 × 80 + 0.10 × 20 + 0.05 × 50 + 0.80 × 20) = 16
5-5-10: 43 – (0.05 × 80 + 0.05 × 20 + 0.10 × 50 + 0.80 × 20) = 17
20-5-10: 45 – (0.20 × 80 + 0.05 × 20 + 0.10 × 50 + 0.65 × 20) = 10

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	1/20	1/20	1/5	1	0	0	0	100	2000
S_2	0	1/10	1/20	1/20	0	1	0	0	180	3600
S_3	0	1/20	1/10*	1/10	0	0	1	0	120	1200 ←
S_4	0	1	0	0	0	0	0	1	30	—
C_j		16	17	10	0	0	0	0		
Solution		0	0	0	100	180	120	30	$Z = 0$	
Δ_j		16	17	10	0	0	0	0		
			↑							

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	1/40	0	3/20	1	0	–1/2	0	40	1,600
S_2	0	3/40	0	0	0	1	–1/2	0	120	1,600
x_2	17	1/2	1	1	0	0	10	0	1,200	2,400
S_4	0	1*	0	0	0	0	0	1	30	30 ←
C_j		16	17	10	0	0	0	0		
Solution		0	1,200	0	40	120	0	30	$Z = 20,400$	
Δ_j		15/2	0	–7	0	0	–170	0		
		↑								

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i
S_1	0	0	0	3/20	1	0	–1/2	–1/40	157/4
S_2	0	0	0	0	0	1	–1/2	–3/40	471/4
x_2	17	0	1	1	0	0	10	–1/2	1,185
x_1	16	1	0	0	0	0	0	1	30
C_j		16	17	10	0	0	0	0	
Solution		30	1185	0	157/4	471/4	0	0	$Z = 20,625$
Δ_j		0	0	–7	0	0	–170	–15/2	

15. Using the given information about profitability and resources, the LPP may be stated as follows:

Maximise $Z = 4{,}000x_1 + 2{,}000x_2 + 5{,}000x_3$ Revenue

Subject to

$12x_1 + 7x_2 + 9x_3 \le 1{,}260$ Labour hours

$22x_1 + 18x_2 + 16x_3 \le 1{,}9008$ Wood

$2x_1 + 4x_2 + 3x_3 \le 396$ Screws

$x_1, x_2, x_3 \ge 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	12	7	9	1	0	0	1,260	140
S_2	0	22	18	16	0	1	0	19,008	1,188
S_3	0	2	4	3*	0	0	1	396	132 ←
C_j		4,000	2,000	5,000	0	0	0		
Solution		0	0	0	1,260	19,008	396		
Δ_j		4,000	2,000	5,000	0	0	0		
				↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	6*	–5	0	1	0	–3	72	12 ←
S_2	0	34/3	–10/3	0	0	1	–16/3	16,896	1,491
x_3	5,000	2/3	4/3	1	0	0	1/3	132	198
C_j		4,000	2,000	5,000	0	0	0		
Solution		0	0	132	72	16,896	0		
Δ_j		2,000/3	–14,000/3	0	0	0	–5,000/3		
		↑							

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_1	4,000	1	–5/6	0	1/6	0	–1/2	12
S_2	0	0	55/9	0	–17/9	1	1/3	16,760
x_3	5,000	0	17/9	1	–1/9	0	2/3	124
C_j		4,000	2,000	5,000	0	0	0	
Solution		12	0	124	0	16,760	0	
Δ_j		0	–37,000/9	0	–1,000/9	0	4,000/3	

(c) From Tableau 3, it is evident that for maximum profit, the company should produce 12 Row boats and 124 Kayaks and no Canoes. The maximum revenue is $4{,}000 \times 12 + 5{,}000 \times 124 = 668{,}000$.

(d) While labour-hours and screws available are fully used, the wood is not used fully. Its spare capacity is 16,760 board feet.

(e) The total wood used to make all of the boats in the optimal solution is $22 \times 12 + 16 \times 124 = 2{,}248$ board feet.

16. The information given in the problem is tabulated below:

	Vehicle Type		
	A	*B*	*C*
Tonnage	10	20	18
Average speed (kmph)	35	30	30
Working hours/day	18	18	21
Cost ('000 Rs)	80	130	150
Crew	3	6	6

The capacity of a vehicle in tonne-kms per day may be obtained by the product of tonnage, average speed, and working hours per day. This works out to be $10 \times 35 \times 18 = 6{,}300$ for *A* $20 \times 30 \times 18 = 10{,}800$ for *B* and $18 \times 30 \times 21 = 11{,}340$ for *C*. Now x_1, x_2, and x_3 be the number of vehicles purchased of types *A*, *B*, and *C* respectively, the LPP may be expressed as:

Maximise $Z = 6{,}300x_1 + 10{,}800x_2 + 11{,}340x_3$ Capacity

Subject to

$80x_1 + 130x_2 + 150x_3 \le 4{,}000$ Budget

$x_1 + x_2 + x_3 \le 30$ Maintenance

$3x_1 + 6x_2 + 6x_3 \le 150$ Crew

$x_1, x_2, x_3 \ge 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	80	130	150	1	0	0	4,000	400/15
S_2	0	1	1	1	0	1	0	30	30
S_3	0	3	6	6*	0	0	1	150	25 ←
C_j		6,300	10,800	11,340	0	0	0		
Solution		0	0	0	4,000	30	150	$Z = 0$	
Δ_j		6,300	10,800	11,340	0	0	0		
				↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	15	–20	0	1	0	–25	250	50
S_2	0	1/2*	0	0	0	1	–1/6	5	10 ←
x_3	11,340	1/2	1	1	0	0	1/6	25	50 ←
C_j		6,300	10,800	11,340	0	0	0		
Solution		0	0	25	250	5	0	$Z = 283{,}500$	
Δ_j		630	–540	0	0	0	–189		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
S_1	0	0	–20	0	1	–10	–70/3	200
x_1	6,300	1	0	0	0	2	–1/3	10
x_3	11,340	0	1	1	0	–1	1/3	20
C_j		6,300	10,800	11,340	0	0	0	
Solution		10	0	20	200	0	0	Z = 289,800
Δ_j		0	–54	0	0	–1,260	–1,680	

From Simplex Tableau 3, it may be observed that the company should buy 10 vehicles of type *A* and 20 vehicles of type *C* in order to maximise the capacity. The capacity is 289,800 tonne-km per day.

17. **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	–4	7	6	–4	1	0	0	20	—
S_2	0	3	–3	4	1	0	1	0	10	10/3
S_3	0	8*	–3	4	2	0	0	1	25	25/8 ←
C_j		7	2	3	4	0	0	0		
Solution		0	0	0	0	20	10	25	Z = 0	
Δ_j		7 ↑	2	3	4	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	11/2*	8	–3	1	0	1/2	65/2	65/11 ←
S_2	0	0	–15/8	5/2	1/4	0	1	–3/8	5/8	—
x_1	7	1	–3/8	1/2	1/4	0	0	1/8	25/8	—
C_j		7	2	3	4	0	0	0		
Solution		25/8	0	0	0	65/2	5/8	0	Z = 175/8	
Δ_j		0	37/8 ↑	–1/2	9/4	0	0	–7/8		

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	2	0	1	16/11	– 6/11	2/11	0	1/11	65/11	—
S_2	0	0	0	115/22	–17/22	15/44	1	–9/44	515/44	—
x_1	7	1	0	23/22	1/22*	3/44	0	7/44	235/44	235/2 ←
C_j		7	2	3	4	0	0	0		
Solution		235/44	65/11	0	0	0	515/44	0	Z = 2,165/44	
Δ_j		0	0	–159/22	105/22 ↑	–37/44	0	–57/44		

Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i
x_2	2	12	1	14	0	1	0	2	70
S_2	0	17	0	23	0	3/2	1	5/2	205/2
x_4	4	22	0	23	1	3/2	0	7/2	235/2
C_j		7	2	3	4	0	0	0	
Solution		0	70	0	235/2	0	205/2	0	$Z = 610$
Δ_j		–105	0	–117	0	–8	0	–18	

18. Let the output of desks I, II, III and IV be x_1, x_2, x_3 and x_4 respectively. The LPP is:
Maximise $Z = 9x_1 + 20x_2 + 15x_3 + 40x_4$
Subject to

$$4x_1 + 9x_2 + 7x_3 + 10x_4 \leq 6{,}000$$
$$x_1 + x_2 + 3x_3 + 40x_4 \leq 4{,}000$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	4	9	7	10	1	0	6,000	600
S_2	0	1	1	3	40*	0	1	4,000	100 ←
C_j		9	20	15	40	0	0		
Solution		0	0	0	0	6,000	4,000	$Z = 0$	
Δ_j		9	20	15	40	0	0		
					↑				

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	15/4	35/4*	25/4	0	1	–1/4	5,000	4,000/7 ←
x_4	40	1/40	1/40	3/40	1	0	1/40	100	4,000
C_j		9	20	15	40	0	0		
Solution		0	0	0	100	5,000	0	$Z = 4{,}000$	
Δ_j		8	19	12	0	0	–1		
			↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	b_i
x_2	20	3/7	1	5/7	0	4/35	–1/35	4,000/7
x_4	40	1/70	0	2/35	1	–1/350	9/350	600/7
C_j		9	20	15	40	0	0	
Solution		0	4,000/7	0	600/7	0	0	$Z = 104{,}000/7$
Δ_j		–1/7	0	–11/7	0	–76/35	–16/35	

19. Introducing necessary surplus and artificial variables, the problem is:

Minimise $Z = 6x_1 + 4x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

Subject to

$$3x_1 + 1/2x_2 - S_1 + A_1 = 12$$
$$2x_1 + x_2 - S_2 + A_2 = 16$$
$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	M	3	1/2	–1	0	1	0	12	4 ←
A_2	M	2	1	0	–1	0	1	16	8
C_j		6	4	0	0	M	M		
Solution		0	0	0	0	12	16		
Δ_j		$6 - 5M$	$4 - 3/2M$	M	M	0	0		
		↑							

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
x_1	6	1	1/6	–1/3	0	1/3	0	4	—
A_2	M	0	2/3*	2/3	–1	–2/3	1	8	12 ←
C_j		6	4	0	0	M	M		
Solution		4	0	0	0	0	8		
Δ_j		0	$3 - 2/3M$	$2 - 2/3M$	M	$-2 + 2/3M$	0		
			↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i
x_1	6	1	1/2	0	–1/2	0	1/2	8
S_2	0	0	1	1	–3/2	–1	3/2	12
C_j		6	4	0	0	M	M	
Solution		8	0	12	0	0	0	$Z = 48$
Δ_j		0	1	0	3	M	$M - 3$	

20. *Phase I:* Introduce surplus and artificial variables to the given problem, assign unit coefficient to the artificial and zero coefficient to the remaining variables to rewrite the problem as under:

Minimise $Z = 0x_1 + 0x_2 + 0S_1 + 0S_2 + A_1 + A_2$

Subject to

$$2x_1 + x_2 - S_1 + A_1 = 4$$
$$x_1 + 7x_2 - S_2 + A_2 = 7$$
$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	1	2	1	–1	0	1	0	4	4
A_2	1	1	7*	0	–1	0	1	7	1 ←
C_j		0	0	0	0	1	1		
Solution		0	0	0	0	4	7		
Δ_j		–3	–8	1	1	0	0		
			↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	1	13/7*	0	–1	1/7	1	–1/7	3	21/13 ←
x_2	0	1/7	1	0	–1/7	0	1/7	1	7
C_j		0	0	0	0	1	1		
Solution		0	1	0	0	0	3		
Δ_j		–13/7	0	1	–1/7	0	8/7		
		↑							

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i
x_1	0	1	0	–7/13	1/13	7/13	–1/13	21/13
x_2	0	0	1	1/13	–14/91	–1/13	14/91	10/13
C_j		0	0	0	0	1	1	
Solution		21/13	10/13	0	0	0	0	
Δ_j		0	0	0	0	1	1	

Phase II: Reconsider Simplex Tableau 3, delete columns headed A_1 and A_2, and replace the C_j row by the coefficients of the original problem. Apply simplex method. This is shown in Table 4, wherein the solution given is found to be optimal and calls for no revision. Thus, optimal solution is: $x_1 = 21/13$, $x_2 = 10/13$, and $Z = 31/13$.

Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
x_1	1	1	0	–7/13	1/13	21/13
x_2	1	0	1	1/13	–14/91	10/13
C_j		1	1	0	0	
Solution		21/13	10/13	0	0	
Δ_j		0	0	6/13	1/13	

21. *Phase I:* Introduce necessary variables. Assign a coefficient of 0 to each of the decision and surplus variable and 1 to each artificial variable.

Minimise $Z = 0x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 + A_1 + A_2$

Subject to

$$2x_1 + 3x_2 + x_3 - S_1 + A_1 = 4$$
$$3x_1 + 2x_2 + x_3 - S_2 + A_2 = 3$$
$$x_1, x_2, x_3, S_1, S_2, A_1, A_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1 1	2	3	1	–1	0	1	0	4	2
A_2 1	3*	2	1	0	–1	0	1	3	1 ←
C_j	0	0	0	0	0	1	1		
Solution	0	0	0	0	0	4	3		
Δ_j	–5 ↑	–5	–2	1	1	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1 1	0	5/3*	1/3	–1	2/3	1	–2/3	2	6/5 ←
x_1 0	1	2/3	1/3	0	–1/3	0	1/3	1	3/2
C_j	0	0	0	0	0	1	1		
Solution	1	0	0	0	0	2	0		
Δ_j	0	–5/3 ↑	–1/3	1	–2/3	0	1/3		

Simplex Tableau 3: Optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b_i
x_2 0	0	1	1/5	–3/5	2/5	3/5	–2/5	6/5
x_1 0	1	0	1/5	2/5	–3/5	–2/5	3/5	1/5
C_j	0	0	0	0	0	1	1	
Solution	1/5	6/5	0	0	0	0	0	
Δ_j	0	0	0	0	0	1	1	

Phase II: Reconsider Simplex Tableau 3. Delete columns headed A_1 and A_2. Also replace the C_j row by co-efficients of the original problem. Solve by simplex.

Simplex Tableau 4: Optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	b_i
x_2 150	0	1	1/5	–3/5	2/5	6/5
x_1 150	1	0	1/5	2/5	–3/5	1/5
C_j	150	150	100	0	0	
Solution	1/5	6/5	0	0	0	$Z = 210$
Δ_j	0	0	40	30	30	

Optimal solution: $x_1 = 1/5$, $x_2 = 6/5$, $Z = 210$

22. *Phase I:* Introducing surplus and artificial variables in the given problem, and assigning zero coefficient to each of the decision and surplus variables, and a coefficient of unity to the artificial variables, we get

Minimise $Z = 0x_1 + 0x_2 + 0S_1 + 0S_2 + A_1 + A_2$

Subject to

$$20x_1 + 30x_2 - S_1 + A_1 = 900$$
$$40x_1 + 30x_2 - S_2 + A_2 = 1,200$$
$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	1	20	30*	–1	0	1	0	900	30 ←
A_2	1	40	30	0	–1	0	1	1,200	40
C_j		0	0	0	0	1	1		
Solution		0	0	0	0	900	1,200		
Δ_j		– 60	– 60 ↑	1	1	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
x_2	0	2/3	1	–1/30	0	1/30	0	30	45
A_2	1	20*	0	1	–1	–1	1	300	15 ←
C_j		0	0	0	0	1	1		
Solution		0	30	0	0	0	300		
Δ_j		–20 ↑	0	–1	1	1	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i
x_2	0	0	1	–1/15	1/30	–1/15	–1/30	20
x_1	0	1	0	1/20	–1/20	–1/20	1/20	15
C_j		0	0	0	0	1	1	
Solution		15	20	0	0	0	0	
Δ_j		0	0	0	0	1	1	

Phase II: The Simplex Tableau 3 is reproduced below after replacing the C_j row by the coefficients from the objective function of the original problem and deleting the columns headed by A_1 and A_2. Then the problem is solved using the simplex method. It may be observed from the table that the solution is an optimal one and no further iterations are called for.

Simplex Tableau: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
x_2	80	0	1	–1/15	1/30	20
x_1	60	1″	0	1/20	–1/20	15
C_j		60	80	0	0	
Solution		15	20	0	0	
Δ_j		0	0	7/3	1/3	

23. (a) Let x_1 and x_2 be the quantity of Ash Trays and Tea Trays, respectively, produced. The problem is:

Maximise $Z = 20x_1 + 30x_2$ Profit (in paise)

Subject to

$10x_1 + 20x_2 \le 30{,}000$ Stamping

$15x_1 + 5x_2 \le 30{,}000$ Forming

$10x_1 + 8x_2 \le 40{,}000$ Painting

$x_1, x_2 \ge 0$

(b) **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	10	20*	1	0	0	30,000	1,500 ←
S_2	0	15	5	0	1	0	30,000	6,000
S_3	0	10	8	0	0	1	40,000	5,000
C_j		20	30	0	0	0		
Solution		0	0	30,000	30,000	40,000	$Z = 0$	
Δ_j		20	30 ↑	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	30	1/2	1	1/20	0	0	1,500	3,000
S_2	0	25/2*	0	–1/4	1	0	22,500	1,800 ←
S_3	0	6	0	–2/5	0	1	28,000	14,000/3
C_j		20	30	0	0	0		
Solution		0	1,500	0	22,500	28,000	$Z = 45{,}000$	
Δ_j		5 ↑	0	–3/2	0	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_2	30	0	1	3/50	–1/25	0	600
x_1	20	1	0	–1/50	2/25	0	1,800
S_3	0	0	0	–7/25	–12/25	1	17,200
C_j		20	30	0	0	0	
Solution		1,800	600	0	0	17,200	$Z = 54{,}000$
Δ_j		0	0	–7/5	–2/5	0	

Thus, optimal daily output = Ash Trays: 1,800, Tea Trays: 600. Daily profit = Rs 540 – Rs 350 (fixed expenses) = Rs 190.

(c) The revised LPP is:

Maximise $Z = 20x_1 + 30x_2$

Subject to

$$10x_1 + 20x_2 \leq 30,000$$
$$15x_1 + 5x_2 \leq 30,000$$
$$10x_1 + 8x_2 \leq 40,000$$
$$16x_1 + 20x_2 \leq 36,000$$
$$x_1, x_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	10	20*	1	0	0	0	30,000	1,500 ←
S_2	0	15	5	0	1	0	0	30,000	6,000
S_3	0	10	8	0	0	1	0	40,000	5,000
S_4	0	16	20	0	0	0	1	36,000	1,800
C_j		20	30	0	0	0	0		
Solution		0	0	30,000	30,000	40,000	36,000	$Z = 0$	
Δ_j		20	30	0	0	0	0		
			↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
x_2	30	1/2	1	1/20	0	0	0	1,500	3,000
S_2	0	25/2	0	–1/4	1	0	0	22,500	1,800
S_3	0	6	0	–2/5	0	1	0	28,000	14,000/3
S_4	0	6*	0	–1	0	0	1	6,000	1,000 ←
C_j		20	30	0	0	0	0		
Solution		0	1,500	0	22,500	28,000	6,000	$Z = 45,000$	
Δ_j		5	0	–3/2	0	0	0		
		↑							

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i
x_2	30	0	1	2/15	0	0	–1/12	1,000
S_2	0	0	0	11/6	1	0	–25/12	10,000
S_3	0	0	0	3/5	0	1	–1	22,000
x_1	20	1	0	–1/6	0	0	1/6	1,000
C_j		20	30	0	0	0	0	
Solution		1,000	1,000	0	10,000	22,000	0	$Z = 50,000$
Δ_j		0	0	–2/3	0	0	–5/6	

Optimal product mix: Ash Trays = 1,000, Tea Trays = 1,000.
Total Profit = Rs = 500 – Rs 350 = Rs 150 per day.

24. If x_1 and x_2 be the respective output of products A and B, the LPP is:

Maximise $Z = 30x_1 + 40x_2$

Subject to

$$4x_1 + 2x_2 \le 100$$
$$4x_1 + 6x_2 \le 180$$
$$x_1 + x_2 \le 40$$
$$x_1 \le 20$$
$$x_2 \ge 10$$
$$x_1, x_2 \ge 0$$

If we let $x_2 = 10 + x_3$, we have the revised problem as:

Maximise $Z = 30x_1 + 40x_3 + 400$

Subject to

$$4x_1 + 2x_3 \le 80;\ 4x_1 + 6x_3 \le 120;\ x_1 + x_2 \le 30;\ x_1 \le 20 \text{ and } x_1, x_3 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1 0	4	2	1	0	0	0	80	40
S_2 0	4	6*	0	1	0	0	120	20
S_3 0	1	1	0	0	1	0	30	20 ←
S_4 0	1	0	0	0	0	1	20	—
C_j	30	40	0	0	0	0		
Solution	0	0	80	120	30	20	$Z = 0$	
Δ_j	30	40 ↑	0	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1 0	8/3*	0	1	–1/3	0	0	40	15 ←
x_3 40	2/3	1	0	1/6	0	0	20	30
S_3 0	1/3	0	0	–1/6	1	0	10	30
S_4 0	1	0	0	0	0	1	20	20
C_j	30	40	0	0	0	0		
Solution	0	20	40	0	10	20	$Z = 800$	
Δ_j	10/3 ↑	0	0	–20/3	0	0		

Simplex Tableau 3: Optimal Solution

Basis	x_1	x_3	S_1	S_2	S_3	S_4	b_i
x_1 30	1	0	3/8	–1/8	0	0	15
x_3 40	0	1	–1/4	1/4	0	0	10
S_3 0	0	0	–1/8	–1/8	1	0	5
S_4 0	0	0	–3/8	1/8	0	1	5
C_j	30	40	0	0	0	0	
Solution	15	10	0	0	5	5	$Z = 850$
Δ_j	0	0	–5/4	–25/4	0	0	

The optimal solution is: $x_1 = 15$, $x_2 = 10 + 10 = 20$ and $Z = 850 + 400 = 1250$.

25. Maximise $Z = 20x_1 + 40x_2$ Total sales

Subject to:

$2x_1 + 4x_2 \leq 100$ Raw material

$-8x_1 + 24x_2 \leq 0$ Sales requirement

$x_1, x_2 \geq 0$

Note: Since the sales volume of product *A* is required to be at least 60 per cent of the total sales, the constraint may be stated as: $20x_1 \geq 0.6\ (20x_1 + 40x_2)$, which simplifies to be $-8x_1 + 24x_2 \leq 0$.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	2	4	1	0	100	25
S_2	0	–8	24*	0	1	0	0 ←
C_j		20	40	0	0		
Solution		0	0	100	0		
Δ_j		20	40	0	0		
			↑				

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	10/3*	0	1	–1/6	100	30 ←
x_2	40	–1/3	1	0	1/24	0	—
C_j		20	40	0	0		
Solution		0	0	100	0		
Δ_j		100/3	0	0	–5/3		
		↑					

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
x_1	20	1	0	3/10	–1/20	30	—
x_2	40	0	1	1/10	1/40*	10	400
C_j		20	40	0	0		
Solution		30	10	0	0		
Δ_j		0	0	–10	0		
					↑		

Simplex Tableau 4: Optimal (alternate) Solution

Basis		x_1	x_2	S_1	S_2	b_i
x_1	20	1	2	7/20	0	50
S_2	0	0	40	4	1	400
C_j		20	40	0	0	
Solution		50	0	0	400	
Δ_j		0	0	–7	0	

The following points may be noted:

(i) The solutions given in the first two tables are both degenerate. However, degeneracy here is temporary.

(ii) In each of tables second and third, only one replacement ratio is considered. The other one involves negative denominator and hence, ignored.

(iii) The problem has multiple optimal solutions as shown in tableau 3 and 4.

26. Let x_1 and x_2 be the output (in tonnes) of the products X and Y respectively. The LPP may be stated as follows:

Maximise $Z = 80x_1 + 120x_2$

Subject to

$$20x_1 + 50x_2 \leq 360$$
$$x_1 + x_2 \leq 9$$
$$x_1 \geq 2$$
$$x_2 \geq 3$$

As this problem involves lower bounds on the values of x_1 and x_2, it can be simplified as follows:

Let $x_1 = 2 + x_3$ and $x_2 = 3 + x_4$

Substituting these relationships, the given problem may be restated as follows:

Maximise $Z = 80x_3 + 120x_4 + 520$

Subject to

$$20x_3 + 50x_4 \leq 170$$
$$x_3 + x_4 \leq 4$$
$$x_3, x_4 \geq 0$$

Now, we can solve this problem. The variables S_1 and S_2 are the slack variables used to convert the inequalities into equations.

Simplex Tableau 1: Non-optimal Solution

Basis		x_3	x_4	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	20	50*	1	0	170	17/5 ←
S_2	0	1	1	0	1	4	4
C_j		80	120	0	0		
Solution		0	0	170	4	$Z = 0 + 520 = 520$	
Δ_j		80	120	0	0		
			↑				

Simplex Tableau 2: Non-optimal Solution

Basis		x_3	x_4	S_1	S_2	b_i	b_i/a_{ij}
x_4	120	2/5	1	1/50	0	17/5	17/2
S_2	0	3/5*	0	–1/50	1	3/5	1 ←
C_j		80	120	0	0		
Solution		0	17/5	0	3/5	$Z = 408 + 520 = 928$	
Δ_j		32	0	–12/5	0		
		↑					

Simplex Tableau 3: Optimal Solution

Basis		x_3	x_4	S_1	S_2	b_i
x_4	120	0	1	1/30	–2/3	3
x_3	80	1	0	–1/30	5/3	1
C_j		80	120	0	0	
Solution		1	3	0	0	$Z = 440 + 520 = 960$
Δ_j		0	0	– 4/3	–160/3	

Thus, optimal solution to the revised problem is:
$x_3 = 1$ and $x_4 = 3$. Accordingly, the solution to the original problem may be obtained as follows:
Output of X, $x_1 = 2 + x_3$ or $2 + 1 = 3$ tonnes,
Output of Y, $x_2 = 3 + x_4$ or $3 + 3 = 6$ tonnes, and
Total profit $= 80 \times 3 + 120 \times 6 =$ Rs 960.

27. Let the production of I_1 and I_2 be x_1 and x_2 units respectively. The LPP is:
Maximise $Z = 40x_1 + 60x_2$
Subject to

$$x_1 + x_2 \leq 40$$
$$2x_1 + x_2 \leq 70$$
$$x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	1	1	1	0	0	40	40
S_2	0	2	1	0	1	0	70	70
S_3	0	1	3*	0	0	1	90	30 ←
C_j		40	60	0	0	0		
Solution		0	0	40	70	90		
Δ_j		40	60	0	0	0		
			↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2/3*	0	1	0	–1/3	10	15 ←
S_2	0	5/3	0	0	1	–1/3	40	24
x_2	60	1/3	1	0	0	1/3	30	90
C_j		40	60	0	0	0		
Solution		0	30	10	40	0	$Z = 1,800$	
Δ_j		20	0	0	0	–20		
		↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_1	40	1	0	3/2	0	–1/2	15
S_2	0	0	0	–5/2	1	1/2	15
x_2	60	0	1	–1/2	0	1/2	25
C_j		40	60	0	0	0	
Solution		15	25	0	15	0	Z = 2,100
Δ_j		0	0	–30	0	–10	

$\therefore$ Optimal mix: I_1 = 15 and I_2 = 25 units. Increase in profit = Rs 2,100 – Rs 1,800 = Rs 300. Idle time on machine M_2 = 15 hours.

28. Let x_1 and x_2 be the number of programmes on TV and radio respectively. The problem is:

Maximise $Z = 5,00,000x_1 + 3,00,000x_2$

Subject to

$$50,000x_1 + 20,000x_2 \leq 2,10,000$$
$$x_1 \geq 3$$
$$x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

Let $x_1 = x_1^* + 3$. The revised problem is:

Maximise $Z = 5,00,000\, x_1^* + 3,00,000x_2 + 15,00,000$

Subject to

$$50,000\, x_1^* + 20,000x_2 \leq 60,000$$
$$x_2 \leq 5$$
$$x_1^*, x_2 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1^*	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	50,000	20,000	1	0	60,000	6/5 ←
S_2	0	0	1	0	1	5	-
C_j		5,00,000	3,00,000	0	0		
Solution		0	0	60,000	5	Z = 15,00,000	
Δ_j		5,00,000	3,00,000	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1^*	x_2	S_1	S_2	b_i	b_i/a_{ij}
x_1^*	5,00,000	1	2/5	1/50,000	0	6/5	3 ←
S_2	0	0	1	0	1	5	5
C_j		5,00,000	3,00,000	0	0		
Solution		6/5	0	0	5	Z = 21,00,000	
Δ_j		0	1,00,000 ↑	– 10	0		

Simplex Tableau 4: Optimal Solution

Basis		x_1^*	x_2	S_1	S_2	b_i
x_2	3,00,000	5/2	1	1/10,000	0	3
S_2	0	–5/2	0	–1/10,000	1	2
C_j		5,00,000	3,00,000	0	0	
Solution		0	3	0	2	Z = 24,00,000
Δ_j		–2,50,000	0	–30	0	

Thus, optimal solution calls for 3 programmes in TV and 3 programmes in Radio. Notice that $x_1 = x_1^* + 3$ or 0 + 3 = 3 and x_2 = 3. This would imply a total reach of 24,00,000, out of which Type A are 15,90,000 while Type B are 8,10,000.

29. Let x_1, x_2 and x_3 be the number of advertisements in magazines *A*, *B* and *C* respectively. The problem is:

Maximise $Z = 1{,}000x_1 + 900x_2 + 280x_3$ Exposure in '000

Subject to

$$10{,}000x_1 + 5{,}000x_2 + 6{,}000x_3 \le 100{,}000 \quad \text{Budget}$$

$$\left.\begin{array}{l} x_1 \ge 2 \\ x_2 \le 5 \\ x_3 \ge 2 \end{array}\right\} \quad \text{Insertion requirement}$$

$$x_1, x_2, x_3 \ge 0$$

To simplify the problem, we set $x_1 = 2 + x_4$ and $x_3 + 2 + x_5$. The revised problem is:

Maximise $Z = 1{,}000x_4 + 900x_2 + 280x_5 + 2{,}560$

Subject to

$$10{,}000x_4 + 5{,}000x_2 + 6{,}000x_5 \le 68{,}000$$

$$x_2 \le 5$$

$$x_4, x_2, x_3 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_4	x_2	x_5	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	10,000*	5,000	6,000	1	0	68,000	6.8 ←
S_2	0	0	1	0	0	1	5	—
C_j		1,000	900	280	0	0		
Solution		0	0	0	68,000	5	Z = 0 + 2,560 = 2,560	
Δ_j		1,000	900	280	0	0		
		↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_4	x_2	x_5	S_1	S_2	b_i	b_i/a_{ij}
x_4	1,000	1	1/2	6/10	1/10,000	0	6.8	13.6
S_2	0	0	1*	0	0	1	5	5 ←
C_j		1,000	900	280	0	0		
Solution		6.8	0	0	0	5	Z = 6,800 + 2,560 = 9,360	
Δ_j		0	400	–320	–1/10	0		
			↑					

Simplex Tableau 3: Optimal Solution

Basis		x_4	x_2	x_5	S_1	S_2	b_i
x_4	1,000	1	0	6/10	1/10,000	–1/2	4.30
x_2	900	0	1	0	0	1	5
C_j		1,000	900	280	0	0	
Solution		4.30	5	0	0	0	Z = 8,800 + 2,560 = 11,360
Δ_j		0	0	–320	–1/10	–400	

Thus, optimal ad-mix is:
Magazine A: 2 + 4.30 = 6.30, Magazine B = 5, Magazine C = 2 + 0 = 0.
Expected exposure = 11,360 (thousand).
Note: A non-integer solution is acceptable in LP.

30. **Simplex Tableau 1: Non-optimal Solution**

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	M	20	30	–1	0	1	0	900	45
A_2	M	40*	30	0	–1	0	1	1,200	30 ←
C_j		120	160	0	0	M	M		
Solution		0	0	0	0	900	1,200		
Δ_j		120 – 60M ↑	160 – 6M	M	M	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	M	0	15*	–1	1/2	1	–1/2	300	20 ←
x_1	120	1	3/4	0	–1/40	0	1/40	30	40
C_j		120	160	0	0	M	M		
Solution		30	0	0	0	300	0		
Δ_j		0	70 – 15M ↑	M	3 – M/2	0	–3 + M/2		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i
x_2	160	0	1	–1/15	1/30	1/15	–1/30	20
x_1	120	1	0	1/20	–1/20	–1/20	1/20	15
C_j		120	160	0	0	M	M	
Solution		15	20	0	0	0	0	Z = 5,000
Δ_j		0	0	14/3	2/3	M – 14/3	M – 2/3	

The solution will be unbounded in case the objective function is of maximisation type.

31. Let x_1 and x_2 respectively be the output of the products A and B. The LPP is:

Maximise $Z = 10x_1 + 12x_2$ Total Profit

Subject to

$2x_1 + 3x_2 \leq 1,500$ Machine M_1

$3x_1 + 2x_2 \leq 1,500$ Machine M_2

$x_1 + x_2 \leq 1,000$ Machine M_3

$x_1, x_2 \geq 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	3*	1	0	0	1,500	500 ←
S_2	0	3	2	0	1	0	1,500	750
S_3	0	1	1	0	0	1	1,000	1,000
C_j		10	12	0	0	0		
Solution		0	0	1,500	1,500	1,000	$Z = 0$	
Δ_j		10	12 ↑	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	12	2/3	1	1/3	0	0	500	750
S_2	0	5/3*	0	–2/3	1	0	500	300 ←
S_3	0	1/3	0	–1/3	0	1	500	1,500
C_j		10	12	0	0	0		
Solution		0	500	0	500	500	$Z = 6,000$	
Δ_j		2 ↑	0	–4	0	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_2	12	0	1	3/5	–2/5	0	300
x_1	10	1	0	–2/5	3/5	0	300
S_3	0	0	0	–1/5	–1/5	1	400
C_j		10	12	0	0	0	
Solution		300	300	0	0	400	$Z = 6,600$
Δ_j		0	0	–16/5	–6/5	0	

Optimal product mix: $x_1 = 300$, $x_2 = 300$. Hours unused on machine $M_3 = 400$. Total Profit = 6,600 + 600 = Rs 7,200.

32. If the output of C_1, C_2 and C_3 be x_1, x_2 and x_3 respectively, the problem is:

Maximise $Z = 6x_1 + 3x_2 + 2x_3$

Subject to

$2x_1 + 2x_2 + 3x_3 \leq 300$

$2x_1 + 2x_2 + x_3 \leq 120$

$x_1, x_2, x_3 \geq 0$

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	b_i	b_i/a_{ij}
S_1 0	2	2	3	1	0	300	150
S_2 0	2*	2	1	0	1	120	60 ←
C_j	6	3	2	0	0		
Solution	0	0	0	300	120	$Z = 0$	
Δ_j	6 ↑	3	2	0	0		

Simplex Tableau 2: Optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	b_i
S_1 0	0	0	2	1	–1	180
x_1 6	1	1	1/2	0	1/2	60
C_j	6	3	2	0	0	
Solution	60	0	0	180	0	$Z = 360$
Δ_j	0	–3	–1	0	–3	

The optimal solution is to produce only 60 units of C_1. The answer would not change by given statement.

33. (a) Since there is no artificial variable in the basis, and all the $C_j - z_j$ values are ≤ 0, the given solution is optimal. The optimal product mix is: $x_1 = 0$, $x_2 = 8/3$ units, and $x_3 = 56/3$ units.
    (b) The given solution is feasible since it involves no artificial variable in the basis.
    (c) The problem does not have any alternate optimal solution since none of the non-basic variables, x_1, S_1, and S_2 has $\Delta_j = 0$.
    (d) The solution given in the table is not degenerate since none of the basic variables has solution value equal to zero.
    (e) The values in the given table under column headed x_1 are 1/3 and 5/6 corresponding to the variables x_2 and x_3 respectively. Thus, 1/3 unit of x_2 and 5/6 unit of x_3 have to be foregone to get one unit of x_1. Now, to obtain six units of x_1, we have to reduce 6 × 1/3 = 2 units of x_2 and 6 × 5/6 = 5 units of x_3.

34. Let S_1, S_2 and A_1 be the necessary surplus, slack and artificial variables.

Simplex Tableau 1

Basis	x_1	x_2	S_1	S_2	A_1	b_i	b_i/a_{ij}
A_1 –M	2	5*	–1	0	1	50	10 ←
S_2 0	4	1	0	1	0	28	28
C_j	10	20	0	0	–M		
Solution	0	0	0	28	50		
Δ_j	10 + 2M	20 + 5M ↑	–M	0	0		

Simplex Tableau 2

Basis		x_1	x_2	S_1	S_2	A_1	b_i	b_i/a_{ij}
x_2	20	2/5	1	–1/5	0	1/5	10	—
S_2	0	18/5	0	1/5*	1	–1/5	18	90
C_j		10	20	0	0	$-M$		
Solution		0	10	0	18	0	$Z = 200$	
Δ_j		2	0	4	0	$-M - 4$		

Simplex Tableau 3

Basis		x_1	x_2	S_1	S_2	A_1	b_i
x_2	20	4	1	0	1	0	28
S_1	0	18	0	1	5	–1	90
C_j		10	20	0	0	$-M$	
Solution		0	28	90	0	0	$Z = 560$
Δ_j		–70	0	0	–20	$-M$	

$\therefore$ Optimal solution is: $x_1 = 0$, $x_2 = 28$ for $Z = 560$.

35. With slack, surplus and artificial variables, the problem is:
Maximise $\pi = 22x + 30y + 25z + 0S_1 + 0S_2 + 0S_3 - MA_1$
Subject to

$$2x + 2y + S_1 = 100$$
$$2x + y + z + S_2 = 100$$
$$x + 2y + 2z - S_3 + A_1 = 100$$
$$x, y, z, S_1, S_2, S_3, A_1 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x	y	z	S_1	S_2	S_3	A_1	b_i	b_i/a_{ij}
S_1	0	2	2	0	1	0	0	0	100	50
S_2	0	2	1	1	0	1	0	0	100	100 ←
A_i	$-M$	1	2*	2	0	0	–1	1	100	50
C_j		22	30	25	0	0	0	$-M$		
Solution		0	0	0	100	100	0	100		
Δ_j		$22 + M$	$30 + 2M$ ↑	$25 + 2M$	0	0	$-M$	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x	y	z	S_1	S_2	S_3	A_1	b_i	b_i/a_{ij}
S_1	0	1	0	–2	1	0	1*	–1	0	0 ←
S_2	0	3/2	0	0	0	1	1/2	–1/2	50	100
y	30	1/2	1	1	0	0	–1/2	1/2	50	–
C_j		22	30	25	0	0	0	$-M$		
Solution		0	50	0	0	50	0	0		$\pi = 1500$
Δ_j		7	0	–5	0	0	15 ↑	$-M - 15$		

Simplex Tableau 3: Non-optimal Solution

Basis		x	y	z	S_1	S_2	S_3	A_1	b_i	b_i/a_{ij}
S_3	0	1	0	–2	1	0	1	–1	0	–
S_2	0	1	0	1*	–1/2	1	0	0	50	50 ←
y	30	1	1	0	1/2	0	0	0	50	–
C_j		22	30	25	0	0	0	$-M$		
Solution		0	50	0	0	50	0	0		$\pi = 1500$
Δ_j		–8	0	25 ↑	–15	0	0	$-M$		

Simplex Tableau 4: Optimal Solution

Basis		x	y	z	S_1	S_2	S_3	A_1	b_i
S_3	0	3	0	0	0	2	1	–1	100
z	25	1	0	1	–1/2	1	0	0	50
y	30	1	1	0	1/2	0	0	0	50
C_j		22	30	25	0	0	0	$-M$	
Solution		0	50	50	0	0	100	0	$\pi = 2,750$
Δ_j		–33	0	0	–5/2	–25	0	$-M$	

Optimal Solution: $x = 0$, $y = 50$, $z = 50$, $\pi = 2,750$

36.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	$-M$	2	3	–1	0	1	0	60	30
A_2	$-M$	4*	3	0	–1	0	1	96	24 ←
C_j		40	35	0	0	$-M$	$-M$		
Solution		0	0	0	0	60	96		
Δ_j		$40 + 6M$	$35 + 6M$ ↑	$-M$	$-M$	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	$-M$	0	3/2*	0	1/2	1	–1/2	12	8 ←
x_1	40	1	3/4	0	–1/4	0	1/4	24	32
C_j		40	35	0	0	$-M$	$-M$		
Solution		24	0	0	0	12	0		
Δ_j		0	$5 + \frac{3}{2}M$ ↑	$-M$	$10 + \frac{M}{2}$	0	$-10 - \frac{M}{2}$		

Simplex Tableau 3: Non-optimal Solution

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
x_2 35	0	1	–2/3	1/3*	2/3	–1/3	8	24
x_1 0	1	0	1/2	–1/2	–1/2	1/2	18	–36
C_j	40	35	0	0	$-M$	$-M$		
Solution	18	8	0	0	0			
Δ_j	0	0	10/3	25/3 ↑	$-M - 10/3$	$-M - 25/3$		

Simplex Tableau 4: Non-optimal Solution

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
S_2 0	0	3	–2	1	2	–1	24	–12
x_1 40	1	3/2	–1/2	0	1/2	0	30	– 60
C_j	40	35	0	0	$-M$	$-M$		
Solution	30	0	0	24	0	0		
Δ_j	0	–25	20 ↑	0	$-M - 20$	$-M$		

It may be observed from Simplex Tableau 4 that the solution is not optimal as all Δ_j values are not less than or equal to zero. However, considering the a_{ij} values of the incoming variable S_1, the replacement ratios are both found to be negative. Accordingly, the procedure terminates. This indicates the problem has unbounded solution.

37. Let x_1 kg of factor *A* and x_2 kg of factor *B* are used. The LPP is:
Maximise $Z = 5x_1 + 6x_2$
Subject to $x_1 + x_2 = 5$, $x_1 \geq 2$, $x_2 \leq 4$, and $x_2 \geq 0$.

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	S_1	A_1	A_2	S_2	b_i	b_i/a_{ij}
A_1 $-M$	1	1	0	1	0	0	5	5
A_2 $-M$	1*	0	–1	0	1	0	2	2 ←
S_2 0	0	1	0	0	0	1	4	—
C_j	5	6	0	$-M$	$-M$	0		
Solution	0	0	0	5	2	4		
Δ_j	$5 + 2M$ ↑	$6 + M$	0	$-M$	$-M$	0		

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_2	S_1	A_1	A_2	S_2	b_i	b_i/a_{ij}
A_1 $-M$	0	1*	1	1	–1	0	3	3 ←
x_1 5	1	0	–1	0	1	0	2	—
S_2 0	0	1	0	0	0	1	4	4
C_j	5	6	0	$-M$	$-M$	0		
Solution	2	0	0	3	0	4		
Δ_j	0 ↑	$6 + M$	$5 + M$	0	$-5 - 2M$	0		

Simplex Tableau 3: Optimal Solution

Basis	x_1	x_2	S_1	A_1	A_2	S_2	b_i
x_2 6	0	1	1	1	–1	0	3
x_1 5	1	0	–1	0	1	0	2
S_2 0	0	0	–1	–1	1	1	1
C_j	5	6	0	$-M$	$-M$	0	
Solution	2	3	0	0	0	1	$Z = 28$
Δ_j	0	0	–1	$-M - 6$	$-M + 6$	0	

Optimal solution: Factor A = 2 kg, Factor B = 3 kg, Profit = Rs 28.

38. To solve the problem using simplex algorithm, we first introduce the necessary slack, surplus, and artificial variables. The augmented LPP is stated below:

Maximise $Z = 2x_1 + 4x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

Subject to

$$2x_1 + x_2 + S_1 = 18$$
$$3x_1 + 2x_2 - S_2 + A_1 = 30$$
$$x_1 + 2x_2 + A_2 = 25$$
$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

Solution to the problem is contained in tables.

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
S_1 0	2	1	1	0	0	0	18	18
A_1 $-M$	3	2	0	–1	1	0	30	15
A_2 $-M$	1	2*	0	0	0	1	25	25/2 ←
C_j	2	4	0	0	$-M$	$-M$		
Solution	0	0	18	0	30	25		
Δ_j	$2 + 4M$	$4 + 4M$ ↑	0	$-M$	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
S_1 0	3/2	0	1	0	0	–1/2	11/2	11/3
A_1 $-M$	2*	0	0	–1	1	–1	5	5/2 ←
x_2 4	1/2	1	0	0	0	1/2	25/2	25
C_j	2	4	0	0	$-M$	$-M$		
Solution	0	25/2	11/2	0	5	0		
Δ_j	$2M$ ↑	0	0	$-M$	0	$-2M - 2$		

Simplex Tableau 3: Optimal Solution

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i
S_1 0	0	0	1	3/4*	–3/4	1/4	7/4
x_1 2	1	0	0	–1/2	1/2	–1/2	5/2
x_2 4	0	1	0	1/4	–1/4	3/4	45/4
C_j	2	4	0	0	$-M$	$-M$	
Solution	5/2	45/4	0	0	0	0	Z = 185
Δ_j	0	0	0	0	$-M$	$-M - 2$	

The Simplex Tableau 3 gives optimal solution as x_1 = 5/2 and x_2 = 45/4, with Z = 185. However, this solution is not unique as a non-basic variable, S_2, has Δ_j = 0. An alternate optimal solution is given here.

Simplex Tableau 4: Optimal Solution (Alternate)

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i
S_2 0	0	0	4/3	1	–1	1/3	7/3
x_1 2	1	0	2/3	0	0	–1/3	11/3
x_2 4	0	1	–1/3	0	0	2/3	533/12
C_j	2	4	0	0	$-M$	$-M$	
Solution	11/3	533/12	0	7/3	0		Z = 1085
Δ_j	0	0	0	0	$-M$	–2	

39. **Simplex Tableau 1: Non-optimal Solution**

Basis	x_1	x_2	S_1	S_2	S_3	S_4	A_1	A_2	b_i	b_i/a_{ij}
S_1 0	4	2	1	0	0	0	0	0	1,600	400
S_2 0	6	5	0	1	0	0	0	0	3,000	500
A_1 $-M$	1	0	0	0	–1	0	1	0	300	300 ←
A_2 $-M$	0	1	0	0	0	–1	0	1	300	—
C_j	10	8	0	0	0	0	$-M$	$-M$		
Solution	0	0	1,600	3,000	0	0	300	300		
Δ_j	$10 + M$ ↑	$8 + M$	0	0	$-M$	$-M$	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_2	S_1	S_2	S_3	S_4	A_1	A_2	b_i	b_i/a_{ij}
S_1 0	0	2*	1	0	4	0	–4	0	400	200*
S_2 0	0	5	0	1	6	0	–6	0	1,200	240
x_1 10	1	0	0	0	–1	0	1	0	300	—
A_2 $-M$	0	1	0	0	0	–1	0	1	300	300
C_j	10	8	0	0	0	0	$-M$	$-M$		
Solution	300	0	400	1,200	0	0	0	300		
Δ_j	0	$8 + M$ ↑	0	0	10	$-M$	$-M - 10$	0		

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	A_1	A_2	b_i	b_i/a_{ij}
x_2	8	0	1	1/2	0	2	0	–2	0	200	—
S_2	0	0	0	–5/2	1	–4	0	4	0	200	50
x_1	10	1	0	0	0	–1	0	1	0	300	300
A_2	$-M$	0	0	–1/2	0	–2	–1	2*	1	100	50 ←
C_j		10	8	0	0	0	0	$-M$	$-M$		
Solution		300	200	0	200	0	0	0	100		
Δ_j		0	0	$-4-\frac{M}{2}$	0	$-2M-6$	$-M$	$M+6$	0		
								↑			

Simplex Tableau 4: Non-optimal Solution (Final)

Basis		x_1	x_2	S_1	S_2	S_3	S_4	A_1	A_2	b_i
x_2	8	0	1	0	0	0	–1	0	1	300
S_2	0	0	0	–3/2	1	0	2	0	–2	0
x_1	10	1	0	1/4	0	0	–1/2	0	–1/2	250
A_1	$-M$	0	01	–1/4	0	–1	–1/2	1	1/2	50
C_j		10	8	0	0	0	0	$-M$	$-M$	
Solution		250	300	0	0	0	0	50	0	
Δ_j		0	0	$\frac{-M-10}{4}$	0	$-M$	$-\frac{M}{2}+3$	0	$-\frac{M}{2}-3$	

In Simplex tableau 4, all Δ_j values are less than, or equal to zero. Hence, the solution is final. However, since an artificial variable is a basic variable, it is not feasible. Thus, the given problem has no feasible solution.

40. With slack variables S_1, S_2 and S_3, the problem may be written as:

Maximise $Z = 50x_1 + 110x_2 + 120x_3 + 0S_1 + 0S_2 + 0S_3$

Subject to

$$3x_1 + 3x_2 + 5x_3 + S_1 = 100$$
$$x_1 + 3x_2 + 4x_3 + S_2 = 80$$
$$2x_1 + 4x_2 + 3x_3 + S_3 = 60$$
$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	3	3	5*	1	0	0	100	20 ←
S_2	0	1	3	4	0	0	0	80	20
S_3	0	2	4	3	0	1	1	60	20
C_j		50	110	120	0	0	0		
Solution		0	0	0	100	80	60		
Δ_j		50	110	120	0	0	0		
				↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3	120	3/5	3/5	1	1/5	0	0	20	100/3
S_2	0	–7/5	3/5*	0	–4/5	1	0	0	0 ←
S_3	0	1/5	11/5	0	–3/5	0	1	0	0
C_j		50	110	120	0	0	0		
Solution		0	0	20	0	0	0	Z = 2,400	
Δ_j		–22	38 ↑	0	–24	0	0		

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3	120	2	0	1	1	–1	0	20	10
x_2	110	–7/3	1	0	–4/3	5/3	0	0	–
S_3	0	16/3*	0	0	7/3	–11/3	1	0	0 ←
C_j		50	110	120	0	0	0		
Solution		0	0	20	0	0	0	Z = 2,400	
Δ_j		200/3 ↑	0	0	80/3	–190/3	0		

Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_3	120	0	0	1	1/8	3/8	–3/8	20
x_2	110	0	1	0	–5/16	1/16	7/16	0
x_1	50	1	0	0	7/16	–11/16	1/16	0
C_j		50	110	120	0	0	0	Z = 2,400
Solution		0	0	20	0	0	0	
Δ_j		0	0	0	–5/2	–35/2	–25/2	

CHAPTER 4

1. Minimise $G = 10y_1 + 2y_2 + 6y_3$
   Subject to

$$y_1 + 2y_2 + 2y_3 \geq 1$$
$$y_1 - 2y_3 \geq -1$$
$$y_1 - y_2 + 3y_3 \geq 3$$
$$y_1, y_2, y_3 \geq 0$$

   y_1, y_2, and y_3 are the dual variables.

2. (a) Minimise $G = 10x_1 - 12x_2$
   Subject to

$$3x_1 + 2x_2 \geq 10$$
$$x_1 - 3x_2 \geq 8$$
$$2x_1 - x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

   (b) Maximise $G = -4y_1 + 13y_2 + 4y_3$
   Subject to

$$y_1 + 2y_2 + y_3 \geq -4$$
$$-2y_1 + 3y_2 - y_3 \geq 3$$
$$y_1, y_2, y_3 \geq 0$$

3. Minimise $G = 4a + 26b + 35c$
   Subject to

$$a + 3b + 10c \geq 5$$
$$a + 8b + 7c \geq 7$$
$$a, b, c \geq 0$$

   where, a, b, c are dual variables.

4. Minimise $G = 4y_1 + 6y_2 + 5y_3 - y_4$
   Subject to $y_1 + y_3 \geq 3$

$$y_2 + y_3 - y_4 \geq -2$$
$$y_1, y_2, y_3, y_4 \geq 0$$

5. Maximise $G = 5y_1 + 4y_2$
   Subject to

$$y_1 + 3y_2 \leq 2$$
$$4y_1 + y_2 \leq 9$$
$$2y_1 + 2y_2 = 3$$
$$y_1, y_2 \geq 0$$

6. Minimise $G = 10y_1 - 15y_2 + 7y_3$
   Subject to

$$y_1 - 4y_2 - y_3 \geq 3$$
$$y_1 + y_2 + y_3 \geq 4$$
$$y_1 + y_2 + y_3 = 7$$

   $y_1, y_2 \geq 0$ y_3: unrestricted in sign

7. Maximise $G = 2y_1^* + 6y_2 - 3y_3$
   Subject to

$$3y_1^* + 4y_2 - 4y_3 \leq 4$$
$$y_1^* + 3y_2 - 2y_3 \leq 1$$

   $y_2, y_3 \geq 0$; y_1^*: unrestricted in sign

8. (a) The dual is:

Minimise $G = 30y_1 + 10y_2 + 0y_3$

Subject to

$$2y_1 + y_2 + y_3 \geq 2$$
$$3y_1 + 2y_2 - y_3 \geq 3$$
$$y_1, y_2, y_3 \geq 0$$

(b) From the given information,

Solution	*Primal: Z-value*	*Dual: G-value*
1	$2 \times 10 + 3 \times 10/3 = 30$	$30 \times 0 + 10 \times 1 + 0 \times 1 = 10$
2	$2 \times 20 + 3 \times 10 = 70$	$30 \times 1 + 10 \times 4 + 0 \times 1 = 70$
3	$2 \times 10/3 + 3 \times 10/3 = 50/3$	$30 \times 1 + 10 \times 5/3 + 0 \times 1/3 = 200/3$

Solution 2 is optimal, therefore, since the values of Z and G are equal for this.

(c) The given problem is restated here with the following adjustments:

(i) Let $x_3 = x_4 - x_5$, where $x_4 \geq 0$, $x_5 \geq 0$.

(ii) The third constraint is multiplied by –1 to convert into ≤ type.

(iii) The first constraint is replaced by a pair of constraints as $x_1 + x_2 + 3x_3 \leq 10$ and $x_1 + x_2 + 3x_3 \geq 10$.

The second of these is then multiplied by –1. The multiplication converts the constraint into ≤ type.
The primal and dual are stated here:

Primal:

Maximise $Z = 7x_1 + 5x_2 - 2x_4 + 2x_5$

Subject to

$$x_1 + x_2 + 3x_4 - 3x_5 \leq 10$$
$$-x_1 - x_2 - 3x_4 + 3x_5 \leq -10$$
$$2x_1 - x_2 + 3x_4 - 3x_5 \leq 16$$
$$-3x_1 - x_2 + 2x_4 - 2x_5 \leq 0$$
$$x_1, x_2, x_4, x_5 \geq 0$$

Dual:

Minimise $G = 10y_1 + 10y_2 + 16y_3 + 0y_4$

Subject to

$$y_1 - y_2 + 2y_3 - 3y_4 \geq 7$$
$$y_1 - y_2 - y_3 - y_4 \geq 5$$
$$3y_1 - 3y_2 + 3y_3 + 2y_4 \geq -2$$
$$-3y_1 + 3y_2 - 3y_3 - 2y_4 \geq 2$$
$$y_1, y_2, y_3, y_4 \geq 0$$

Now, putting $y_1 - y_2 = y$ and combining the last two constraints to replace by one involving '=' sign, we can rewrite the dual as:

Minimise $G = 10y + 16y_3 + 0y_4$

Subject to

$$y + 2y_3 - 3y_4 \geq 7$$
$$y - y_3 - y_4 \geq 5$$
$$3y + 3y_3 + 2y_4 = 2$$
$$y_3, y_4 \geq 0, \; y \text{ unrestricted in sign}$$

9. (i) The resource availability in the three production processes I, II and III is 15 × 200 = 3,000; 30 × 200 = 6,000; and 15 × 200 = 3,000 hours respectively. If x_1, x_2 and x_3 be the output of the models A, B and C respectively, the problem is:

Maximise $Z = 7{,}500x_1 + 15{,}000x_2 + 30{,}000x_3$

Subject to

$$60x_1 + 100x_2 + 200x_3 \leq 3{,}000$$

$$100x_1 + 240x_2 + 360x_3 \le 6{,}000$$
$$80x_1 + 100x_2 + 160x_3 \le 3{,}000$$
$$x_1, x_2, x_3 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	60	100	200	1	0	0	3,000	15
S_2	0	100	240	360	0	1	0	6,000	50/3 ←
S_3	0	80	100	160	0	0	1	3,000	75/4
C_j		7,500	15,000	30,000	0	0	0		
Solution		0	0	0	3,000	6,000	3,000	$Z = 0$	
Δ_j		7,500	15,000	30,000 ↑	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3	30,000	3/10	1/2	1	1/200	0	0	15	30
S_2	0	–8	60	0	–9/5	1	0	600	10 ←
S_3	0	32	20	0	–4/5	0	1	600	30
C_j		7,500	15,000	30,000	0	0	0		
Solution		0	0	15	0	600	600	$Z = 450{,}000$	
Δ_j		–1,500	0 ↑	0	–150	0	0		

From Tableau 2, the optimal product mix is: Models *A* and *B*: none, and Model *C*: 15 cars. Maximum profit obtainable is Rs 450,000.

(ii) The shadow prices of the resources are:

Process I: Rs 150/worker-day

Process II and III: nil

Range of validity:

Process I : 0 – 10,000/3 [3,000 – 3,000, 3,000 – (–100/3)]

Process II : 5,400 – ∞ [6,000 – 600]

Process III : 2,400 – ∞ [3,000 – 600]

(iii) Let y_1, y_2 and y_3 be the dual variables. The dual is:

Minimise $G = 3{,}000y_1 + 6{,}000y_2 + 3{,}000y_3$

Subject to

$$60y_1 + 100y_2 + 80y_3 \ge 7{,}500$$
$$100y_1 + 240y_2 + 100y_3 \ge 15{,}000$$
$$200y_1 + 360y_2 + 160y_3 \ge 30{,}000$$
$$y_1, y_2, y_3 \ge 0$$

Optimal solution to the dual is: $y_1 = 150$, $y_2 = y_3 = 0$.

(iv) The optimal solution in Simplex Tableau 2 is not unique. An alternate optimal is given in Simplex Tableau 3.

Simplex Tableau 3: Alternate Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_3	30,000	11/30	0	1	1/50	–1/120	0	10
x_2	15,000	–2/15	1	0	–3/100	1/60	0	10
S_3	0	104/3	0	0	–1/5	–1/3	1	400
C_j		7,500	15,000	30,000	0	0	0	
Solution		0	10	10	0	0	400	Z = 450,000
Δ_j		–1,500	0	0	–150	0	0	

10. (a) Let x_1, x_2 and x_3 be the number of Tables, Chairs and Book cases to be produced. The LPP is:

Maximise $Z = 30x_1 + 20x_2 + 12x_3$

Subject to
$$8x_1 + 4x_2 + 3x_3 \leq 640$$
$$4x_1 + 6x_2 + 2x_3 \leq 540$$
$$x_1 + x_2 + x_3 \leq 100$$
$$x_1, x_2, x_3 \geq 0$$

(b) Let S_1, S_2 and S_3 be the slack variables.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	8	4	3	1	0	0	640	80 ←
S_2	0	4	6	2	0	1	0	540	135
S_3	0	1	1	1	0	0	1	100	100
C_j		30	20	12	0	0	0		
Solution		0	0	0	640	540	100	Z = 0	
Δ_j		30	20	12	0	0	0		
		↑							

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_1	30	1	1/2	3/8	1/8	0	0	80	160
S_2	0	0	4	1/2	–1/2	1	0	220	55
S_3	0	0	1/2	5/8	–1/8	0	1	20	40 ←
C_j		30	20	12	0	0	0		
Solution		80	0	0	0	220	20	Z = 2,400	
Δ_j		0	5	3/4	–15/4	0	0		
			↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_1	30	1	0	–1/4	1/4	0	–1	60
S_2	0	0	0	–9/2	1/2	1	–8	60
x_2	20	0	1	5/4	–1/4	0	2	40
C_j		30	20	12	0	0	0	
Solution		60	40	0	0	60	0	Z = 2,600
Δ_j		0	0	–11/2	–5/2	0	–10	

Optimal product mix is: Tables = 60, Chairs = 40, Book Cases = 0.
Maximum profit contribution = Rs 2,600.

(c) Shadow prices of resources:
Timber: Rs 2.50 per cubic foot
Assembly Department man-hours: Nil
Finishing Department = Rs 10 per man-hour

(d) Sensitivity of the optimal solution:
For Tables (x_1): Rs 20 – 40 [30 – 10, 30 + 10]
For Chairs (x_2): Rs 15.60 –30 [20 – 4.5, 20 + 10]
For Resources:
Timber: [640 – 120, 640 – (–160)] i.e. 520 – 800
Assembly: [540 – 60, ∞] i.e. 480 – ∞
Finishing: [100 – 20, 100 – (–7.5) i.e. 80 – 107.50

(e) Other information:
1. The optimal product-mix does not include book cases. Its production will result in a net loss of Rs 11/2 per unit.
2. The optimal solution is unique.

11. (a) Let x_1 and x_2 be the output of products A and B respectively.

Maximise $Z = 800x_1 + 500x_2$

Subject to
$$2x_1 + 3x_2 \le 42$$
$$7x_1 + 7x_2 \le 70$$
$$7x_1 + 5x_2 \le 70$$
$$x_1, x_2 \ge 0$$

With slack variables S_1, S_2 and S_3, the solution follows:

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	3	1	0	0	42	21
S_2	0	7	7	0	1	0	70	10 ←
S_3	0	7	5	0	0	1	70	10
C_j		800	500	0	0	0		
Solution		0	0	42	70	70		
Δ_j		800	500	0	0	0		
		↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
S_1	0	0	1	1	–2/7	0	22
x_1	800	1	1	0	1/7	0	10
S_3	0	0	–2	0	–1	1	0
C_j		800	500	0	0	0	
Solution		10	0	22	0	0	Z = 8,000
Δ_j		0	–300	0	–800/7	0	

Optimum output : Product A - 10 units
Product B - Nil

with y_1, y_2 and y_3 as the dual variables, the dual is:

Minimise $G = 42y_1 + 70y_2 + 70y_3$

Subject to

$2y_1 + 7y_2 + 7y_3 \geq 800$

$3y_1 + 7y_2 + 5y_3 \geq 500$

$y_1, y_2, y_3 \geq 0$

The optimal solution is degenerate. The third of the constraints here is redundant in terms of the solution obtained. Accordingly, the shadow price of zero for department III capacity is valid from 70 to infinity, while for department II capacity, the shadow price of Rs 114.29 is valid from 0 to 70. Any reduction in capacity of department upto 22 hours and any increase in it would not cause a change in profit.

12. (a) From the given information, the LPP may be stated as:

Maximise $Z = 124x + 80y$

Subject to $150x + 90y \leq 13{,}500$

$100x + 120y \leq 12{,}000$

$y \leq 75$

$x, y \geq 0$

The constraints are plotted graphically in adjacent figure. The feasible region is bounded by the points *OABCD*. An evaluation of these points yields optimal solution at *C* where $x = 60$ and $y = 50$. Thus, we have

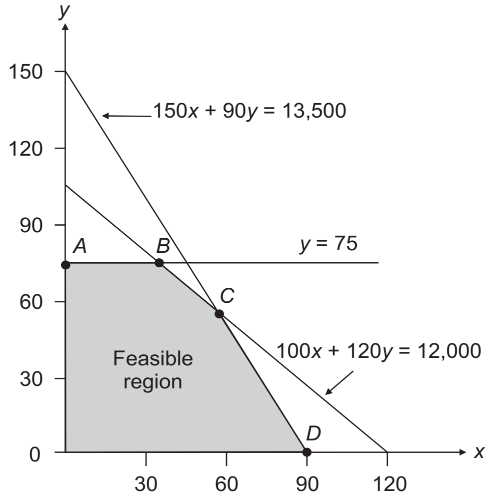
Revised contribution

$= 60 \times 124 + 50 \times 80 =$ Rs 11,440

Current contribution

$= 30 \times 124 + 75 \times 80 =$ Rs 9,720

$\therefore$ Increase = Rs 1,720



Graphic determination of optimal mix

(b) Since the sale of *y* is restricted to 75, increased capacity may be used to maximise production of product *X*. From figure above, the maximum output of *X* = 120 units.

	Hours required	*Hours available*	*Additional hours*
Deptt. 1: $\frac{120 \times 150}{60}$	300	225	75
Deptt. 2: $\frac{120 \times 100}{60}$	200	200	—

	Contribution = 120 × 124 =	Rs 14,880.0
less	Additional cost = 75 × 0.5 =	37.5
		14,842.5
less	Contribution per (a) above	11,440.0
	Increased contribution	3,402.5

Suggestion: Increase deptt. 1 hour by 75.

(c)

	Department 1	*Department 2*
Total hours	225	200
Hours needed for 30 units of *Y*	45	60
Hours available for *X*	180	140
Hours per unit of *X*	2.5	5/3
Output of *X*	72 units	84 units

Thus, maximum output of X = 72 units
Revised contribution = 124 × 72 + 80 × 30
= Rs 11,328
Contribution as per (a) = Rs 11,440
∴ Decrease in contribution = Rs 11,440 – 11,328
= Rs 112

Now,

Sales quota for Y = 75
Production as per (a) = 50
Unsold quota = 25

Thus, under the product plan determined in (a), 25 units of the quota remain unsold. Since the company derives no benefit from this element of the quota at present, it could be sold for a minimum price of zero. The remaining 20 units of quota (50 – 30) should be sold to negate the decrease in contribution of Rs 112.

Thus, minimum price = $\frac{\text{Rs } 112}{20}$ = Rs 5.60 per unit.

13. Let x_1, x_2 and x_3 be the number of units of lamps A, B and C produced. Using the given information, we may state the LPP as follows:

Maximise $Z = 120x_1 + 190x_2 + 210x_3$

Subject $0.1x_1 + 0.2x_2 + 0.3x_3 \leq 80$
$0.2x_1 + 0.3x_2 + 0.4x_3 \leq 120$
$0.1x_1 + 0.1x_2 + 0.1x_3 \leq 100$
$x_1, x_2, x_3 \geq 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0.1	0.2	0.3*	1	0	0	80	800/3 ←
S_2	0	0.2	0.3	0.4	0	1	0	120	300
S_3	0	0.1	0.1	0.1	0	0	1	100	1,000
C_j		120	190	210	0	0	0		
Solution		0	0	0	80	120	100	$Z = 0$	
Δ_j		120	190	210	0	0	0		
				↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3	210	0.33	0.67	1.00	3.33	0.00	0.00	266.67	400
S_2	0	0.07	0.03*	0.00	–1.33	1.00	0.00	13.33	400 ←
S_3	0	0.07	0.03	0.00	– 0.33	0.00	1.00	73.33	2,200
C_j		120	190	210	0	0	0		
Solution		0	0	266.67	0	13.33	73.33	$Z = 56{,}000$	
Δ_j		50	50	0	700	0	0		
			↑						

Simplex Tableau 3: Non-optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3 210	–1.00	0.00	1.00	30.00*	–20.00	0.00	0.00	0 ←
x_2 190	2.00	1.00	0.00	–40.00	30.00	0.00	400.00	—
S_3 0	0.00	0.00	0.00	1.00	–1.00	1.00	60.00	60
C_j	120	190	210	0	0	0		
Solution	0	400	0	0	0	60	Z = 76,000	
Δ_j	–50	0	0	1,300	–1,500	0		
				↑				

Simplex Tableau 4: Optimal Solution

Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i
S_1 0	–0.03	0.00	0.03	1.00	–0.67	0.00	0.00
x_2 190	0.67	1.00	1.33	0.00	3.33	0.00	400.00
S_3 0	0.03	0.00	–0.03	–0.00	–0.33	1.00	60.00
C_j	120	190	210	0	0	0	
Solution	0	400	0	0	0	60	Z = 76,000
Δ_j	–6.67	0.00	–43.33	0.00	–633.33	0.00	

Thus, optimal product mix is: Model *A*—nil, Model *B*—400 and Model *C*—nil.

The shadow prices of the resources are given by Δ_j values for the slack variables in Simplex Tableau 4. These are: Assembly: nil; Wiring: Rs 633.33 per hour, and Packaging: nil.

Dual: Let y_1, y_2 and y_3 be the dual variables. The dual is:

Minimise $G = 80y_1 + 120y_2 + 100y_3$

Subject to
$$0.1y_1 + 0.2y_2 + 0.1y_3 \geq 120$$
$$0.2y_1 + 0.3y_2 + 0.1y_3 \geq 190$$
$$0.3y_1 + 0.4y_2 + 0.1y_3 \geq 210$$
$$y_1, y_2, y_3 \geq 0$$

For the optimal solution to the primal, we can obtain optimal values of the dual variables as:
$$y_1 = 0,\ y_2 = 633.33 \text{ and } y_3 = 0$$

With these, the objective function value of the dual problem is:
$$G = 80 \times 0 + 120 \times 633.33 + 100 \times 0 = 76{,}000$$

which is identical to the objective function value of the dual.

14. Let the output of waste cans, filing cabinets, correspondence boxes, and lunch boxes be x_1, x_2, x_3 and x_4 respectively. From the given information, we may state the LPP as follows:

Maximise $Z = 20x_1 + 400x_2 + 90x_3 + 20x_4$ — Revenue

Subject to
$$6x_1 + 2x_3 + 3x_4 \leq 225 \quad \text{Sheet metal } A$$
$$10x_2 \leq 300 \quad \text{Sheet metal } B$$
$$4x_1 + 8x_2 + 2x_3 + 3x_4 \leq 190 \quad \text{Labour}$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Dual: The dual, with variables y_1, y_2 and y_3 is:

Minimise $G = 225y_1 + 300y_2 + 190y_3$

Subject to
$$6y_1 + 4y_3 \geq 20$$
$$10y_2 + 8y_3 \geq 400$$
$$2y_1 + 2y_3 \geq 90$$
$$3y_1 + 3y_3 \geq 20$$
$$y_1, y_2, y_3 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1 0	6	0	2	3	1	0	0	225	—
S_2 0	0	10	0	0	0	1	0	300	30
S_3 0	4	8*	2	3	0	0	1	190	95/4 ←
C_j	20	400	90	20	0	0	0		
Solution	0	0	0	0	225	300	190	$Z = 0$	
Δ_j	20	400 ↑	90	20	0	0	0		

Simplex Tableau 2: Optimal Solution

Basis	x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i
S_1 0	6	0	2	3	1	0	0	225
S_2 0	–5	0	–5/2	–15/2	0	1	–5/4	125/2
x_2 400	1/2	1	1/4	3/8	0	0	1/8	95/4
C_j	20	400	90	20	0	0	0	
Solution	0	95/4	0	0	225	125/2	0	$Z = 9,500$
Δ_j	—180	0	–10	–130	0	0	–50	

Thus, the optimal solution is: $x_1 = 0$, $x_2 = 95/4$, $x_3 = x_4 = 0$, for $Z = 9,500$. From the Δ_j row of the tableau, the optimal values of the dual variables may be obtained from the columns of slack variables. Thus, $y_1 = 0$, $y_2 = 0$, and $y_3 = 50$, and $G = 9,500$, the same as that of the primal problem.

15. (a) To obtain the objective function (to maximise total contribution), we first need to calculate contribution margin for each tonne of the different products. This, in turn, requires the calculation of cost of materials. The material cost per tonne is shown calculated below:

$$X_1: 0.1 \times 150 + 0.1 \times 60 + 0.2 \times 120 + 0.6 \times 10 = \text{Rs } 51$$
$$X_2: 0.1 \times 150 + 0.2 \times 60 + 0.1 \times 120 + 0.6 \times 10 = \text{Rs } 45$$
$$X_3: 0.1 \times 150 + 0.1 \times 60 + 0.1 \times 120 + 0.6 \times 10 = \text{Rs } 54$$

Now, since Contribution = Selling price – (Material cost + Manufacturing cost), we have

Product	*Contribution per tonne*
X_1	83 – (51 + 11) = Rs 21
X_2	81 – (45 + 11) = Rs 25
X_3	81 – (54 + 11) = Rs 16

The LPP may be stated as follows:

Maximise $Z = 21X_1 + 25X_2 + 16X_3$

Subject to

$$0.1X_1 + 0.1X_2 + 0.2X_3 \le 1,200$$
$$0.1X_1 + 0.2X_2 + 0.1X_3 \le 2,000$$
$$0.2X_1 + 0.1X_2 + 0.1X_3 \le 2,200$$
$$X_1, X_2, X_3 \ge 0$$

(b) The slack variables X_4, X_5 and X_6 represent the amount of scarce resources, namely, nitrate, phosphate, and potash respectively, remaining unutilised. They convert inequalities into equations. Thus,

$$0.1X_1 + 0.1X_2 + 0.2X_3 + X_4 = 1,200$$
$$0.1X_1 + 0.2X_2 + 0.1X_3 + X_5 = 2,000$$
$$0.2X_1 + 0.1X_2 + 0.1X_3 + X_6 = 2,200$$
$$X_1, X_2, X_3, X_4, X_5, X_6 \ge 0$$

Simplex Tableau

Basis		X_1	X_2	X_3	X_4	X_5	X_6	b_i
X_4	0	0.1	0.1	0.2	1	0	0	1,200
X_5	0	0.1	0.2	0.1	0	1	0	2,000
X_6	0	0.2	0.1	0.1	0	0	1	2,200
C_j		21	25	16	0	0	0	
Solution		0	0	0	1,200	2,000	2,200	
Δ_j		21	25	16	0	0	0	

(c) The entering variable is the one with largest Δ_j value. In this case, it is X_2, with $\Delta_j = 25$. To determine the leaving variable, we first calculate replacement ratios b_i/a_{ij} (using the a_{ij} values of the entering variable). This gives the values as 1,200/0.1 = 12,000; 2,000/0.2 = 10,000; and 2,200/0.1 = 22,000. The smallest non-negative ratio being 10,000, the leaving variable is X_5.

(d) It is evident from the given table that

(i) Optimal product-mix is: 4,000 tonnes of X_1
8,000 tonnes of X_2
nil of X_3

(ii) Total contribution = 21 × 4,000 + 25 × 8,000 + 16 × 0 = Rs 284,000 per month

(iii) The optimal mix uses all the nitrate and phosphate but leaves 600 tonnes of potash unused.

(iv) The shadow prices of the resources are:
Nitrate : Rs 170 per tonne
Phosphate : Rs 40 per tonne
Potash : Nil

(v) Each tonne of X_3 produced would reduce the contribution by Rs 22.

(e) (i) Since nitrate availability is constraining the solution, any increase in its availability will change the optimal solution. The elements in the column headed X_4 in the final tableau show the changes which will result from each extra tonne of nitrate per month. With 100 extra tonnes per month, the new values will be

X_1: 4,000 + (20 × 100) = 6,000
X_2: 8,000 + (–10 × 100) = 7,000
X_6: 600 + (–3 × 100) = 300
Z: 284,000 + (170 × 100) = 301,000

Thus, the new optimal solution is to make 6,000 tonnes of X_1 and 7,000 tonnes of X_2 per month for a contribution of Rs 301,000.

(ii) Under the current optimal policy, X_3 is not produced, as its production would reduce the total contribution at the rate of Rs 22 per tonne. The changes in the solution for each tonne of X_3 produced and sold every month are given by elements in the column headed X_3, of the optimal solution tableau. The changes resulting from 200 tonnes would be:

X_1: 4,000 – (3 × 200) = 3,400
X_2: 8,000 – (–1 × 200) = 8,200
X_6: 600 – (–0.4 × 200) = 680
Z: 284,000 – (22 × 200) = 279,600

Thus, production of 200 tonnes of X_3 would mean production of 3,400 tonnes of X_1, 8,200 tonnes of X_2 (besides, of course, 200 tonnes of X_3) for a contribution of Rs 279,600.

16.

Variable	y_1	y_2	y_3	y_4	y_5	y_6
Solution	0	4/15	1/15	0	0	0
Δ_j	–80/3	0	0	–160/3	–8/3	–20/3

Objective function value = 76/3.

17. (a) Let the quantity of scrap metal purchased from suppliers X and Y be x_1 and x_2 quintals, respectively. With the given information, the LPP may be stated as:

Minimise $C = 2x_1 + 4x_2$

Subject to

$$x_1 + x_2 \geq 200$$
$$\frac{1}{4}x_1 + \frac{3}{4}x_2 \geq 100$$
$$\frac{1}{10}x_1 + \frac{1}{5}x_2 \leq 35$$
$$x_1, x_2 \geq 0$$

(b) The dual of the problem is given below:

Maximise $Z = 200y_1 + 100y_2 - 35y_3$

Subject to

$$y_1 + \frac{1}{4}y_2 + \frac{1}{10}y_3 \leq 2$$
$$y_1 + \frac{3}{4}y_2 + \frac{1}{5}y_3 \leq 4$$
$$y_1, y_2, y_3 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		y_1	y_2	y_3	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	1*	1/4	–1/10	1	0	2	2 ←
S_2	0	1	3/4	–1/5	0	1	4	4
C_j		200	100	–35	0	0		
Solution		0	0	0	2	4		
Δ_j		200 ↑	100	–35	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		y_1	y_2	y_3	S_1	S_2	b_i	b_i/a_{ij}
y_1	200	1	1/4	–1/10	1	0	2	8
S_2	0	0	1/2*	–1/10	–1	1	2	4 ←
C_j		200	100	–35	0	0		
Solution		2	0	0	0	2	$Z = 400$	
Δ_j		0 ↑	50	–15	–200	0		

Simplex Tableau 3: Optimal Solution

Basis		y_1	y_2	y_3	S_1	S_2	b_i
y_1	200	1	0	–1/10	3/2	–1/2	1
y_2	100	0	1	–1/5	–2	2	4
C_j		200	100	–35	0	0	
Solution		1	4	0	0	0	$Z = 600$
Δ_j		0	0	–15	–100	–100	

In Simplex Tableau 3, the Δ_j values of the slack variables are equal to –100 and –100. Thus, the solution to the primal problem as would minimise the total cost is:

$$x_1 = 100 \quad \text{and} \quad x_2 = 100$$

which gives total cost as 2 × 100 + 4 × 100 = Rs 600

18. (a) The linear programming model, using the given notation, is stated below:

Maximise	$Z = 400x_1 + 200x_2 + 100x_3$	Contribution
Subject to	$2x_1 + 3x_2 + 2.5x_3 \leq 1{,}920$	Process 1
	$3x_1 + 2x_2 + 2x_3 \leq 2{,}200$	Process 2
	$x_1 \leq 200$	Alpha sales
	$x_1, x_2, x_3 \geq 0$	

(b) Initial simplex tableau is given here:

Initial Simplex Tableau

Basis		x_1	x_2	x_3	x_4	x_5	x_6	b_I
x_4	0	2	3	2.5	1	0	0	1,920
x_5	0	3	2	2	0	1	0	2,200
x_6	0	1	0	0	0	0	1	200
C_j		400	200	100	0	0	0	
Solution		0	0	0	1,920	2,200	200	

The slack variable x_4 represents unused hours of process 1; x_5 represents unused hours of process 2; and x_6 indicates the unused sales potential.

(c) The b_i column gives the optimum production plan:
Alpha (x_1): 200 units; Beta (x_2): 506.7 units and Gamma (x_3): nil. The total contribution margin, Z = Rs 181,333.3
Resource utilisation is:
Process 1: all hours used.
Process 2: 586.7 hours unused.
The shadow prices in the last row indicate the following:
(i) x_3: 66.7 implies that any Gamma produced would lead to a fall of Rs 66.67 per unit.
(ii) x_4: 66.7 means that extra hours in process 1 would increase contribution by Rs 66.67 per hour.
(iii) x_6: 266.7 signifies that every Alpha sale above 200 would increase contribution by Rs 266.7.

(d) (i) For an increase of 20 hours in process 1, we may use values under column x_4 as multipliers to get the revised values. This is shown below:

Variable	*Original value*	*Multiplier*	*Revised value*	
x_2	506.7	0.33	506.7 + 0.33 × 20 =	513.3
x_5	586.7	–0.67	586.7 + (–0.67 × 20) =	573.3
x_1	200.0	0.00	200.0 + (0 × 20) =	200.0
Z	181,333.3	66.70	181,333.3 + (66.7 × 20) =	182,666.7

Thus, an increase of 20 hours in process 1 leads to an increase in contribution by Rs 1,333.4. Output of Beta (x_2) increases by 6.6 units and 13.4 more hours of process 2 will be used.

(ii) For an increase of 10 units Alpha (x_1) production, we used values given in column headed x_6.

Variable	Original value	Multiplier	Revised value	
x_2	506.7	−0.67	506.7 + (−0.67 × 10) =	500.3
x_5	586.7	−0.67	586.7 + (−0.67 × 10) =	570.3
x_1	200.0	1.00	200.0 + (1 × 10) =	210.0
Z	181,333.3	266.70	181,333.3 + 266.7 × 10 =	184,000.3

Accordingly, contribution increases by Rs 2,667 to Rs 184,000.3. Production of Beta (x_2) increases by 6.7 units, production of alpha by 10 units, and 16.7 more process 2 hours will be used.

(iii) For introducing 10 units of Gamma, the contribution reduces by Rs 666.7 to Rs 180,666.7, output of Beta (x_2) falls by 8.3 units, and 3.3 more process 2 hours will be used. The calculations, using a_{ij} values of column x_3, are given below:

Variable	Original value	Multiplier	Revised value	
x_2	506.7	0.83	506.7 − (0.83 × 10) =	498.4
x_5	586.7	0.33	586.7 − (0.33 × 10) =	583.4
x_1	200.0	0.00	200.0 − (0 × 10) =	200.0
Z	181,333.3	66.67	181,333.3 − 66.67 × 10 =	180,666.6

19. (a) Let x_1 and x_2 be the number of units of foods F_1 and F_2 respectively, purchased by the housewife. The LPP is:

Minimise $Z = 3x_1 + 2x_2$ Total cost

Subject to

$14x_1 + 4x_2 \geq 60$ Vitamin A

$10x_1 + 8x_2 \geq 40$ Vitamin B

$4x_1 + 16x_2 \geq 32$ Vitamin C

$x_1, x_2 \geq 0$

(b) Let y_1, y_2 and y_3 be the dual variables. The dual is:

Maximise $G = 60y_1 + 40y_2 + 32y_3$

Subject to

$14y_1 + 10y_2 + 4y_3 \leq 3$

$4y_1 + 8y_2 + 16y_3 \leq 2$

$y_1, y_2, y_3 \geq 0$

The optimal values of the dual variables y_1, y_2 and y_3 would indicate the imputed values of one unit of each of vitamins A, B and C respectively. Obviously, the total value imputed to 14 units of A, 10 units of B, and 4 units of C should not exceed Rs 3 because each unit of food F_1 contains as much quantity of the three vitamins and costs Rs 3. Similarly, a unit of food F_2 contains, respectively, 4, 8 and 16 units of vitamins A, B and C, and costs Rs 2. Thus, the combined imputed value of these quantities of vitamins should not exceed Rs 2. The total value would equal $60y_1 + 40y_2 + 32y_3$, the maximum.

(c) The solution to the dual is given here.

Simplex Tableau 1: Non-optimal Solution

Basis		y_1	y_2	y_3	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	14*	10	4	1	0	3	3/14 ←
S_2	0	4	8	16	0	1	2	1/2
C_j		60	40	32	0	0		
Solution		0	0	0	3	2		
Δ_j		60 ↑	40	32	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		y_1	y_2	y_3	S_1	S_2	b_i	b_i/a_{ij}
y_1	60	1	5/7	2/7	1/14	0	3/14	3/4
S_2	0	0	36/7	104/7*	–2/7	1	8/7	1/13 ←
C_j		60	40	32	0	0		
Solution		3/14	0	0	0	8/7	$G = 90/7$	
Δ_j		0	–20/7	104/7	–30/7	0		
				↑				

Simplex Tableau 3: Optimal Solution

Basis		y_1	y_2	y_3	S_1	S_2	b_i
y_1	60	1	8/13	0	1/13	–1/52	5/26
y_3	32	0	9/26	1	–1/52	7/104	1/13
C_j		60	40	32	0	0	
Solution		5/26	0	1/13	0	0	$G = 14$
Δ_j		0	–8	0	–4	–1	

Thus, optimal solution to this problem is:

$$y_1 = 5/26,\ y_2 = 0, \text{ and } y_3 = 1/13.$$

The optimal values of the variables of the primal problem are obtained from the Δ_j row as follows:

$$x_1 = 4 \quad \text{and} \quad x_2 = 1, \quad \text{and} \quad Z = 3 \times 4 + 2 \times 1 = 14$$

The objective function values for the primal and the dual are both seen to be equal, at 14.

20. Let the output be: x_1 cases of 60-watt soft-light bulbs, x_2 cases if 60-watt regular bulbs and x_3 cases of 100-watt bulbs. The LPP is:

Maximise $\quad Z = 70x_1 + 50x_2 + 50x_3$

Subject to

$$x_1 + x_2 + x_3 \le 25$$
$$2x_1 + x_2 + x_3 \le 40$$
$$x_1 + x_2 \le 25$$
$$x_3 \le 60$$
$$x_1, x_2, x_3 \ge 0$$

Let S_1, S_2, S_3 and S_4 be the slack variables.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	1	1	1	1	0	0	0	25	25
S_2	0	2*	1	1	0	1	0	0	40	20 ←
S_3	0	1	1	0	0	0	1	0	25	25
S_4	0	0	0	1	0	0	0	1	60	—
C_j		70	50	50	0	0	0	0		
Solution		0	0	0	25	40	25	60	$Z = 0$	
Δ_j		70	50	50	0	0	0	0		
		↑								

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	0	1/2*	1/2	1	–1/2	0	0	5	10 ←
x_1	70	1	1/2	1/2	0	1/2	0	0	20	40
S_3	0	0	1/2	–1/2	0	–1/2	1	0	5	10
S_4	0	0	0	1	0	0	0	1	60	—
C_j		70	50	50	0	0	0	0		
Solution		20	0	0	5	0	5	60	Z = 1,400	
Δ_j		0	15 ↑	15	0	–35	0	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i
x_2	50	0	1	1	2	–1	0	0	10
x_1	70	1	0	0	–1	1	0	0	15
S_3	0	0	0	–1	–1	0	1	0	0
S_4	0	0	0	1	0	0	0	1	60
C_j		70	50	50	0	0	0	0	
Solution		15	10	0	0	0	0	60	Z = 1,550
Δ_j		0	0	0	–30	–20	0	0	

The solution in Simplex Tableau 3 is optimal. This is:
$x_1 = 15$, $x_2 = 10$ and $x_3 = 0$. The solution is degenerated.
The dual is:

Minimise $G = 25y_1 + 40y_2 + 25y_3 + 60y_4$

Subject to
$$y_1 + 2y_2 + y_3 \geq 70$$
$$y_1 + y_2 + y_3 \geq 50$$
$$y_1 + y_2 + y_4 \geq 50$$
$$y_1, y_2, y_3, y_4 \geq 0$$

Here the dual variables y_1, y_2, y_3 and y_4 indicate the following:

y_1: Imputed value of line 1 per hour
y_2: Imputed value of line 2 per hour
y_3: Imputed value of the combined demand for 60-watt soft lite and 60-watt regular bulbs
y_4: Imputed value of the demand for 100-watt bulbs

From the optimal solution tableau, the optimal values of the dual variables are:
y_1 = Rs 30/hour, y_2 = Rs 20/hour, y_3 = nil and y_4 = nil.

21. Let x_1, x_2 and x_3 be the number of shipments (per 100 units) of transistors, resistors, and electron tubes respectively. According to the given data, the problem is:

Maximise $Z = 100x_1 + 60x_2 + 40x_3$ Total profit

Subject to

$x_1 + x_2 + x_3 \leq 100$ Engineering time
$10x_1 + 4x_2 + 5x_3 \leq 600$ Labour time
$2x_1 + 2x_2 + 6x_3 \leq 300$ Administration time
$x_1, x_2, x_3 \geq 0$

To determine the optimal mix, we solve this problem by simplex method. The variables S_1, S_2 and S_3 are the slack variables used to convert the constraints into '=' type.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	1	1	1	1	0	0	100	100
S_2	0	10*	4	5	0	1	0	600	60 ←
S_3	0	2	2	6	0	0	1	300	150
C_j		100	60	40	0	0	0		
Solution		0	0	0	100	600	300		
Δ_j		100	60	40	0	0	0		
		↑							

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	3/5*	1/2	1	–1/10	0	40	200/3 ←
x_1	100	1	2/5	1/2	0	1/10	0	60	150
S_3	0	0	6/5	5	0	–1/5	1	180	150
C_j		100	60	40	0	0	0		
Solution		60	0	0	40	0	180	Z = 6,000	
Δ_j		0	20	–10	0	–10	0		
			↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_2	60	0	1	5/6	5/3	–1/6	0	200/3
x_1	100	1	0	1/6	–2/3	1/6	0	100/3
S_3	0	0	0	4	–2	0	1	100
C_j		100	60	40	0	0	0	
Solution		100/3	200/3	0	0	0	100	Z = 22,000/3
Δ_j		0	0	–80/3	–100/3	–20/3	0	

Thus, optimal mix is: $x_1 = 100/3$, $x_2 = 200/3$, and $x_3 = 0$.
The maximum profit = 100 × 100/3 + 60 × 200/3 + 40 × 0 = Rs 22,000/3 or Rs 7,333.33.
Dual: The dual to the given LPP is given below.

Minimise $G = 100y_1 + 600y_2 + 300y_3$

Subject to

$$y_1 + 10y_2 + 2y_3 \geq 100$$
$$y_1 + 4y_2 + 2y_3 \geq 60$$
$$y_1 + 5y_2 + 6y_3 \geq 40$$
$$y_1, y_2, y_3 \geq 0$$

From the information given in Simplex Tableau 3, it may be observed that the marginal profitability of the three rersources is: Engineering time: Rs 100/3 per hour; Labour time: Rs 20/3 per hour and Administrative time: nil. These are the minimum rentals that this firm would seek if the capacity were to be rented out. The minimum total rental would be: 100 × 100/3 + 600 × 20/3 + 300 × 0 = Rs 22,000/3, the same as the maximum profit.

22. (a) Profit per unit:

Product A; Rs 15 – 11 = Rs 4

Product B; Rs 20 – 12 = Rs 8

Product C; Rs 16 – 10 = Rs 6

If x_1 units of product A, x_2 units of product B, and x_3 units of product C are produced, the LPP may be stated as follows:

Maximise $Z = 4x_1 + 8x_2 + 6x_3$

Subject to

$$x_1 + 3x_2 + 2x_3 \leq 160$$
$$3x_1 + 4x_2 + 2x_3 \leq 120$$
$$2x_1 + x_2 + 2x_3 \leq 80$$
$$x_1, x_2, x_3 \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	3	2	1	0	0	160	160/3
S_2	0	3	4*	2	0	1	0	120	30 ←
S_3	0	2	1	2	0	0	1	80	80
C_j		4	8	6	0	0	0		
Solution		0	0	0	160	120	80		
Δ_j		4	8	6	0	0	0		
			↑						

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	–5/4	0	1/2	1	–3/4	0	70	140
x_2	8	3/4	1	1/2	0	1/4	0	30	60
S_3	0	5/4	0	3/2*	0	–1/4	0	50	100/3 ←
C_j		4	8	6	0	0	0		
Solution		0	30	0	70	0	50	$Z = 240$	
Δ_j		–2	0	2	0	–2	0		
				↑					

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
S_1	0	–5/3	0	0	1	–2/3	–1/3	160/3
x_2	8	1/3	1	0	0	1/3	–1/3	40/3
x_3	6	5/6	0	1	0	–1/6	2/3	100/3
C_j		4	8	6	0	0	0	
Solution		0	40/3	100/3	160/3	0	0	$Z = 920/3$
Δ_j		–11/3	0	0	0	–5/3	–4/3	

Thus, optimal solution to the problem is:

Product A: nil,

Product B: 40/3 units, and

Product C: 100/3 units.

Total profit = 8 × 40/3 + 6 × 100/3 = Rs 920/3.

(b) Shadow prices are:

Centre 1 : nil

Centre 2 : Rs 5/3 per hour

Centre 3 : Rs 4/3 per hour

(c) RHS Ranging :

b_i	a_{ij} *for* S_1	*Ratio*	a_{ij} *for* S_2	*Ratio*	a_{ij} *for* S_3	*Ratio*
160/3	1	160/3	–2/3	–80	–1/3	–160
40/3	0	∞	1/3	40	–1/3	–40
100/3	0	∞	–1/6	–200	2/3	50

Range for centre 1: 160 – 160/3 = $106\frac{2}{3}$; $106\frac{2}{3}$ to ∞

Range for centre 2: 120 – 40 = 80, 120 + 80 = 200 ; 80 to 200

Range for centre 3: 80 – 50 = 30, 80 + 40 = 120 ; 30 to 120

(d)

Δ_j :	–11/3	0	0	0	–5/3	–4/3
for x_2 :	1/3	1	0	0	1/3	–1/3
Ratio :	–11	0	–	–	–5	4
for x_3 :	5/6	0	1	0	–1/6	2/3
Ratio :	–22/5	–	0	–	10	–2

∴ Range for x_1 : –∞ to $7\frac{2}{3}$

Range for x_2 : 8 – 5 = 3, 8 + 4 = 12 3 to 12

Range for x_3 : 6 – 2 = 4, 6 + 10 = 16 4 to 16

(e) Minimise G = $160y_1 + 120y_2 + 80y_3$

Subject to

$$y_1 + 3y_2 + 2y_3 \geq 4$$
$$3y_1 + 4y_2 + y_3 \geq 8$$
$$2y_1 + 2y_2 + 2y_3 \geq 6$$
$$y_1, y_2, y_3 \geq 0$$

Optimal values : $y_1 = 0$, $y_2 = 5/3$ and $y_3 = 4/3$ for $G = 920/3$

23. (i) Yes. The tableau represents an optimal solution since all $c_j - z_j$ values are ≤ 0, and there is no artificial variable in the basis.

(ii) No, because all the non-basic variables have $c_j - z_j < 0$.

(iii) No, since none of the basic variables has solution value equal to zero.

(iv) Yes. This solution is feasible because there is no artificial variable in the basis.

(v) Here $S_1 = 0$ and $S_2 = 2$. Thus, machine A is used to the full capacity.

(vi) For x_1, we have $c_j - z_j = -1$. Thus, the price of the product x_1 should be increased by at least Re 1 to ensure no reduction in profit.

(vii) $x_1 = 0$, $x_2 = 6$. Total profit = 3 × 0 + 4 × 6 = Rs 24.

(viii) Since marginal profitability of machine A is Rs 4 per hour, a reduction of 2 hours capacity in a week would cause a reduction of 2 × 4 = Rs 8 in profit.

(ix) Machine A: Rs 4/hour; Machine B: nil

(x) Machine A: Rs 4 per hour; machine B: nil.

24. Let x_1, x_2 and x_3 be the output of the products *A*, *B* and *C* respectively. The LPP is:

Maximise $Z = 20x_1 + 6x_2 + 8x_2$

Subject to

$$8x_1 + 2x_2 + 3x_3 \le 250$$
$$4x_1 + 3x_2 \le 150$$
$$2x_1 + x_3 \le 50$$
$$x_1, x_2, x_3 \ge 0$$

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i / a_{ij}
S_1	0	8	2	3	1	0	0	250	125/4
S_2	0	4	3	0	0	1	0	150	75/2
S_3	0	2	0	1	0	0	1	50	25 ←
C_j		20	6	8	0	0	0		
Solution		0	0	0	250	150	50		
Δ_j		20 ↑	6	8	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i / a_{ij}
S_1	0	0	2	–1	1	0	–4	50	25
S_2	0	0	3	–2	0	1	–2	50	50/3 ←
x_1	20	1	0	1/2	0	0	1/2	25	—
C_j		20	6	8	0	0	0		
Solution		25	0	0	50	50	0	$Z = 500$	
Δ_j		0	6	–2 ↑	0	0	–10		

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i / a_{ij}
S_1	0	0	0	1/3	1	–2/3	–8/3	50/3	50
x_2	6	0	1	–2/3	0	1/3	–2/3	50/3	—
x_1	20	1	0	1/2	0	0	1/2	25	50 ←
C_j		20	6	8	0	0	0		
Solution		25	50/3	0	50/3	0	0	$Z = 600$	
Δ_j		0	0	2 ↑	0	–2	–6		

When x_1 is the outgoing variable:

Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
S_1	0	–2/3	0	0	1	–2/3	–3	0
x_2	6	4/3	1	0	0	1/3	0	50
x_3	8	2	0	1	0	0	1	50
C_j		20	6	8	0	0	0	
Solution		0	50	50	0	0	0	Z = 700
Δ_j		–4	0	0	0	–2	–8	

When S_1 is the outgoing variable:

Simplex Tableau 5: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i / a_{ij}
x_3	8	0	0	1	3	–2	–8	50	—
x_2	6	0	1	0	2	–1	–6	50	—
x_1	20	1	0	0	–3/2	1	9/2	0	0 ←
C_j		20	6	8	0	0	0		
Solution		0	50	50	0	0	0	Z = 700	
Δ_j		0	0	0	–6	2	10 ↑		

Simplex Tableau 6: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_3	8	16/9	0	1	1/3	–2/9	0	50
x_2	6	4/3	1	0	0	1/3	0	50
S_3	0	2/9	0	0	–1/3	2/9	1	0
C_j		20	6	8	0	0	0	
Solution		0	50	50	0	0	0	Z = 700
Δ_j		–20/9	0	0	–8/3	–2/9	0	

Optimal solution: $x_1 = 0$, $x_2 = 50$, $x_3 = 50$; $Z = 700$.

Shadow Prices:

(a) From Tableau 4:
Milling: nil, Lathe: Rs 2/hour, Grinder = Rs 8/hour
Ranges
Milling: 0 ÷ 1 = 0, 50 ÷ 0 = ∞, 50 ÷ 0 = ∞
Range: 250 – ∞
Lathe: 0 ÷ (–2/3) = 0, 50 ÷ 1/3 = 150, 50 ÷ 0 = ∞
Range: 150 – 150, 150 + 0 or 0 – 150
Grinder: 0 ÷ (–3) = 0, 50 ÷ 0 = ∞, 50 ÷ 1 = 50
Range: 50 – 50, 50 + 0 or 0 – 50

(b) From Tableau 6:
Milling: Rs 8/9 per hour, Lathe: Rs 2/9 per hour, Grinder = Rs nil
Ranges
Milling: 50 ÷ 1/3 = 150, 50 ÷ 0 = ∞, 0 ÷ (–1/3) = 0

Range: 250 – 150, 250 + 0 or 100 – 250
Lathe: 50 ÷ (–2/9) = –225, 50 ÷ 1/3 = 150, 0 ÷ 2/9 = 0
Range: 150 – 0, 150 – (–225) or 150 – 375
Grinder: 50 ÷ 0 = ∞, 50 ÷ 0 = ∞, 0 ÷ 1 = 0
Range: 50 – 0, ∞ or 50 – ∞

Note: When optimal solution is degenerate, the ranges are not unique.

25. We first solve the primal problem by simplex method. For this, introducing necessary slack, surplus and artificial variables, we get

Maximise $Z = 8x_1 + 6x_2 + 0S_1 + 0S_2 - MA_1$

Subject to
$$x_1 - x_2 + S_1 = 3/5$$
$$x_1 - x_2 - S_2 + A_1 = 2$$
$$x_1, x_2, S_1, S_2, A_1 \geq 0$$

From the Simplex Tableau 2, we observe that infeasibility exists since the solution is final but has an artificial variable in the basis.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	A_1	b_i	b_i/a_{ij}
S_1	0	1*	–1	1	0	0	3/5	3/5 ←
A_1	$-M$	1	–1	0	–1	1	2	2
C_j		8	6	0	0	$-M$		
Solution		0	0	3/5	0	2		
Δ_j		$8 + M$ ↑	$6 - M$	0	$-M$	0		

Simplex Tableau 2: Final Solution (Infeasible)

Basis		x_1	x_2	S_1	S_2	A_1	b_i	b_i/a_{ij}
x_1	8	1	–1	0	1	0	3/5	—
A_1	$-M$	0	0	–1	–1	1	7/5	—
C_j		8	6	0	0	$-M$		
Solution		3/5	0	0	0	7/5		
Δ_j		0	14 ↑	$-M$	$-M - 8$	0		

Note that this solution is final not in terms of Δ_j values but in the sense that the key column values are all ≤ 0. With none of these being positive, the solution process terminates.

Dual The dual to the given problem is:

Minimise $G = \frac{3}{5} y_1 - 2y_2$

Subject to
$$y_1 - y_2 \geq 8$$
$$-y_1 + y_2 \geq 6$$
$$y_1, y_2 \geq 0$$

With surplus variables S_1 and S_2, and artificial variables A_1 and A_2, the solution is given in the two tables. It is evident from the second table that the solution has an artificial variable in the basis and has all $\Delta_j \geq 0$. Hence, infeasibility is present. Hence, the given statement in the problem.

Simplex Tableau 1: Non-optimal Solution

Basis		y_1	y_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	M	1	–1	–1	0	1	0	8	—
A_2	M	–1	1	0	–1	0	1	6	6
C_j		3/5	–2	0	0	M	M		
Solution		0	0	0	0	8	6		
Δ_j		3/5	–2 ↑	M	M	0	0		

Simplex Tableau 2: Final Solution (Infeasible)

Basis		y_1	y_2	S_1	S_2	A_1	A_2	b_i
A_1	M	0	0	–1	–1	1	1	14
y_2	–2	–1	1	0	–1	0	1	6
C_j		3/5	–2	0	0	M	M	
Solution		0	6	0	0	14	0	
Δ_j		7/5	0	M	$M - 2$	0	2	

26. (a) Based on the statement of the problem, it may be concluded that the constraints of the type $A \leq$ Demand (for A), etc. have been used. The shadow prices, accordingly, would relate to the slack variables that are introduced into the constraints and indicate the amount by which the total contribution will change given a unit change in demand. In case of product A, the demand constraint is binding and if demand increases by one unit then contribution would increase by two units. For B, demand is not a limiting factor since it has a zero shadow price. Product C does not appear in the final solution since each unit produced of it would reduce the profit by Rs 3.

(b) Given the product price and the cost data, the information about shadow prices may be used to indicate which products should figure in the optimum production plan. The shadow prices also show by how much the product cost and/or prices should change in order that the currently non-profitable products may become profitable.

27. The given information is reproduced below:

C_j	x_j	x_1	x_2	S_1	S_2	S_3	b_i
21/2	x_2	0	1	3/5	–2/5	0	300
17/2	x_1	1	0	–2/5	3/5	0	300
0	S_3	0	0	–1/5	–1/5	1	400
C_j		17/2	21/2	0	0	0	
Solution		300	300	0	0	400	
Δ_j		0	0	–29/10	–9/10	0	

(a) The given solution is optimal since all $\Delta_j \leq 0$ (being a maximisation problem) and feasible.

(b) The solution is feasible since there is no artificial variable in the basis.

(c) The given solution is unique because none of the non-basic variables has $\Delta_j = 0$.

(d) Objective function value, $Z = \frac{17}{2} \times 300 + \frac{21}{2} \times 300$ = Rs 5,700.

(e) Shadow prices:
Resource 1 (S_1) = Rs 2.9/unit
Resource 2 (S_2) = Rs 0.9/unit
Resource 3 (S_3) = nil

(f) Since the shadow price for this resource is Rs 2.9 per unit, this is the maximum amount the company be willing to pay for each unit of production capacity.

(g) We first obtain validity range of the shadow price for S_2 as follows:

b_i	a_{ij}	b_i/a_{ij}	
300	–2/5	–750	least negative
300	3/5	500	least positive
400	–1/5	–2,000	

∴ Lower limit = 500 and Upper limit = 750.

Since the increase in demand is only 20 units (which is within the validity range), the shadow price is constant.

Thus, increase in contribution = 20 × 0.9 = Rs 18

New contribution level = 5.700 + 18 = Rs 5,718

New product-mix:

$$x_2 = 300 - (20 \times 2/5) = 292, \text{ and}$$
$$x_1 = 300 + (20 \times 3/5) = 312$$

28. (a) Let x_1, x_2, x_3 and x_4 be the output of producs A, B, C and D respectively. The LPP is:

Maximise $Z = 4x_1 + 6x_2 + 3x_3 + x_4$

Subject to
$$1.5x_1 + 2x_2 + 4x_3 + 3x_4 \le 550$$
$$4x_1 + x_2 + 2x_3 + x_4 \le 700$$
$$2x_1 + 3x_2 + x_3 + 2x_4 \le 200$$
$$x_1, x_2, x_3, x_4 \ge 0$$

To solve the problem by simplex method, we introduce slack variables S_1, S_2 and S_3. The problem is:

Minimise $Z = 4x_1 + 6x_2 + 3x_3 + x_4 + 0S_1 + 0S_2 + 0S_3$

Subject to
$$1.5x_1 + 2x_2 + 4x_3 + 3x_4 + S_1 = 550$$
$$4x_1 + x_2 + 2x_3 + x_4 + S_2 = 700$$
$$2x_1 + 3x_2 + x_3 + 2x_4 + S_3 = 200$$
$$x_1, x_2, x_3, x_4, S_1, S_2, s_3 \ge 0$$

The solution is given in tables that follow.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	3/2	2	4	3	1	0	0	550	275
S_2	0	4	1	2	1	0	1	0	700	700
S_3	0	2	3*	1	2	0	0	1	200	200/3 ←
C_j		4	6	3	1	0	0	0		
Solution		0	0	0	0	550	700	200	$Z = 0$	
Δ_j		4	6	3	1	0	0	0		
			↑							

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	1/6	0	10/3*	5/3	1	0	–2/3	1,250/3	125 ←
S_2	0	10/3	0	5/3	1/3	0	1	–1/3	1,900/3	380
x_2	6	2/3	1	1/3	2/3	0	0	1/3	2,00/3	200
C_j		4	6	3	1	0	0	0		
Solution		0	200/3	0	0	1,250/3	1,900/3	0	$Z = 400$	
Δ_j		0	0	1	–3	0	0	–2		
				↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_i
x_3	3	1/20	0	1	1/2	3/10	0	–1/5	125
S_2	0	13/4	0	0	–1/2	–1/2	1	0	425
x_2	6	13/20	1	0	1/2	–1/10	0	6/15	25
C_j		4	6	3	1	0	0	0	
Solution		0	25	125	0	0	425	0	
Δ_j		–1/20	0	0	–7/2	–3/10	0	–9/5	$Z = 525$

Optimal product-mix : $x_1 = 0$, $x_2 = 25$, $x_3 = 125$, and $x_4 = 0$

Total maximum profit contribution = 4 × 0 + 6 × 25 + 3 × 125 + 1 × 0 = Rs 525

(b) No, since none of the non-basic variables has Δj equal to zero.

(c) Minimise G = $550y_1 + 700y_2 + 200y_3$

Subject to $1.5y_1 + 4y_2 + 2y_3 \geq 4$

$2y_1 + y_2 + 3y_3 \geq 6$

$4y_1 + 2y_2 + y_3 \geq 3$

$3y_1 + y_2 + 2y_3 \geq 1$

$y_1, y_2, y_3 \geq 0$

$y_1 = 3/10$, $y_2 = 0$, $y_3 = 9/5$, $G = 525$

(d) Shadow prices : Machine I : Rs 3/10 per hour,

Machine II : Nil

Machine III : Rs 9/5 per hour

Machine III should be given priority.

(e) Re 0.05 per unit

(f) Yes, since price increase is more than Δj value of 5 paise.

29. (a) From the given simplex tableau, we have: $x_1 = 120$, $x_2 = 300$, $S_1 = S_2 = 0$, $S_3 = 240$.

Also, Z = 80 × 120 + 100 × 300 = Rs 39,600.

(b) The dual to the given problem is:

Minimise $G = 720y_1 + 1{,}800y_2 + 900y_3$

Subject to $y_1 + 5y_2 + 3y_3 \geq 80$

$2y_1 + 4y_2 + y_3 \geq 100$

$y_1, y_2, y_3 \geq 0$

(c) Tracing the values from the Δ_j row of the given simplex tableau, we get $y_1 = 30$, $y_2 = 10$, and $y_3 = 0$. This gives G = 720 × 30 + 1,800 × 10 + 900 × 0 = Rs 39,600.

(d) The marginal profitability of capacity of machining and fabrication is Rs 30 and Rs 10 per hour, respectively. It is nil for the assembly.

(e) Since the cost of overtime in the fabricating department is greater than the marginal profitability (15 > 10), it is not advisable to work overtime in this department. It is worth, however, to work overtime in machining. For this, dividing b_i by the a_{ij}'s column headed S_1, we get 300 ÷ 5/6 = 360; 120 ÷ (–2/3) = –180; and 240 ÷ 7/6 = 1,440/7. The least negative of these being –180, we can work overtime up to 180 hours.

(f) For machining: same as in (e) above. For fabrication, we have 300 ÷ (–1/6) = –1,800, 120 ÷ (1/3) = 360, and 240 ÷ (–5/6) = –288. Since the least negative value is –288, we can work overtime up to 288 hours.

(g) The range of profit over which the given solution would be valid can be obtained as under:

Δ_j	0	0	–30	–10	0
a_{2j}	1	0	–2/3	1/3	0
Ratio	0	—	45	–30	—
			↑ least positive	↑ least negative	

The solution would be valid over the range 80 – 30 to 80 + 45, or 50 to 125. Since the profit would increase to Rs 100, there would be no change in the plan.

(h) From the given information,

Δ_j	0	0	–30	–10	0
a_{1j}	0	1	5/6	–1/6	0
Ratio	—	0	–36	60	—
			↑	↑	
			least positive	least negative	

The solution would be valid over the range 100 – 36 to 100 + 60, or 64 to 160.

(i) The minimum profit obtainable would be equal to the summation of the products of the capacity requirements and the corresponding marginal profitabilities. This equals 2 × 30 + 3 × 10 + 2 × 0 = Rs 90.

30. (i) Let S_1, S_2 and S_3 be the slack variables.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	4	1	0	0	1,000	250
S_2	0	6	2	0	1	0	1,200	600
S_3	0	0	1*	0	0	1	200	200 ←
C_j		30	80	0	0	0		
Solution		0	0	1,000	1,200	200	Z = 0	
Δ_j		30	80	0	0	0		
			↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	0	1	0	–4	200	100 ←
S_2	0	6	0	0	1	–2	800	400/3
x_2	80	0	1	0	0	1	200	—
C_j		30	80	0	0	0	Z = 16,000	
Solution		0	200	200	800	0		
Δ_j		30	0	0	0	–80		
		↑						

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_1	30	1	0	1/2	0	–2	100
S_2	0	0	0	–3	1	10	200
x_2	80	0	1	0	0	1	200
C_j		30	80	0	0	0	
Solution		100	200	0	200	0	Z = 19,000
Δ_j		0	0	–15	0	–20	

Optimal product mix: Lawn Mowers 100

Snow Blowers 200

Optimal profit: Rs 19,000

(ii) Shadow prices: Labour hours : Rs 15/hour
Steel : Re 0
Snowblower engines: Rs 20/engine

Snowblower engines have the highest marginal value.

(iii) Labour hour : 800 – 1,067[1,000 – 200, 1,000 – (–200/3)]
Steel : 1,000 – ∞ [1,200 –200]
Engines : 180 – 250 [200 – 20, 200 – (–50)]

(iv) x_1: 0 – 40 [30 – 30 = 0, 30 + 10 = 40]
x_2: 60 – ∞ [80 – 20 = 60]

(v) Let y_1, y_2 and y_3 be the dual variables. The dual is:

Minimise $G = 1{,}000y_1 + 1{,}200y_2 + 200y_3$

Subject to
$$2y_1 + 6y_2 \geq 30$$
$$4y_1 + 2y_2 + y_3 \geq 80$$
$$y_1, y_2, y_3 \geq 0$$

Solution to the dual: $y_1 = 15$, $y_2 = 0$ and $y_3 = 20$.

CHAPTER 5

1. **Initial Solution (VAM): Optimal**

Origin	*Destination*					*Capacity*	u_i
	D_1	D_2	D_3	D_4	D_5		
O_1	12 (–1)	4 (0)	9 [25]	5 [30]	9 (–5)	55	4
O_2	8 [10]	1 [20]	6 [15]	6 (–4)	7 (–6)	45	1
O_3	1 [30]	12 (–18)	4 (–5)	7 (–12)	7 (–13)	30	–6
O_4	10 (–2)	15 (–14)	6 [10]	9 (–7)	1 [40]	50	1
Req.	40	20	50	30	40	180	
v_j	7	0	5	1	0		

Total cost = Rs 695
The solution is not unique since a cell O_1D_2 has $\Delta_{ij} = 0$.
An alternate optimal solution is:
$O_1D_2 = 20$, $O_1D_3 = 5$, $O_1D_4 = 30$, $O_2D_1 = 10$, $O_2D_3 = 35$, $O_3D_1 = 30$, $O_4D_3 = 10$ and $O_4D_5 = 40$.

2. The given solution is reproduced in the table and is found to be non-optimal as all Δ_{ij}'s are not less than, or equal to, zero. This solution involves a total cost of Rs 1,335.

Proposed Solution: Non-optimal

Source	*Destination*					*Supply*	u_i
	1	*2*	*3*	*4*	*5*		
1	15 [25]	7 (6)	12 (6)	8 [30]	12 (1)	55	0
2	11 (–5)	4 [20]	9 [25]	9 (–10)	10 (–6)	45	–9
3	4 [15]	15 (–13)	7 [15]	10 (–13)	10 (–8)	30	–11
4	13 (–7)	18 (–14)	9 [10]	12 (–13)	4 [40]	50	–9
Demand	40	20	50	30	40	180	
v_j	12	10	15	5	10		

Now initial solution to the problem using VAM is contained in the table below. This solution involves a total cost of Rs 1,235 and is found to be optimal.

Initial Feasible solution: Optimal

Source	*Destination* 1	2	3	4	5	*Supply*	u_i
1	12 (−1)	4 (0)	[25] 9	[30] 5	9 (−5)	55	0
2	[10] 8	[20] 1	[15] 6	6 (−4)	7 (−6)	45	–3
3	[30] 1	12 (−18)	4 (−5)	7 (−12)	7 (−13)	30	–10
4	10 (−2)	15 (−14)	[10] 6	9 (−7)	[40] 1	50	–30
Demand	40	20	50	30	40	180	
v_j	11	4	9	5	4		

3. In this problem, $AD = 30$ while $AS = 34$. Thus, it is an unbalanced problem. A dummy rolling mill, M_6, is introduced with zero cost elements, as shown in table. In this table, initial solution using VAM is also given.

The given solution has seven occupied cells while the required number is 3 + 6 – 1 = 8. Thus, it is degenerate. Accordingly, an ε is placed in the cell F_1M_6, which is as independent cell.

Initial Solution: Degenerate, Non-optimal

	M_1	M_2	M_3	M_4	M_5	M_6	*Supply*	u_i
F_1	4 (2) +	[4] 2	3 (1)	[4] 2	6 (−5)	[ε] 0 –	8	0
F_2	5 (1)	4 (−2)	5 (−1)	[4] 2	[8] 1	0 (0)	12	0
F_3	[4] 6 –	5 (−3)	[6] 4	7 (−5)	7 (−6)	[4] 0 +	14	0
Demand	4	4	6	8	8	4	34	
v_j	6	2	4	2	1	0		

Now, the solution is tested for optimality and found to be non-optimal. The cell F_1M_1 has the largest Δ_{ij} value. Beginning with this cell, a closed path is drawn and a revised solution is obtained.

This is tested for optimality and found to be optimal. Thus, optimum shipping schedule is: F_1M_2 : 4, F_1M_4 : 4, F_2M_4 : 4, F_2M_5 : 8, F_3M_1 : 4 and F_3M_3 : 6. Total cost = Rs 80.

Improved Solution: Optimal

	M_1	M_2	M_3	M_4	M_5	M_6	*Supply*	u_i
F_1	4 [ε]	2 [4]	3 (–1)	2 [4]	6 (–5)	0 (–2)	8	0
F_2	5 (–1)	4 (–2)	5 (–3)	2 [4]	1 [8]	0 (–2)	12	0
F_3	6 [4]	5 (–1)	4 [6]	7 (–3)	7 (–4)	0 [4]	14	2
Demand	4	4	6	8	8	4	34	
v_j	4	2	2	2	1	–2		

4. The initial feasible solution using VAM is given in table below. On testing, it is found to be optimal. However, since Δ_{11} and Δ_{31} are each equal to zero, the solution is not unique and multiple optima exist.

Initial Feasible Solution: Optimal

	1	*2*	*3*	*4*	*Supply*	u_i
1	8 (0) +	8 [7] –	5 (–1)	12 (–1)	7	0
2	6 [3] –	9 (–3)	11 (–9)	9 [4] +	7	–2
3	10 (0)	15 (–5)	6 [8]	13 [2]	10	2
4	6 (–1)	8 (–3)	7 (–6)	8 [6]	6	–3
5	11 (–1)	10 [3] +	11 (–5)	13 [2] –	5	2
6	8 [6]	14 (–6)	5 (–1)	12 (–1)	6	0
Demand	9	10	8	14	41	
v_j	8	8	4	11		

Total cost = 8 × 7 + 6 × 3 + 9 × 4 + 6 × 8 + 13 × 2 + 8 × 6 + 10 × 3 + 13 × 2 + 8 × 6 = Rs 336

One alternate optimal solution may be obtained through the closed loop shown in the table. The solution is: $x_{11} = 2$, $x_{12} = 5$, $x_{21} = 1$, $x_{24} = 6$, $x_{34} = 2$, $x_{44} = 6$, $x_{52} = 5$, and $x_{61} = 6$.

5. **Initial Solution (VAM): Non-optimal**

Plant	*Warehouse* *P*	*Q*	*R*	*S*	*Dummy*	*Prod. ('000)*	u_i
X	15 (–4)	[18] 13	[2] 14	16 (–3)	0 (–8)	20	–8
Y	16 (–6)	14 (–2)	[2] 13 +	[14] 12 –	0 (–9)	16	–9
Z	[15] 19	20 (1)	[8] 22 –	18 (3) +	[2] 0	25	0
Demand ('000)	15	18	12	14	2	61	
v_j	19	21	22	21	0		

Total cost = Rs 917,000

Revised Solution: Optimal

Plant	*Warehouse* *P*	*Q*	*R*	*S*	*Dummy*	*Prod. ('000)*	u_i
X	15 (–1)	[18] 13	[2] 14	16 (–3)	0 (–5)	20	–5
Y	16 (–3)	14 (–2)	[10] 13	[6] 12	0 (–6)	16	–6
Z	[15] 19	20 (–2)	22 (–3)	[8] 18	[2] 0	25	0
Demand ('000)	15	18	12	14	2	61	
v_j	19	18	19	18	0		

Total cost = Rs 893,000

6. The initial solution to this problem is given in table. This solution, obtained by VAM is tested for optimality and is found to be optimal.

Initial solution: Optimal

Source	*Distination* *1*	*2*	*3*	*4*	*Supply*	u_i
1	15 (0) +	[10] 18 –	[20] 22	16 (0)	30	0
2	15 (–2)	19 (–3)	[5] 20	[35] 14	40	–2
3	[20] 13 –	[10] 16 +	23 (–3)	17 (–3)	30	–2
Demand	20	20	25	35	100	
v_j	15	18	22	16		

However, since all Δ_{ij} values are not negative, the solution is not unique optimal. Alternate optimal solutions may be obtained by drawing a closed path beginning with (i) the cell 1,1 and (ii) cell 1,4. In table, a closed path beginning with cell 1,1 is drawn and a revised solution is obtained as shown in table given below. Similarly, another solution may be worked out by starting from the cell 1,4.

Alternate Optimal Solution

Source	*Destination*				*Supply*	u_i
	1	*2*	*3*	*4*		
1	[10] 15	18 (0)	[20] 22	16 (0)	30	0
2	15 (–2)	19 (–3)	[5] 20	[35] 14	40	–2
3	[10] 13	[20] 16	23 (–3)	17 (–3)	30	–2
Demand	20	20	25	35	100	
v_j	15	18	22	16		

7. Here *AD* = 145 units and *AS* = 105 units. The *AD* being greater than the *AS*, a total of 40 units of demand is obviously going to be unsatisfied. Since the penalty for unsatisfied demand is given, the cost elements for the row representing dummy factory (needed to balance the problem) would not be taken to be zero. The given penalties instead would be taken. The restructured problem is given in the table. The initial solution by VAM is also given, which is found to be optimal.

Initial Solution (VAM): Optimal

Factory	*Destination*			*Supply*	u_i
	1	*2*	*3*		
A	8 (–2)	[10] 4	10 (–4)	10	0
B	[20] 9	[10] 7	[50] 9	80	3
C	[15] 6	5 (–1)	8 (–2)	15	0
Dummy	[40] 4	5 (–3)	6 (–2)	40	–2
Demand	75	20	50	145	
v_j	6	4	6		

Total cost = Rs 830 + Rs 160 (penalty) = Rs 990

8. **Initial Solution (VAM): Non-optimal Solution**

Plant	*Distribution Centres* D_1	D_2	D_3	D_4	*Supply*	u_i
P_1	20 [5]	30 (–26)	50 (–53)	17 [2]	7	0
P_2	70 (–7)	35 (12)	40 [7]	60 [3] (–)	10	43
P_3	40 (–12)	12 [8] (–)	60 (–55)	25 [10] (+)	18	8
Demand	5	8	7	15	35	
v_j	20	4	–3	17		

Revised Solution: Optimal

Plant	*Distribution Centres* D_1	D_2	D_3	D_4	*Supply*	u_i
P_1	20 [5]	30 (–26)	50 (–41)	17 [2]	7	0
P_2	70 (–19)	35 [3]	40 [7]	60 (–12)	10	31
P_3	40 (–12)	12 [5]	60 (–43)	25 [13]	18	8
Demand	5	8	7	15	35	
v_j	20	4	9	17		

Total cost = Rs 904

Maximum savings = 1200 – 904 = Rs 296

9. **Initial Solution (VAM): Non-optimal**

Plant	*Distribution Centres* W	X	Y	Z	*Production*	u_i
A	500 [ε]	1000 (–700)	150 [10]	800 (–400)	10	400
B	200 [8]	700 (–700)	500 (–600)	100 [4]	12	100
C	600 [1] –	400 [7] +	100 (150)	900 (–400)	8	500
Dummy	0 (200)	0 [2] –	0 (–150)	0 (100)	2	100
Demand	9	9	10	4	32	
v_j	100	–100	–250	0		

(Note: Supply and demand in ’000 units)

Improved Solution: Optimal

Plant	*Distribution Centres*				*Production*	u_i
	W	*X*	*Y*	*Z*		
A	500	1000 (–500)	[10] 150	800 (–400)	10	400
B	[8] 200	700 (–500)	500 (–650)	[4] 100	12	100
C	600 (–200)	[8] 400	100 (–50)	900 (–600)	8	300
Dummy	[1] 0	[1] 0	0 (–350)	0 (–100)	2	–100
Demand	9	9	10	4	32	
v_j	100	100	–250	0		

Total Cost = Total Tonne Miles × Cost per Tonne-mile
= 6700,000 × 10 = Rs 67,000,000

When *CX* is not allowed:

Optimal Solution

Plant	*Distribution Centres*				*Production*	u_i
	W	*X*	*Y*	*Z*		
A	[8] 500	1000 (0)	[2] 150	800 (–400)	10	400
B	[1] 200	7 700	500 (–650)	[4] 100	12	100
C	600 (–150)	*M*	[8] 100	900 (–550)	8	350
Dummy	0 (–500)	[2] 0	0 (–850)	0 (–600)	2	–600
Demand	9	9	10	4	32	
v_j	100	600	–250	0		

Total Cost = Total Tonne Miles × Cost per Tonne-mile
= 10,600,000 × 10 = Rs 106,000,000

10. (a)

Initial Solution (VAM): Optimal

Factory	Warehouse X	Warehouse Y	Warehouse Z	Warehouse W	Supply	u_i
A	[60] 25	55 (–21)	40 (–25)	60 (–16)	60	44
B	35 (–14)	[50] + 30	50 (–39)	[90] – 40	140	40
C	[30] 36	[ε] – 45	[120] 26	66 (–11)	150	55
D	35 (–14)	[50] 30	41 (–30)	50 (–10)	50	40
Dummy	0 (–19)	0 (–10)	0 (–29)	[50] 0	50	0
Demand	90	100	120	140	450	
v_j	–19	–10	–29	0		

Total Cost = Rs 12,300

(b) From the solution in (a) the Δ_{ij} value for the cell $C – W$ is seen to be equal to –11. If the cost on this route is reduced to Rs 50, the Δ_{ij} would work out to be +5, so that every unit moved through this route would reduce the cost by Rs 5. To obtain improved solution, we draw closed path as shown in the table. However, only ε moves through closed path. Hence, effectively, no cost reduction can be achieved.

11. (i) and (ii) The given solution is reproduced in table below. The test of optimality shows that this solution is not optimal since the route *CX* shows a positive Δ_{ij} value.

Initial Solution: Non-optimal

Factory	Stockist X	Stockist Y	Stockist Z	Capacity	u_i
A	[31] 4 –	[25] 8 +	8 (–4)	56	0
B	[41] 16	24 (–4)	[41] 16	82	12
C	8 (4)	[77] 16 –	24 (–12)	77	8
Demand	72	102	41	215	
v_j	4	8	4		

Beginning with the cell *CX*, a closed path is drawn and a revised solution is obtained. This is shown in table below. The solution is found to be an optimal one. The solution is:

Rout	*Units*	*Cost*
A to *Y*	56	56 × 8 = 448
B to *X*	41	41 × 16 = 656
B to *Z*	41	41 × 16 = 656
C to *X*	31	31 × 8 = 248
C to *Y*	46	46 × 16 = 736

Revised Solution: Optimal

Factory	*Stockist*			*Capacity*	u_i
	X	*Y*	*Z*		
A	4 (–4)	8 [56]	8 (–8)	56	0
B	16 [41]	24 (0)	16 [41]	82	16
C	8 [31]	16 [46]	24 (–16)	77	8
Demand	72	102	41	215	
v_j	0	8	0		

(iii) The problem has multiple optimal solutions. This is because a cell (*B*, *Y*) has $\Delta_{ij} = 0$. An alternate optimal solution, obtained by drawing a closed path beginning with this cell and making adjustments, is produced in table that follows.

Alternate Optimal Solution

Factory	*Stockist*			*Capacity*	u_i
	X	*Y*	*Z*		
A	4 (–4)	8 [56]	8 (–8)	56	0
B	16 (0)	24 [41]	16 [41]	82	16
C	8 [72]	16 [5]	24 (–16)	77	8
Demand	72	102	41	215	
v_j	0	8	0		

(iv) The significance of multiple optimal solutions lies in the fact that they provide the management with operational flexibility in terms of solving the problem at hand. They provide alternatives to the management that are equally effective.

(v) If 20 units are considered necessary to send from *A* to *Z*, then the cost would increase by Rs 8×20 = Rs 160. The revised shipping plan can be obtained by drawing a closed path, starting from the cell *A*, *Z*. The resulting schedule is given in table given here.

Revised Solution

Factory	*Stockist*			*Capacity*
	X	*Y*	*Z*	
A	4	36 8	20 8	56
B	16	61 24	21 16	82
C	72 8	5 16	24	77
Demand	72	102	41	215

(vi) The route *A*, *Z* is unoccupied in terms of the optimal solution to the problem. An increase in per unit cost would not change this status. Hence, the solution will not change.

12. The given solution is reproduced here and tested for optimality.

	D_1	D_2	D_3	D_4	*Total*	u_i
S_1	10 (–1)	200 6	100 18	23 (–15)	300	0
S_2	150 4	9 (–8)	50 13	10 (–7)	200	–5
S_3	7 (–1)	13 (–10)	50 15	350 5	400	–3
Total	150	200	200	350	900	
v_j	9	6	18	8		

(i) The given solution is not degenerate because the number of occupied cells is 6, which is equal to $m + n - 1$.

(ii) The solution is tested for optimality and found to be optimal as all Δij values are less than zero. Further, the solution is unique since none of the Δij values is equal to zero.

(iii) Rs 15, since $\Delta ij = -15$.

(iv) It would increase the cost by Rs 8 per unit transported on this route.

13. (a) **Initial Solution (VAM): Optimal**

Shop	*Warehouse* I	II	III	IV	V	*Supply*	u_i
A	20 (–3)	[15] 18	[85] 18	21 (–5)	19 (–1)	100	18
B	21 (0)	[20] 22	23 (–1)	[105] 20	24 (–2)	125	22
C	[60] 18	[45] 19	21 (2)	18 (–1)	[70] 19	175	19
Demand	60	80	85	105	70	400	
v_j	–1	0	0	–2	0		

Total Cost = Rs 7605

(b) The solution in (a) is not unique. An alternate solution is: $x_{12} = 15$, $x_{13} = 85$, $x_{21} = 20$, $x_{24} = 105$, $x_{31} = 40$, $x_{32} = 65$ and $x_{35} = 70$.

(c) The dual is:

Maximise $Z = 100u_1 + 125u_2 + 175u_3 + 60v_1 + 80v_2 + 85v_3 + 105v_4 + 70v_5$

Subject to

$$u_1 + v_1 \le 20$$
$$u_1 + v_2 \le 18$$
$$u_1 + v_3 \le 18$$
$$u_1 + v_4 \le 21$$
$$u_1 + v_5 \le 19$$
$$u_2 + v_1 \le 21$$
$$u_2 + v_2 \le 22$$
$$u_2 + v_3 \le 23$$
$$u_2 + v_4 \le 20$$
$$u_2 + v_5 \le 24$$
$$u_3 + v_1 \le 18$$
$$u_3 + v_2 \le 19$$
$$u_3 + v_3 \le 21$$
$$u_3 + v_4 \le 18$$
$$u_3 + v_5 \le 19$$

$u_1, u_2, u_3, v_1, v_2, v_3, v_4, v_5$: Unrestricted in sign

Optimal values of dual variables are:

$u_1 = 18$, $u_2 = 22$, $u_3 = 19$, $v_1 = -1$, $v_2 = 0$, $v_3 = 0$, $v_4 = -2$ and $v_5 = 0$. With these, $Z = 7605$.

14. **Initial Solution (VAM): Non-optimal**

Company	*Depot* 1	2	3	4	*Dummy*	*Supply*	u_i
1	5.00 (–0.75)	[165000] 5.00 −	[110000] 4.50 +	5.50 (–1.00)	0 (–0.50)	2,75,000	–0.50
2	[110000] 4.75 −	[55000] 5.50 +	6.00 (–1.00)	6.00 (–1.50)	[385000] 0	5,50,000	0
3	4.25 (0.50)	6.75 (–1.25)	[220000] 5.00 −	[440000] 4.50	0 (0)	6,60,000	0
Demand	1,10,000	2,20,000	3,30,000	4,40,000	3,85,000	1,485,000	
v_j	4.75	5.50	5.00	4.50	0		

Revised Solution: Optimal

Company	*Depot* 1	2	3	4	*Dummy*	*Supply*	u_i
1	5.00 (–1.25)	[55000] 5.00	[220000] 4.50	5.50 (–1.50)	0 (–0.50)	2,75,000	–0.50
2	4.75 –0.50	[165000] 5.50	6.00 (–1.00)	6.00 (–1.50)	[385000] 0	5,50,000	0
3	[110000] 4.25	6.75 (–1.25)	[110000] 5.00	[440000] 4.50	0 (0)	6,60,000	0
Demand	1,10,000	2,20,000	3,30,000	4,40,000	3,85,000	1,485,000	
v_j	4.25	5.50	5.00	4.50	0		

Total Cost = Rs 5170,000

Dual: Let u_1, u_2, u_3, v_1, v_2, v_3, v_4 and v_5 be the dual variables. With these, the dual is:

Maximise $Z = 275{,}000u_1 + 550{,}000u_2 + 660{,}000u_3 + 110{,}000v_1 + 220{,}000v_2 + 330{,}000v_3 + 440{,}000v_4 + 385{,}000v_5$

Subject to

$u_1 + v_1 \leq 5.00$	$u_2 + v_4 \leq 6.00$
$u_1 + v_2 \leq 5.00$	$u_2 + v_5 \leq 0.00$
$u_1 + v_3 \leq 4.50$	$u_3 + v_1 \leq 4.25$
$u_1 + v_4 \leq 5.50$	$u_3 + v_2 \leq 6.75$
$u_1 + v_5 \leq 0.00$	$u_3 + v_3 \leq 5.00$
$u_2 + v_1 \leq 4.75$	$u_3 + v_4 \leq 4.50$
$u_2 + v_2 \leq 5.50$	$u_3 + v_5 \leq 0.00$
$u_2 + v_3 \leq 6.00$	

u_1, u_2, u_3, v_1, v_2, v_3, v_4, v_5 unrestricted in sign. Substituting u_i and v_j values from the table, Z = 5170,000.

15. (a) **Initial Solution (VAM): Non-optimal**

Source	*Destination* 1	2	3	*Supply*	u_i
1	12 (–12)	8 [10] +	2 [30] –	40	2
2	9 (–7)	10 [30]	9 (–5)	30	4
3	7 [20]	15 [10] –	6 (3)	30	9
Demand	20	50	30	100	
v_j	–2	6	0		

Improved Solution: Optimal

Source	*Destination* 1	2	3	*Supply*	u_i
1	12 (–9)	8 [20]	2 [20]	40	2
2	9 (–4)	10 [30]	9 (–5)	30	4
3	7 [20]	15 (–3)	6 [10]	30	6
Demand	20	50	30	100	
v_j	1	6	0		

Total: 700

(b) With u_1, u_2, u_3, v_1, v_2 and v_3 as dual variables, the dual is:
Maximise $Z = 40u_1 + 30u_2 + 30u_3 + 20v_1 + 50v_2 + 30v_3$
Subject to

$$u_1 + v_1 \le 12$$
$$u + v_2 \le 8$$
$$u_1 + v_3 \le 2$$
$$u_2 + v_1 \le 9$$
$$u_2 + v_2 \le 10$$
$$u_2 + v_3 \le 9$$
$$u_3 + v_1 \le 7$$
$$u_3 + v_2 \le 15$$
$$u_3 + v_3 \le 6$$

u_i, v_j: Unrestricted in sign, i = 1, 2, 3 j = 1, 2, 3
With u_i and v_j values from the table, we have
$Z = 40 \times 2 + 30 \times 4 + 30 \times 6 + 20 \times 1 + 50 \times 6 + 30 \times 0 = 700.$

16. (a) and (b) The proposed solution is tested for optimality in table. It is found to be non-optimal. To improve this solution, a closed path is drawn beginning with the cell *CS*.

The revised solution is given in second table. It is found to be optimal. The minimum transportation cost involved is Rs 149. The optimal schedule is: *A* to *Q*: 12; *A* to *R*: 2; *A* to *S*: 8; *B* to *R*: 15; *C* to *P*: 7; and *C* to *S*: 1.

Proposed Solution: Non-optimal

Warehouse	*Market*				*Supply*	u_i
	P	*Q*	*R*	*S*		
A	6 (–4)	3 [12]	5 [1] +	4 [9] –	22	0
B	5 (–6)	9 (–9)	2 [15]	7 (–6)	15	–3
C	5 [7]	7 (–1)	8 [1] –	6 (1) +	8	3
Req.	7	12	17	9	45	
v_j	2	3	5	4		

Total cost = 3 × 12 + 5 × 1 + 4 × 9 + 2 × 15 + 5 × 7 + 8 × 1 = Rs 150

Improved Solution: Optimal

Warehouse	*Market*				*Supply*	u_i
	P	*Q*	*R*	*S*		
A	6 (–3)	3 [12]	5 [2]	4 [8]	22	0
B	5 (–5)	9 (–9)	2 [15]	7 (–6)	15	–3
C	5 [7]	7 (–2)	8 (–1)	6 [1]	8	2
Req.	7	12	17	9	45	
v_j	3	3	5	4		

Total cost = 3 × 12 + 5 × 2 + 4 × 8 + 2 × 15 + 5 × 7 + 6 × 1 = Rs 149

(c) For the route *C* to *Q*, we have $\Delta_{ij} = -2$. This implies that the rate should be reduced by at least Rs 2 per unit by the carrier to get the business.

17. The initial solution, using VAM, is presented in table. When tested for optimality, it is found to be optimal. The solution is not unique, however, since Δ_{21} and Δ_{54} are both equal to zero. The optimal solution involves a total cost of Rs 2,340.

Initial Basic Feasible Solution: Optimal

	Store					*Surplus*	u_i
	S_1	S_2	S_3	S_4	S_5		
W_1	9 [10]	12 (–2)	10 (–4)	10 [90]	6 [50]	150	0
W_2	5 (0)	18 (–12)	12 (–10)	11 (–5)	2 [30]	30	–4
W_3	10 (–8)	*M*	7 (–8)	3 [120]	20 (–21)	120	–7
W_4	5 [70]	6 [40]	2 [20]	*M*	8 (–6)	130	–4
W_5	0 (–1)	0 [20]	0 (–4)	0 (0)	0 (–4)	20	–10
Req.	80	60	20	210	80	450	
v_j	9	10	6	10	6		

Total cost = 9 × 10 + 10 × 90 + 6 × 50 + 2 × 30 + 3 × 120 + 5 × 70 + 6 × 40 + 2 × 20 = Rs 2,340

18. This problem can be conceived as a transportation problem by taking the sources as the cash inflows for the various months as *AR* Oct, *AR* Nov, and *AR* Dec along with the bank loan, while taking the destinations as the accounts payable for the three months *AP* Oct, *AP* Nov, and *AP* Dec. The cost elements can be derived as follows:
   (i) Money available in a month but not used until the following month earns an interest of 1 per cent. Accordingly, the cost is taken to be 0 when money is used in the same month, –1 when used in the next month, and –2 when used in the month following that.
   (ii) Payments can be delayed only by one month. A two-month delay is, thus, infeasible and attracts a very large penalty (M).
   (iii) Interest on bank loan is taken as 7.5, 5.0 and 2.5 per cent for loan taken in October, November, and December respectively.

   The information is shown tabulated in table given here.

Transportation Problem: Funds Management

Sources	*Destination*			*Supply*
	AP Oct	*AP Nov*	*AP Dec*	
AR Oct	0	–1	–2	18
AR Nov	2	0	–1	27
AR Dec	—	2	0	35
Loan	7.5	5.0	2.5	15
Demand	20	32	43	95

The initial solution using VAM is presented in table below and tested for optimality. The solution is found to be optimal.

Initial Solution: VAM

Sources	*Destinations*			*Supply*	u_i
	AP Oct	*AP Nov*	*AP Dec*		
AR Oct	0 [18]	–1 (–1)	–2 (–2)	18	0
AR Nov	2 [2]	0 [25]	–1 (–1)	27	2
AR Dec	*M*	2 [7]	0 [28]	35	4
Loan	7.5 (–1)	5.0 (–0.5)	2.5 [15]	15	6.5
Demand	20	32	43	95	
v_j	0	–2	–4		

The optimal solution implies:

(a) Use Rs 18 lakh of receipts of accounts receivable (*A*/*R*) in October to pay off accounts payable (*A*/*P*) of October.

(b) Use Rs 27 lakh of receipts of *A*/*R* in November to pay off Rs 25 lakh of *A*/*P* of November and Rs 2 lakh of *A*/*P* of October.

(c) Use Rs 35 lakh of *A*/*R* in December to pay off Rs 28 lakh of *A*/*P* of December and Rs 7 lakh of *A*/*P* of November.

(d) Use Rs 15 lakh of bank loan in December to pay off for *A*/*P* of December.

19. (a) Yes, the solution is feasible because all the rim requirements (demand and supply) are satisfied by it.

(b) No. It has the required number of 3 + 4 – 1 = 6 occupied cells.

(c) Yes, it is optimal since it has all Δ_{ij}'s less than, or equal to, zero. It is unique since any of the Δ_{ij} values for the unoccupied cells is not equal to zero.

(d) Rs 15 (given by the Δ_{ij} value).

(e) Optimal values of the dual variables are: $u_1 = 10$, $u_2 = 5$, $u_3 = 7$, $v_1 = 1$, $v_2 = –2$, $v_3 = 10$, and $v_4 = 0$.
The objective function values are given here:
For primal: $8 \times 250 + 20 \times 150 + 6 \times 200 + 15 \times 100 + 17 \times 100 + 7 \times 400 = 12{,}200$.
For dual: $400 \times 10 + 300 \times 5 + 500 \times 7 + 200 \times 1 – 250 \times 2 + 350 \times 10 + 400 \times 0 = 12{,}200$.

(f) The u_i value at each source indicates value of the product at ith origin while v_j is indicative of its delivered value at particular destination. For a given route, the delivered value of the product at the destination plus the value at the source involved in that route, cannot be greater than the cost of transporting a unit on that route.

(g) The cost would increase by Rs 8.

(h) Marginal gain = 25 per cent of Rs 12 = Rs 3 per unit. Marginal cost = Rs 7 (given by the Δ_{ij} value). Since $MG < MC$, it would not be advisable to accept the offer.

(i) The indicated adjustment in the output would cause a change of $2 \times 5 - 2 \times 10 = -10$. Any such adjustment which brings a reduction, like here, would imply that the cost would reduce while a positive change would cause the cost to rise. Thus, the total cost here would reduce by Rs 10.

(j) The management should concentrate in distribution centre D_2 since it has the smallest opportunity cost.

20. To derive the profit (or loss) obtainable by selling at a particular agency a unit produced at a particular plant, we proceed as follows. First, the cost of producing a unit at the plant is determined, and to this is added the appropriate shipping cost value, considering the agency to which it is sent. This total cost is then subtracted from the sales price at which it would be sold at that agency to get the unit profit. For example, an item produced in plant 1 and sold at agency 3 would involve a total cost of 18 + 7 = 25, where it can be sold for Rs 31. Thus, the profit value corresponding to the cell 1, 3 would be 31 – 25 = 6. Similarly, other values are determined as follows.

Plant	*Agency* 1	2	3	4	*Supply*
1	12	12	6	15	400
2	0	7	1	10	300
3	9	11	7	11	800
Demand	300	400	300	500	1,500

To solve this problem for maximisation, we first convert it into an opportunity loss matrix by subtracting each cell value from the largest value, 15. Then it is solved as a minimisation problem. This is shown in table below. The table also gives the initial solution using VAM.

Initial Feasible Solution: Non-optimal

Plant	*Agency* 1	2	3	4	*Supply*	u_i
1	3 (2) +	3 (0)	9 (2)	0 [400] −	400	0
2	15 (–5)	8 [200] −	14 (–2)	5 [100] +	300	5
3	6 [300] −	4 [200] +	8 [300]	4 (–3)	800	1
Demand	300	400	300	500	1,500	
v_j	5	3	7	0		

Total profit = 15 × 400 + 7 × 200 + 10 × 100 + 9 × 300 + 11 × 200 + 7 × 300 = Rs 15,400

The solution in the table is seen to be non-optimal. An improved solution is given below which is tested to be optimal.

Improved solution: Optimal

Plant	*Agency* 1	2	3	4	*Supply*	u_i
1	3 [200]	3 (–2)	9 (–4)	0 [200]	400	0
2	15 (–7)	8 (–2)	14 (–4)	5 [300]	300	5
3	6 [100]	4 [400]	8 [300]	4 (–1)	800	3
Demand	300	400	300	500	1,500	
v_j	3	1	5	0		

Total profit = 12 × 200 + 15 × 200 + 10 × 300 + 9 × 100 + 11 × 400 + 7 × 300 = Rs 15,800

21. From the given information, it is evident that the problem is an unbalanced one. A dummy row is introduced and the problem is balanced as shown below.

Investment made at the beginning of year	*Investment type* A	B	C	D	E	*Rupees available (in '000)*
	Net return data					
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Dummy	0	0	0	0	0	1,000
Maximum rupee investment ('000)	750	600	500	800	1,000	

Being a maximisation problem, it is first converted into a minimisation problem. The opportunity loss matrix is presented in table below. The loss entries here are expressed in paise for simplicity of presentation.

The initial solution using VAM is also given in the table. The solution is degenerate as the number of occupied cells is eight, as against the required nine (= 5 + 5 –1). To remove degeneracy, an ε is placed in the cell 1, 2. From the Δ_{ij} values calculated, it is evident that the solution is not optimal. Accordingly, a revised solution may be obtained for which a closed path in drawn starting from the cell 5, 2, which has largest Δ_{ij} value.

Opportunity Loss Matrix: Initial Solution

Year	*Investment*					*Availability*	u_i
	A	*B*	*C*	*D*	*E*		
1	20 (–20)	[ε] 10 –	40 (–50)	25 (–45)	[500] 00 +	500	0
2	45 (–20)	[600] 35	60 (–45)	40 (–35)	50 (–25)	600	25
3	70 (0)	75 (5)	70 (–10)	[750] 50	80 (–10)	750	70
4	[250] 85	88 (7)	[500] 75	[50] 65	90 (–5)	800	85
Dummy	[500] 100	100 (10)	100 (–10)	100 (–20)	[500] 100 –	1,000	100
Max. Investment	750	600	500	800	1,000	3,650	
v_j	0	10	–10	–20	0		

The revised solution is given here. Since all Δ_{ij}'s in this solution are seen to be less than, or equal to, zero, it is optimal.

Revised Solution: Optimal

Year	*Investment*					*Availability*	u_i
	A	*B*	*C*	*D*	*E*		
1	20 (–20)	10 (–10)	40 (–50)	25 (–45)	[500] 00	500	0
2	45 (–10)	[600] 35	60 (–35)	40 (–25)	50 (–15)	600	35
3	70 (0)	75 (–5)	70 (–10)	[750] 50	80 (–10)	750	70
4	[250] 85	88 (–3)	[500] 75	[50] 65	90 (–5)	800	85
Dummy	[500] 100	[ε] 100	100 (–10)	100 (–20)	[500] 100	1,000	100
Max. Investment	750	600	500	800	1,000	3,650	
v_j	0	0	–10	–20	0		

The optimal allocation is:

Year	*Investment type*	*Amount (000 Rs)*
1	*E*	500
2	*B*	600
3	*D*	750
4	*A*	250
4	*C*	500
4	*D*	50

22. **Option 1: When plant is situated at *C***

Opportunity Loss Matrix – Initial Solution (VAM): Optional

Plant	*Product*				*Capacity*	u_i
	P_1	P_2	P_3	*Dummy*		
A	2 [500]	13 (–4)	17 (–17)	37 [100]	600	0
B	7 (–5)	9 [800]	12 (–12)	37 [200]	1,000	0
C	17 (–15)	12 (–3)	0 [600]	37 [200]	800	0
Demand	500	800	600	500	2,400	
v_j	2	9	0	37		

Total profit = Rs 62,100

Option 2: When plant is situated at *D*

Opportunity Loss Matrix – Initial Solution (VAM): Optimal

Plant	*Product*				*Capacity*	u_i
	P_1	P_2	P_3	*Dummy*		
A	0 [500]	11 (–4)	15 (–5)	35 [100]	600	0
B	5 (–5)	7 [ε]	10 [600]	35 [400]	1,000	0
D	11 (–15)	3 [800]	7 (–1)	35 (–4)	800	–4
Demand	500	800	600	500	2,400	
v_j	0	7	10	35		

Total profit = Rs 58,100

Conclusion: Plant should be setup in city *C*.

23. The total amount required by five projects is Rs 750 thousand. Since a private bank can give any amount of credit, the amount allocated to this is 750 – (400 + 250) = Rs 100 thousand. The initial solution using VAM is given in table below.

Initial Feasible Solution: Optimal

Bank	*Project* P	Q	R	S	T	*Avail.*	u_i
Pvt.	20 (–2)	[100] 18 –	18 (0)	17 (–1)	17 (0)	100	2
Nat.	[200] 16	[50] 16 +	[150] 16 –	15 (–1)	16 (–1)	400	0
Co-op.	15 (0)	15 (0)	[50] 15 +	[125] 13	[75] 14 –	250	–1
Req.	200	150	200	125	75	750	
v_j	16	16	16	14	15		

The solution is tested and found to be an optimal one. It involves a total interest of 100 × 18% + 200 × 16% + 50 × 16% + 150 × 16% + 50 × 15% + 125 × 13% + 75 × 14% = Rs 116.25 thousand or Rs 1,16,225.

The solution is no unique, however, since some of the unoccupied cells have $\Delta_{ij} = 0$. A closed path drawn from each cell with $\Delta_{ij} = 0$ would yield an alternate optimal solution. One such solution is obtainable by starting with the cell 1, 5, as shown in the table. The alternate optimal solution is shown below.

Revised Solution: Alternate Optimal

Bank	*Project* P	Q	R	S	T	*Avail.*	u_i
Pvt.	20 (–2)	[25] 18	18 (0)	17 (–1)	[75] 17	100	0
Nat.	[200] 16	[125] 16	[75] 16	15 (–1)	16 –1	400	–2
Co-op.	15 (0)	15 (0)	[125] 15	[125] 13	14 (0)	250	–3
Req.	200	150	200	125	75	750	
v_j	18	18	18	16	17		

24. This problem is an unbalanced one since the amount available is Rs 230 lakh while the investment requirement is Rs 210 lakh. The problem is restated adding a dummy investment as shown here.

Investment made at the beginning of year	*Net return data (in paise) of selected investments*					*Amount available (lakh)*
	P	*Q*	*R*	*S*	*Dummy*	
1	95	80	70	60	0	70
2	75	65	60	50	0	40
3	70	45	50	40	0	90
4	60	40	40	30	0	30
Maximum Investment (lakh)	40	50	60	60	20	230

To solve this problem, it is first converted into an equivalent minimisation problem, as shown in table below. The initial solution, using VAM, is also given in the table.

Opportunity Loss Matrix: Initial Solution

Year	*Investment*					*Avail.*	u_i
	P	*Q*	*R*	*S*	*Dummy*		
1	0 [40]	15 [30]	25 (−5)	35 (−5)	95 (−35)	70	0
2	20 (−5)	30 [20]	35 [20]	45 (0)	95 (−20)	40	15
3	25 (0)	50 (−10)	45 [40]	55 [50]	95 (−10)	90	25
4	35 (0)	55 (−5)	55 (0)	65 [10]	95 [20]	30	35
Invest.	40	50	60	60	20	230	
v_j	0	15	20	30	60		

According to the solution obtained, the optimal investment plan is:

Year	*Investment*	*Net return*
1	Rs 40 lakh in *P*	Rs 40 lakh × 0.95 = Rs 38,00,000
	Rs 30 lakh in *Q*	Rs 30 lakh × 0.80 = Rs 24,00,000
2	Rs 20 lakh in *Q*	Rs 20 lakh × 0.65 = Rs 13,00,000
	Rs 20 lakh in *R*	Rs 20 lakh × 0.60 = Rs 12,00,000
3	Rs 40 lakh in *R*	Rs 40 lakh × 0.50 = Rs 20,00,000
	Rs 50 lakh in *S*	Rs 50 lakh × 0.40 = Rs 20,00,000
4	Rs 10 lakh in *S*	Rs 10 lakh × 0.30 = Rs 3,00,000
		Total Rs 1,30,00,000

25. By subtracting the wage scales for various applicant categories, we shall first obtain the efficiency matrix. Thus, for category value *A*, we shall subtract 1,000 from each of the values 1,000, 1,200, 1,000, and 1,500 respectively. Similarly, other values can be determined as shown below.

Relative Efficiency Matrix

Category	*Skill requirement level*							*Applicants*
	A	*B*	*C*	*D*	*E*	*F*	*Dummy*	
I	0	0	300	100	0	–50	0	54
II	200	150	0	50	0	–100	0	57
III	100	0	0	100	100	100	0	45
IV	500	400	400	100	0	0	0	74
Req.	25	29	31	40	33	17	55	230

Here, since the number of applicants exceeds the requirement, a column headed *dummy* has been introduced to balance the two. To solve this problem for maximisation of efficiency, we convert the problem into a minimisation problem, obtain the initial solution using VAM, and test the solution for optimality. The optimality test indicates that the solution is optimal.

Initial Feasible Solution: Optimal

Cat.	*Skill Requirement Level*							*App.*	u_i
	A	*B*	*C*	*D*	*E*	*F*	*Dummy*		
I	500	500	[11] 200	[40] 400	[3] 500	550	500	54	0
II	300	350	500	450	[2] 500	600	[55] 500	57	0
III	400	500	500	400	[28] 400	[17] 400	500	45	–100
IV	[25] 0	[29] 100	[20] 100	400	500	500	500	74	–100
Req.	25	29	31	40	33	17	55	230	
v_j	100	200	200	400	500	500	500		

According to the given values of u_i and v_j, we have
$\Delta_{11} = -400$, $\Delta_{12} = -300$, $\Delta_{16} = -50$, $\Delta_{17} = 0$, $\Delta_{21} = -200$, $\Delta_{22} = -150$, $\Delta_{23} = -300$, $\Delta_{24} = -50$, $\Delta_{26} = -100$, $\Delta_{31} = -400$, $\Delta_{32} = -400$, $\Delta_{33} = -400$, $\Delta_{34} = -100$, $\Delta_{37} = -100$, $\Delta_{44} = -100$, $\Delta_{45} = -100$, $\Delta_{46} = -100$, $\Delta_{47} = -100$.
The optimal solution, then, is to select $X_{IC} = 11$, $X_{ID} = 40$, $X_{IE} = 3$, $X_{IIE} = 2$, $X_{IIIE} = 28$, $X_{IVA} = 25$, $X_{IVB} = 29$, and $X_{IVC} = 20$.

26. **Production Planning Problem**

Prod. Month	*Supply month*						*Capacity*
	Jan	*Feb*	*Mar*	*Apr*	*May*	*Dummy*	
Jan	24	29	34	39	44	0	250
Feb	*M*	27	32	37	42	0	225
Mar	*M*	*M*	32	37	42	0	250
Apr	*M*	*M*	*M*	30	35	0	200
May	*M*	*M*	*M*	*M*	34	0	225
Demand	200	250	150	80	120	350	1,150

Production Planning-Optimal Solution

Prod. Month	*Supply month*						*Capacity*	u_i
	Jan	*Feb*	*Mar*	*Apr*	*May*	*Dummy*		
Jan	[200] 24	[25] 29	34 (–2)	39 (–9)	44 (–10)	[25] 0	250	0
Feb	*M*	[225] 27	32 (–2)	37 (–9)	42 (–10)	0 (–2)	225	–2
Mar	*M*	*M*	[150] 32	37 (–7)	42 (–8)	[100] 0	250	0
Apr	*M*	*M*	*M*	[80] 30	35 (–1)	[120] 0	200	0
May	*M*	*M*	*M*	*M*	[120] 34	[105] 0	225	0
Demand	200	250	150	80	120	350	1,150	
v_j	24	29	32	30	34	0		

Total cost = Rs 22,880

27. The cost matrix based on the given information, along with initial solution to the problem, is given in table below. The solution involves a total cost equal to Rs 2,230 and is non-optimal.

The improved solution is given in table that follows, which is found to be optimal, involving a total cost equal to Rs 2,210. It is not unique. The two optimal production plans are:

Plan I:

Production	*For use in*
Month 1 : 40 units	Month 1 : 20 units, Month 2 : 20 units
Month 2 : 30 units	Month 2 : 10 units, Month 3 : 20 units
Month 3 : 30 units	Month 3 : 30 units
Month 4 : 40 units	Month 4 : 40 units

Plan II:

Production	*For use in*
Month 1 : 40 units	Month 1 : 20 units, Month 2 : 20 units
Month 2 : 30 units	Month 2 : 30 units
Month 3 : 30 units	Month 3 : 30 units
Month 4 : 40 units	Month 4 : 40 units

Initial Basic Feasible Solution: Non-optimal

Month	*Month*					*Supply*	u_i
	1	*2*	*3*	*4*	*Dummy*		
1	[20] 14	[20] 15	16 (0)	17 (0)	0 (0)	40	0

(*Contd*)

(*Contd*)

2	*M*	16 [10]	17 [20]	18 [20] −	0 (1) +	50	1
3	*M*	*M*	15 [30]	16 (0)	0 (–1)	30	–1
4	*M*	*M*	*M*	17 [20] +	0 [30] –	50	0
Demand	20	30	50	40	30	170	
v_j	14	15	16	17	0		

Total cost = Rs 2,230

Improved Solution: Optimal

Month	*Month*					*Supply*	u_i
	1	*2*	*3*	*4*	*Dummy*		
1	14 [20]	15 [20]	16 (0)	17 (–1)	0 (–1)	40	0
2	*M*	16 [10]	17 [20]	18 (–1)	0 [20]	50	1
3	*M*	*M*	15 [30]	16 (–1)	0 (–2)	30	–1
4	*M*	*M*	*M*	17 [40]	0 [10]	50	1
Demand	20	30	50	40	30	170	
v_j	14	15	16	16	–1		

Total cost = Rs 2,210

28. Using given information, the cost matrix has been developed as shown in table. Here JN indicates January normal time, JOT indicates January overtime and so on, while JS shows January standard while JD shows January deluxe, and so on. The initial solution using VAM is obtained and, upon testing, found to be non-optimal. Subsequent tables δ are prepared to obtain optimal solution. The optimal solution involves a total cost of Rs 6,43,000.

Initial Feasible Solution: Non-optimal

	JS	JD	FS	FD	MS	MD	Dummy	Out.	u_i
JN	[50] 400 +	[100] 750	406 (4)	760 (0)	412 (8)	[150] 770 –	0 (–30)	300	0
JOT	[200] 450 –	820 (–20)	456 (4)	830 (–20)	462 (8)	840 (–20)	0 (20)	200	50
FN	M	M	[100] 400	[180] 750	406 (4)	[20] 760	0 (–40)	300	–10
FOT	M	M	[200] 450	820 (–20)	456 (4)	830 (–20)	0 (10)	200	40
MN	M	M	M	M	[295] 400 –	[5] 750 +	0 (–50)	300	–20
MOT	M	M	M	M	[25] 450 +	820 (–20)	[175] 0 –	200	30
Dem.	250	100	300	180	320	175	175	1,500	
v_j	400	750	410	760	420	770	–30		

Improved Solution 1: Non-optimal

	JS	JD	FS	FD	MS	MD	Dummy	Out.	u_i
JN	[200] 400	[100] 750	406 (–16)	760 (–20)	412 (–12)	770 (–20)	0 (–50)	300	0
JOT	[50] 450	820 (–20)	456 (–16)	830 (–40)	462 (–12)	840 (–40)	[150] 0	200	50
FN	M	M	[100] 400 +	[180] 750	406 (4)	[20] 760 –	0 (–40)	300	10
FOT	M	M	[200] 450 –	820 (–20)	456 (4)	830 (–20)	0 (10)	200	60
MN	M	M	M	M	[145] 400 –	[155] 750 +	0 (–50)	300	0
MOT	M	M	M	M	[175] 450 +	820 (–20)	[25] 0 –	200	50
Dem.	250	100	300	180	320	175	175	1,500	
v_j	400	750	390	740	400	750	–50		

Improved Solution 2: Optimal

	JS	*JD*	*FS*	*FD*	*MS*	*MD*	*Dummy*	*Out.*	u_i
JN	[200] 400	[100] 750	406 (–6)	760 (–10)	412 (–12)	770 (–20)	0 (–50)	300	0
JOT	[50] 450	820 (–20)	456 (–6)	830 (–30)	462 (–12)	840 (–40)	[150] 0	200	50
FN	*M*	*M*	[120] 400	[180] 750	406 (–6)	760 (–10)	0 (–50)	300	0
FOT	*M*	*M*	[180] 450	820 (–20)	456 (–6)	830 (–30)	[20] 0	200	50
MN	*M*	*M*	*M*	*M*	[125] 400	[175] 750	0 (–50)	300	0
MOT	*M*	*M*	*M*	*M*	[195] 450	820 (–20)	[5] 0	200	50
Dem.	250	100	300	180	320	175	175	1,500	
v_j	400	750	400	750	400	750	–50		

29. (a) The given data are presented in table here. Using VAM, the initial solution is found and presented in the table.

Initial Solution: Non-optimal

	B_1	B_2	B_3	B_4	B_5	*Avail.*	u_i
A_1	71 (–45)	70 (–31)	57 (11)	[400] 21 –	50 (–29)	400	0
A_2	[280] 55	[180] 68	[60] 97 –	[280] 50 +	53 (–3)	800	29
A_3	58 (–58)	50 (–37)	[400] 42	58 (–63)	27 (–32)	400	–26
A_4	66 (–28)	[180] 51	93 (–13)	35 (–2)	[220] 33	400	12
Req.	280	360	460	680	220	2,000	
v_j	26	39	68	21	21		

The solution is tested for optimality and is found to be non-optimal. The cell 1, 3 is found to have $\Delta_{ij} > 0$. Thus, beginning with this cell, a closed path is drawn as shown in the table. The revised solution is given in the following table. This is seen to be optimal. The solution involves a total cost of Rs 88,440.

Revised Solution: Optimal

	B_1	B_2	B_3	B_4	B_5	*Avail.*	u_i
A_1	71 (–45)	70 (–31)	60 / 57	340 / 21	50 (–29)	400	0
A_2	280 / 55	180 / 68	97 (–11)	340 / 50	53 (–3)	800	29
A_3	58 (–47)	50 (–26)	400 / 42	58 (–52)	27 (–21)	400	–15
A_4	66 (–28)	180 / 51	93 (–24)	35 (–2)	220 / 33	400	12
Req.	280	360	460	680	220	2,000	
v_j	26	39	57	21	21		

(b) The problem is presented as a transhipment problem in table below. The initial solution is found to be non-optimal. Successively improved solutions are given in the following tables.

From table, we observe the optimal solution to be: A_1 to B_4 : 680 units; A_2 to A_1 : 280 units; A_2 to B_1 : 280 units; A_2 to B_2 : 240 units; A_3 to B_3 : 400 units; A_4 to B_2 : 120 units: A_4 to B_5 : 280 units; and B_5 to B_3 : 60 units. Total cost Rs 85,260.

Initial Feasible Solution: Non-optimal

	A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4	B_5	*Supply*	u_i
A_1	0 [–0]	20 (–49)	23 (–8)	19 (–31)	71 (–45)	70 (–31)	57 [60] –	21 [340] +	50 (–29)	400	0
A_2	20 (9)	0 [–0]	34 (10)	17 (0)	55 [280]	68 [180] +	97 (–11)	50 [340] –	53 (–3)	800	29
A_3	23 (–38)	34 (–78)	0 [–0]	55 (–82)	58 (–47)	50 (–26)	42 [400]	58 (–52)	27 (–21)	400	–15
A_4	19 (–7)	17 (–34)	55 (–28)	0 [–0]	66 (–28)	51 [180] –	93 (–24)	35 (–2)	33 [220] +	400	12
B_1	71 (–97)	55 (–110)	58 (–69)	66 (–104)	0 [–0]	48 (–35)	28 (3)	24 (–29)	38 (–43)	0	–26
B_2	70 (–109)	68 (–136)	50 (–74)	51 (–102)	48 (–61)	0 [–0]	39 (–21)	54 (–72)	45 (–63)	0	–39
B_3	57 (–114)	97 (–183)	42 (–84)	93 (–162)	28 (–59)	39 (–57)	0 [–0]	36 (–72)	25 (–61)	0	–57
B_4	21 (–42)	50 (–100)	58 (–64)	35 (–68)	24 (–19)	54 (–36)	36 (0)	0 [–0]	52 (–52)	0	–21
B_5	50 (–71)	53 (–103)	27 (–33)	33 (–66)	38 (–33)	45 (–27)	25 (11) +	52 (–52)	0 [–0] –	0	–21
Demand	0	0	0	0	280	360	460	680	220	2,000	
v_j	0	–29	15	–12	26	39	57	21	21		

Improved Solution: Non-optimal

	A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4	B_5	Supply	u_i
A_1	[–0] 0 –	20 (–49)	23 (–19)	19 (–31)	71 (–45)	70 (–31)	57 (–11)	[400] 21 +	50 (–29)	400	0
A_2	20 (9) +	[–0] 0	34 (–1)	17 (0)	[280] 55	[240] 68	97 (–22)	[280] 50 –	53 (–3)	800	29
A_3	23 (–27)	34 (–67)	[–0] 0	55 (–71)	58 (–36)	50 (–15)	[400] 42	58 (–41)	27 (–10)	400	–4
A_4	19 (–7)	17 (–34)	55 (–39)	[–0] 0	66 (–28)	[120] 51	93 (–35)	35 (–2)	[280] 33	400	12
B_1	71 (–97)	55 (–110)	58 (–80)	66 (–104)	[–0] 0	48 (–35)	28 (–8)	24 (–29)	38 (–43)	0	–26
B_2	70 (–109)	68 (–136)	50 (–85)	51 (–102)	48 (–61)	[–0] 0	39 (–32)	54 (–72)	45 (–63)	0	–39
B_3	57 (–103)	97 (–172)	42 (–84)	93 (–151)	28 (–48)	39 (–46)	[–0] 0	36 (–61)	25 (–50)	0	–46
B_4	21 (–42)	50 (–100)	58 (–75)	35 (–68)	24 (–19)	54 (–36)	36 (–11)	[–0] 0	52 (–52)	0	–21
B_5	50 (–71)	53 (–103)	27 (–44)	33 (–66)	38 (–33)	45 (–27)	[60] 25	52 (–52)	[–60] 0	0	–21
Demand	0	0	0	0	280	360	460	680	220	2,000	
v_j	0	–29	4	–12	26	39	46	21	21		

Improved Solution: Optimal

	A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4	B_5	*Supply*	u_i
A_1	[–280] 0	20 (–40)	23 (–10)	19 (–22)	71 (–106)	70 (–22)	57 (–2)	[680] 21	50 (–20)	400	0
A_2	[280] 20	[–0] 0	34 (–1)	17 (0)	[280] 55	[240] 68	97 (–22)	50 (–9)	53 (–3)	800	20
A_3	23 (–36)	34 (–67)	[–0] 0	55 (–71)	58 (–36)	50 (–15)	[400] 42	58 (–50)	27 (–10)	400	–13
A_4	19 (–16)	17 (–34)	55 (–39)	[–0] 0	66 (–28)	[120] 51	93 (–35)	35 (–11)	[280] 33	400	3
B_1	71 (–106)	55 (–110)	58 (–80)	66 (–104)	[–0] 0	48 (–35)	28 (–8)	24 (–38)	38 (–43)	0	–35
B_2	70 (–118)	68 (–136)	50 (–85)	51 (–102)	48 (–61)	[–0] 0	39 (–32)	54 (–81)	45 (–63)	0	–48
B_3	57 (–112)	97 (–172)	42 (–84)	93 (–151)	28 (–48)	39 (–46)	[–0] 0	36 (–70)	25 (–50)	0	–55
B_4	21 (–42)	50 (–91)	58 (–66)	35 (–59)	24 (–10)	54 (–27)	36 (–2)	[–0] 0	52 (–43)	0	–21
B_5	50 (–80)	53 (–103)	27 (–44)	33 (–66)	38 (–33)	45 (–27)	[60] 25	52 (–61)	[–60] 0	0	–30
Demand	0	0	0	0	280	360	460	680	220	2,000	
v_j	0	–20	13	–3	35	48	55	21	30		

30. (a) From the given information, the cost matrix is shown below. Also given in the table is the initial feasible solution using VAM. The solution is tested and found to be optimal, though not unique. The solution are: $x_{12} = 40$, $x_{13} = 200$, $x_{21} = 80$, $x_{22} = 80$; and $x_{12} = 120$, $x_{13} = 120$, $x_{21} = 80$, $x_{23} = 80$. Total cost = 36,400.

Initial Feasible Solution: Optimal

	W_1	W_2	W_3	*Capacity*	u_i
P_1	100 (–30)	[40] 90	[200] 60	240	0
P_2	[80] 120	[80] 140	110 (0)	160	50
Req.	80	120	200	400	
v_j	70	90	60		

(b) The given problem is represented as a transhipment problem in table given below. The optimal solution obtained in (a) above is reproduced. Upon testing, it is found to be optimal in this case as well. In addition to the above two optimal solutions, another one can be traced. This is: P_1 to $W_3(x_{15}) = 240$, P_2 to $W_1(x_{23}) = 80$, P_2 to $W_2(x_{24}) = 80$, and W_3 to $W_2(x_{54}) = 40$ units.

Initial Feasible Solution: Optimal

	P_1	P_2	W_1	W_2	W_3	*Capacity*	u_i
P_1	[–0] 0	80 (–130)	100 (–30)	[40] 90	[200] 60	240	0
P_2	80 (–30)	[–0] 0	[80] 120	[80] 140	110 (0)	160	50
W_1	100 (–170)	120 (–240)	[–0] 0	60 (–40)	80 (–90)	0	–70
W_2	90 (–180)	140 (–280)	60 (–80)	[–0] 0	30 (–60)	0	–90
W_3	60 (–120)	110 (–220)	80 (–70)	30 (0)	[–0] 0	0	–60
Req.	0	0	80	120	200	400	
v_j	0	–50	70	90	60		

31. The optimal solution to the given problem, assuming it to be a transportation problem, is shown in table below.

Table 1 : Optimal solution as a TP

To → / From ↓	*D*	*E*	*F*	*Supply*	u_i
A	6 (−7)	[50] 4	[35] 1	50	0
B	[20] 3	[20] 8	7 (−2)	40	4
C	4 (−5)	[60] 4	2 (−1)	60	0
Demand	20	95	35	150	
v_j	−1	4	1		

Now, we consider the solution to the problem as a transhipment problem. The solution is shown in Tables 1 through 5. The optimal solution involves a total cost of Rs 405.

Table 2 Initial Feasible Solution: Non-optimal

Terminal	*A*	*B*	*C*	*D*	*E*	*F*	*Supply*	u_i
A	[−0] 0	3 (−7)	2 (−2)	6 (−7)	[15] 4	[35] 1	50	0
B	3 (1)	[−0] 0	4 (0)	[20] 3 +	[20] 8 −	7 (−2)	40	4
C	2 (−2)	4 (−8)	[−0] 0	4 (−5)	[60] 4	2 (−3)	60	0
D	6 (−5)	3 (−6)	4 (−3)	[−0] 0 −	2 (3) +	5 (−3)	0	1
E	4 (−8)	8 (−16)	4 (−8)	2 (−7)	[−0] 0	1 (−4)	0	−4
F	1 (−2)	7 (−12)	2 (−3)	5 (−7)	1 (2)	[−0] 0	0	−1
Demand	0	0	0	20	95	35	150	
v_j	0	−4	0	−1	4	1		

Total cost: 4 × 15 + 1 × 35 + 3 × 20 + 8 × 20 + 4 × 60 = Rs 555

Table 3 Improved Solution: Non-optimal

Terminal	*A*	*B*	*C*	*D*	*E*	*F*	*Supply*	u_i
A	0 [–0]	3 (–4)	2 (–2)	–6 (–4)	4 [15] –	1 [35] +	50	0
B	3 (–2)	0 [–0]	4 (–5)	3 [40]	8 (3)	7	40	1
C	2 (–2)	4 (–5)	0 [–0]	4 (–2)	4 [60]	2 (–1)	60	0
D	6 (–8)	3 (–6)	4 (–6)	0 [–20]	2 [20]	5 (–6)	0	–2
E	4 (–8)	8 (–13)	4 (–8)	2 (–4)	0 [–0]	1 (–4)	0	–4
F	1 (–2)	7 (–9)	2 (–3)	5 (–4)	1 (2) +	0 [–0] –	0	–1
Demand	0	0	0	20	95	35	150	
v_j	0	–1	0	2	4	1		

Total cost: 4 × 15 + 1 × 35 + 3 × 40 + 4 × 60 + 2 × 60 = Rs 495

Table 4 Improved Solution: Non-optimal

Terminal	*A*	*B*	*C*	*D*	*E*	*F*	*Supply*	u_i
A	0 [–0]	3 (–6)	2 (–4)	6 (–6)	4 (–2)	1 [50]	50	0
B	3 (0)	0 [–0]	4 (–5)	3 [40]	8 (–3)	7 (–3)	40	3
C	2 (0)	4 (–5)	0 [–0]	4 (–2)	4 [60] –	2 (1) +	60	2
D	6 (–6)	3 (–6)	4 (–6)	0 [–20]	2 [20]	5 (–4)	0	0
E	4 (–6)	8 (–13)	4 (–8)	2 (–4)	0 [–0]	1 (–2)	0	–2
F	1 (–2)	7 (–11)	2 (–5)	5 (–6)	1 [15] +	0 [–15] –	0	–1
Demand	0	0	0	20	95	35	150	
v_j	0	–3	–2	0	2	1		

Total cost: 1 × 50 + 3 × 40 + 4 × 60 + 2 × 20 + 1 × 75 = Rs 465

Table 5 Improved Solution: Optimal

Termimal	*A*	*B*	*C*	*D*	*E*	*F*	*Supply*	u_i
A	0 [–0]	3 (–6)	2 (–3)	6 (–6)	4 (–2)	1 [50]	50	0
B	3 (0)	0 [–0]	4 (–2)	3 [40]	8 (–3)	7 (–3)	40	3
C	2 (–1)	4 (–6)	0 [–0]	4 (–3)	4 (–1)	2 [60]	60	1
D	6 (–6)	3 (–6)	4 (–5)	0 [–20]	2 [20]	5 (–4)	0	0
E	4 (–6)	8 (–13)	4 (–7)	2 (–4)	0 [–0]	1 (–2)	0	–2
F	1 (–2)	7 (–11)	2 (–4)	5 (–6)	1 [75]	0 [–75]	0	–1
Demand	0	0	0	20	95	35	150	
v_j	0	–3	–1	0	2	1		

Total cost = 1 × 50 + 3 × 40 + 2 × 60 + 2 × 20 + 1 × 75 = Rs 405

CHAPTER 6

1. (a) Formulation of assignment problem as transportation problem:

Transportation Problem

Worker	*Job*			*Supply*
	1	*2*	*3*	
1	3	4.5	7	1
2	6	4	6	1
3	5	*M*	4	1
Demand	1	1	1	3

(b) Assignment problem as a linear programming problem:

Minimise $Z = 3x_{11} + 4.5x_{12} + 7x_{13} + 6x_{21} + 4x_{22} + 6x_{23} + 5x_{31} + Mx_{32} + 4x_{33}$

Subject to

$x_{11} + x_{12} + x_{13} = 1$
$x_{21} + x_{22} + x_{23} = 1$
$x_{31} + x_{32} + x_{33} = 1$
$x_{11} + x_{21} + x_{31} = 1$
$x_{12} + x_{22} + x_{32} = 1$
$x_{13} + x_{23} + x_{33} = 1$
$x_{ij} = 0$ or 1, for $i = 1, 2, 3$; $j = 1, 2, 3$

2. The problem is:

Worker	*Job 1*	*Job 2*	*Job 3*
A	8	5	6
B	6	9	4
C	5	7	7

Assignments:

A–1	*B*–2	*C*–3	24
A–1	*B*–3	*C*–2	19
A–2	*B*–1	*C*–3	18
A–2	*B*–3	*C*–1	14*
A–3	*B*–1	*C*–2	19
A–3	*B*–2	*C*–1	20

3. Enumeration Method:

A–1	*B*–2	*C*–3	15
A–1	*B*–3	*C*–2	12
A–2	*B*–1	*C*–3	16
A–2	*B*–3	*C*–1	9*
A–3	*B*–1	*C*–2	20
A–3	*B*–2	*C*–1	16

HAM:

RCT–1

2	[0]	5
5	2	[0]
[0]	1	2

A–2, *B*–3, *C*–1

4. **Reduced-Cost Table 1**

0	2	3	1	7
4	0	3	6	2
4	1	0	2	5
2	5	0	4	3
2	0	4	3	6

Reduced-Cost Table 2

0	2	3	0	5
4	0	3	5	0
4	1	0	1	3
2	5	0	3	1
2	0	4	2	4

Reduced-Cost Table 3

0	2	4	0	5
4	0	4	5	0
3	0	0	0	2
1	4	0	2	0
2	0	5	2	4

The optimal assignment is:

Brand	*Set up*	*Cost*
B_1	S_1	4
B_2	S_5	5
B_3	S_4	6
B_4	S_3	7
B_5	S_2	5
	Total	27

5. (a) To determine the optimal assignment pattern that minimises the total time taken, we apply Hungarian method to the given matrix. The row reductions are shown in Reduced-Cost Table 1 while the column reductions are presented in Reduced-Cost Table 2. Note that although the given data are in hours, the word 'Cost' is used in a broad sense while presenting the results in Reduced-Cost tables.

Reduced-Cost Table 1

5	7	1	1	0
5	0	5	5	4
0	1	6	7	4
0	4	1	1	3
4	4	7	2	0

Reduced-Cost Table 2

5	7	0	0	0
5	0	4	4	4
0	1	5	6	4
0	4	0	0	3
4	4	6	1	0

The minimum number of lines covering all zeros is five here, which equals the matrix order. Thus, assignments are made. Here, there are two alternate solutions as:

Alternative 1			*Alternative 2*		
Job	*Employee*	*Time*	*Job*	*Employee*	*Time*
1	*C*	3	1	*D*	3
2	*B*	1	2	*B*	1
3	*A*	3	3	*A*	3
4	*D*	2	4	*C*	2
5	*E*	2	5	*E*	2

(b) **Cost Matrix**

Job	*Employee*				
	A	*B*	*C*	*D*	*E*
1	105	135	45	39	26
2	90	15	90	78	65
3	45	60	135	130	91
4	15	75	30	26	52
5	90	90	135	52	26

RCT–1

79	109	19	13	0
75	0	75	63	50
0	15	90	85	46
0	60	15	11	37
64	64	109	26	0

RCT–2

79	109	4	2	0
75	0	60	52	50
0	15	75	74	46
0	60	0	0	37
64	64	94	15	0

RCT–3

79	107	2	0	0
77	0	60	52	52
0	13	73	72	46
2	60	0	0	39
64	62	92	13	0

Optimal assignments: 1–*D*, 2–*B*, 3–*A*, 4–*C*, 5–*E*. Total Cost = 155

6. **RCT–1**

25	0	50	40	50
25	0	65	50	50
22	0	67	52	57
25	0	75	45	55
25	0	65	47	50

RCT–2

3	0	0	0	0
3	0	15	10	0
0	0	17	12	7
3	0	25	5	5
3	0	15	7	0

RCT–3

3	3	0	0	3
0	0	12	7	0
0	3	17	12	10
0	0	22	2	5
0	0	12	4	0

RCT–4

5	5	0	0	5
0	0	10	5	0
0	3	15	10	10
0	0	20	0	5
0	0	10	2	0

Alternate optimal assignments:

Job	*Machine*	*Machine*
J_1	M_3	M_3
J_2	M_2	M_5
J_3	M_1	M_1
J_4	M_4	M_4
J_5	M_5	M_2

Total Cost = 365

7. **RCT–1**

3	2	0	1	2	3
6	4	0	1	5	4
0	0	3	2	1	1
0	4	6	5	1	4
0	0	0	0	0	0
0	0	0	0	0	0

RCT–2

3	1	0	0	1	2
6	3	0	0	4	3
1	0	4	2	0	0
0	3	6	4	0	3
1	0	1	0	0	0
1	0	1	0	0	0

The problem has multiple optimal solutions. One such solution is given here. Patrol units 1, 3, 4 and 5 should respond. Average response time is 3.5 minutes.

8. To solve this problem, we first balance it by introducing two rows with zero elements, dummy stores. The problem is restated below stating the bids in units of Rs 10,000s.

Store	*Construction company*					
	1	*2*	*3*	*4*	*5*	*6*
1	853	880	875	824	891	867
2	789	774	774	765	793	783
3	820	813	824	806	835	817
4	843	846	862	833	844	855
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Reduced-Cost Table 1

29	56	51	0	67	43
24	9	9	0	28	18
14	7	18	0	29	11
10	13	29	0	11	22
0	0	0	0	0	0
0	0	0	0	0	0

Reduced-Cost Table 2

22	49	44	0	60	36
17	2	2	0	21	11
7	0	11	0	22	4
3	6	22	0	4	15
0	0	0	7	0	0
0	0	0	7	0	0

Reduced-Cost Table 3

20	47	42	0	58	34
15	0	0	0	19	9
7	0	11	2	22	4
1	4	20	0	2	13
0	0	0	9	0	0
0	0	0	9	0	0

Reduced-Cost Table 4

19	47	42	0	57	33
14	0	0	0	18	8
6	0	11	2	21	3
0	4	20	0	1	12
0	1	1	10	0	0
0	1	1	10	0	0

The optimal assignment is:

Store	*Company*	*Cost (Rs lakh)*
1	4	82.4
2	3	77.4
3	2	81.3
4	1	84.3
	Total	325.4

9. **Reduced-Cost Table 1**

120	110	90	110	70	0	0	0
70	130	100	120	60	0	0	0
0	10	110	10	30	0	0	0
140	0	0	60	20	0	0	0
90	30	0	70	0	0	0	0
10	20	30	10	20	0	0	0
40	120	40	0	20	0	0	0
80	10	70	30	40	0	0	0

Reduced-Cost Table 2

110	100	80	100	60	0	0	0
60	120	90	110	50	0	0	0
0	10	110	10	30	10	10	10
140	0	0	60	20	10	10	10
90	30	0	70	0	10	10	10
0	10	20	0	10	0	0	0
40	120	40	0	20	10	10	10
70	0	60	20	30	0	0	0

Optimal assignments are:

T_1	—;	T_5	5;
T_2	—;	T_6	—;
T_3	1;	T_7	4;
T_4	3;	T_8	2;

Total cost = 380

10.(i) **Cost Matrix (Revised)**

Project	*Contractor*				
	A	*B*	*C*	*D*	*E*
1	18	25	22	26	25
2	26	29	26	27	24
3	28	31	30	*M*	31
4	26	28	27	26	29
5	0	0	0	0	0

Reduced-Cost Table 1

Contractor				
A	*B*	*C*	*D*	*E*
0	7	4	8	7
2	5	2	3	0
0	3	2	*M*	3
0	2	1	0	3
0	0	0	0	0

Reduced-Cost Table 2

0	6	3	8	7
2	4	1	3	0
0	2	1	*M*	3
0	1	0	0	3
1	0	0	1	1

Reduced-Cost Table 3

0	5	2	7	7
2	3	0	2	0
0	1	0	*M*	3
1	0	0	0	4
2	0	0	1	2

Optimal assignment schedule is: 1–*A*; 2–*E*; 3–*C*; 4–*D*.

(ii) Mimimum Total Cost = 18 + 24 + 30 + 26 = Rs 98 thousand.

11. **RCT–1**

0	3	1	2
M	4	3	0
0	*M*	2	0
4	4	2	0

RCT–2

0	0	0	2
M	1	2	0
0	*M*	1	0
4	1	1	0

RCT–3

0	0	0	3
M	0	1	0
0	*M*	1	1
3	0	0	0

Alternate optimal assignments:

Clerk	*Job*	*Job*
I	*C*	*B*
II	*B*	*D*
III	*A*	*A*
IV	*D*	*C*

Total time = 18 hours

12. **Reduced-Cost Table 1**

0	2	6	1	2
3	0	M	1	0
M	4	7	4	0
7	1	5	0	1
0	0	0	0	0

Optimal assignment is:

Machine	*Place*	*Cost (Rs)*
M_1	*A*	9
M_2	*B*	9
M_3	*E*	7
M_4	*D*	7
	Total	32

13. The given information is presented below where time taken by various swimmers is given in seconds. The swimmer-swimming style combinations not feasible are indicated by *M*. Further, a dummy style has been added to balance the problem. Based on this, Reduced-Cost Table 1 is obtained where each row is considered and its least value is subtracted from every value. Lines are drawn to cover zeros. Since four lines cover all zeros, which is less than $n(= 5)$, assignments cannot be made.

Swimming Time Matrix

Swimmer	*Style*				
	Back-stroke	*Breaststroke*	*Freestyle*	*Butterfly*	*Dummy*
Anand	69	75	*M*	*M*	0
Bhaskar	*M*	76	61	80	0
Chandru	70	80	65	72	0
Dorai	*M*	*M*	*M*	71	0
Easwar	80	76	66	70	0

Reduced-Cost Table 1

0	0	M	M	0
M	1	0	10	0
1	5	4	2	0
M	M	M	1	0
11	1	5	0	0

Reduced-Cost Table 2

0	[0]	M	M	1
M	1	[0]	11	1
[0]	4	3	2	0
M	M	M	1	[0]
10	0	4	[0]	0

Optimal assignment is:

Anand : Breaststroke
Bhaskar : Freestyle
Chandru : Back-stroke
Easwar : Butterfly

Dorai would be left out of the relay.

14. (i) Due to requirement (c), the following bids are not acceptable: 3–*A*, 4–*D*, and 2–*E*.

(ii) After introducing a column for dummy contract and replacing the cost element for each prohibited assignment by *M*, the cost matrix is given here:

Bidder	*Contract*					
	A	*B*	*C*	*D*	*E*	*F*
1	7	8	8	12	7	0
2	9	13	10	14	*M*	0
3	*M*	7	6	13	11	0
4	17	17	7	*M*	8	0
5	8	12	7	15	16	0
6	10	10	10	16	8	0

Reduced-Cost Table 1

0	1	2	0	0	0
2	6	4	2	*M*	0
M	0	0	1	4	0
10	10	1	*M*	1	0
1	5	1	3	9	0
3	3	4	4	1	0

Reduced-Cost Table 2

0	1	2	0	0	1
1	5	3	1	*M*	0
M	0	0	1	4	1
9	9	0	*M*	0	0
0	4	0	2	8	0
2	2	3	3	0	0

The least number of lines covering all zeros matches with the matrix order. Hence, assignments are shown made. The successful bidders are:

1–*D*, 3–*B*, 4–*C*, 5–*A*, and 6–*E*.

(iii) Contract assignment cost = 12 + 7 + 7 + 8 + 8
= Rs 42 thousand.

(iv) If requirement (c) is waived, the cost matrix would appear as:

Bidder	*Contract*					
	A	*B*	*C*	*D*	*E*	*F*
1	7	8	8	12	7	0
2	9	13	10	14	5	0
3	3	7	6	13	11	0
4	17	17	7	8	8	0
5	8	12	7	15	16	0
6	10	10	10	16	8	0

Reduced-Cost Table 3

4	1	2	4	2	0
6	6	4	6	0	0
0	0	0	5	6	0
14	10	1	0	3	0
5	5	1	7	11	0
7	3	4	8	3	0

Reduced-Cost Table 4

3	0	1	3	2	0
5	5	3	5	0	0
0	0	0	5	7	1
14	10	1	0	4	1
4	4	0	6	11	0
6	2	3	7	3	0

The total number of lines covering all zeros in RCT 4 is six that equals *n*. Hence, assignments are made. The optimal assignment pattern is:

1–*B*, 2–*E*, 3–*A*, 4–*D*, and 5–*C*.

Total Cost = 8 + 5 + 3 + 8 + 7 = Rs 31 thousand.

15. (a) The given problem appears to be an unbalanced one. However, a careful consideration suggests that it is not so since two jobs can be done internally. Thus, the completed table is given here in which the cost row for 'internal' is included twice.

		Job					
		1	*2*	*3*	*4*	*5*	*6*
Firm	1	48	72	36	52	50	65
	2	44	67	41	53	48	64
	3	46	69	40	55	45	68
	4	43	73	37	51	44	62
	I_1	50	65	35	50	46	63
	I_2	50	65	35	50	46	63

Reduced-Cost Table 1

12	36	0	16	14	29
3	26	0	12	7	23
6	29	0	15	5	28
6	36	0	14	7	25
15	30	0	15	11	28
15	30	0	15	11	28

Reduced-Cost Table 2

9	10	0	4	9	6
0	0	0	0	2	0
3	3	0	3	0	5
3	10	0	2	2	2
12	4	0	3	6	5
12	4	0	3	6	5

Reduced-Cost Table 3

7	8	0	2	7	4
0	0	2	0	2	0
3	3	2	3	0	5
1	8	0	0	0	0
10	2	0	1	4	3
10	2	0	1	4	3

Reduced-Cost Table 4

6	7	0	1	6	3
0	0	3	0	2	0
3	3	3	3	0	5
1	8	1	0	0	0
9	1	0	0	3	2
9	1	0	0	3	2

Reduced-Cost Table 5

5	6	[0]	1	5	2
[0]	0	4	1	2	0
3	3	4	4	[0]	5
1	8	2	1	0	[0]
8	[0]	0	0	2	1
8	0	0	[0]	2	1

Firm	*Job*	*Cost*
1	3	36
2	1	44
3	5	45
4	6	62
I_1	2	65
I_2	4	50
	Total	302

(0,000 Rs)

16. **Revised-Time Matrix**

Operator	*Job* 1	2	3	4	5	6
1	6	2	5	2	6	0
2	2	5	8	7	7	0
3	7	8	6	9	8	0
4	6	2	3	4	5	0
5	9	3	8	9	7	0
6	4	7	4	6	8	0

Reduced-Cost Table 1

Job 1	2	3	4	5	6
4	0	2	0	1	0
0	3	5	5	2	0
5	6	3	7	3	0
4	0	0	2	0	0
7	1	5	7	2	0
2	5	1	4	3	0

Reduced-Cost Table 2

5	0	2	[0]	1	1
[0]	2	4	4	1	0
5	5	2	6	2	[0]
5	0	0	2	[0]	1
7	[0]	4	6	1	0
2	4	[0]	3	2	0

Optimal assignment schedule is:

Operator	*Job*	*Time*
1	4	2
2	1	2
3	6	— (*Dummy*)
4	5	5
5	2	3
6	3	4

Determination of dual variable values: For determining the dual variables, the given problem is formulated and expressed as a transportation problem. The optimal solution is substituted into it. The degeneracy of the solution is removed by placing necessary number of epsilons as shown. The u_i and v_j values represent the optimal values of the dual variables. These are presented in the following table.

Obtaining Dual Variable Values

	1	*2*	*3*	*4*	*5*	*6*	*SS*	u_i
1	6	2	5	[1] 2	6	[ε] 0	1	0
2	[1] 2	5	8	7	7	[ε] 0	1	0
3	7	8	6	9	8	[1] 0	1	0
4	6	2	3	4	[1] 5	[ε] 0	1	0
5	9	[1] 3	8	9	7	[ε] 0	1	0
6	4	7	[1] 4	6	8	[ε] 0	1	0
DD	1	1	1	1	1	1	6	
v_j	2	3	4	2	5	0		

DD: Demand, *SS:* Availability

17. Since the problem is of maximisation type, we first convert it into a minimisation type. The relative inefficiency matrix is obtained by subtracting each score from 50. After this, the problem is solved using Hungarian method.

Relative Inefficiency Matrix

30	24	8
26	18	0
18	16	6

Reduced-Cost Table 1

22	16	0
26	18	0
12	10	0

Reduced-Cost Table 2

10	6	0
14	8	0
0	0	0

Reduced-Cost Table 3

4	[0]	~~0~~
8	2	[0]
[0]	~~0~~	(6)

The optimal assignment pattern is:
P_1: Education, P_2: Housing, P_3: Health.
Total performance score = 26 + 50 + 32 = 108

18. As a first step, we balance the given problem by introducing a dummy salesman. This is shown below. Now, to solve it, we transform it into an equivalent minimisation problem. For this, we subtract each element of the matrix from the largest value, which is 85. This is expressed as opportunity Loss Matrix and given alongside.

Sales (Rs in lakh)

Salesman	*Market*			
	I	*II*	*III*	*IV*
A	80	70	75	72
B	75	75	80	85
C	78	78	82	78
D	0	0	0	0

Opportunity Loss Matrix

Market			
I	*II*	*III*	*IV*
5	15	10	13
10	10	5	0
7	7	3	7
85	85	85	85

Reduced-Cost Table 1

[0]	10	5	8
10	10	5	[0]
4	4	[0]	4
0	[0]	0	0

The result of optimal assignment is:

Salesman	*Market*	*Sales (Rs lakh)*
A	*I*	80
B	*IV*	85
C	*III*	82
	Total	247

19. **Opportunity Loss Matrix**

49	33	61	10	29
40	27	50	38	52
24	19	0	40	30
63	47	24	34	31
III	III	III	III	III

Reduced-Cost Table 1

39	23	51	0	19
13	0	23	11	25
24	19	0	40	30
39	23	0	10	7
0	0	0	0	0

Reduced-Cost Table 2

32	23	51	[0]	12
6	[0]	23	11	18
17	19	[0]	40	23
32	23	0	10	[0]
[0]	7	7	7	0

Optimal assignments:

1–*D*, 2–*B*, 3–*C*, 4–*E*
Maximum profit = 376
Decline job *A*.

20. **Opportunity Loss Matrix**

35	15	10	25
5	15	0	15
10	15	13	5
7	5	13	10

Reduced-Cost Table 1

25	5	0	15
5	15	0	15
5	10	8	0
2	0	8	5

Reduced-Cost Table 2

23	5	0	15
3	15	0	15
3	10	8	0
0	0	8	5

Reduced-Cost Table 3

20	2	0	15
0	12	0	15
0	7	8	0
0	0	11	8

Optimal assignment: *A–Y*, *B–W*, *C–Z*, *D–X*
Total monthly sales = 145 + 150 + 150 + 150 = Rs 595 lakh.

21. **Sales Data**

Salesman	*Sales territories*				
	I	*II*	*III*	*IV*	*V*
A	75	80	85	70	90
B	91	71	82	75	85
C	78	90	85	80	80
D	65	75	88	85	90
E	0	0	0	0	0

Opportunity Loss matrix

Sales territories				
I	*II*	*III*	*IV*	*V*
16	11	6	21	1
0	20	9	16	6
13	1	6	11	11
26	16	3	6	1
91	91	91	91	91

Reduced-Cost Table 1

15	10	5	20	0
0	20	9	16	6
12	0	5	10	10
25	15	2	5	0
0	0	0	0	0

Reduced-Cost Table 2

13	8	3	18	0
0	20	9	16	8
12	0	5	10	12
23	13	0	3	0
0	0	0	0	2

The optimal assignment schedule, accordingly, is:

Salesman	:	*A*	*B*	*C*	*D*
Territory	:	*V*	*I*	*II*	*III*

Total Sales : 90 + 91 + 90 + 88 = Rs 359 lakh.

When D cannot be assigned Territory III: The solution can be obtained by replacing element three in the opportunity Loss Matrix given earlier, lying on the intersection of *D*-III, by *M*. It is given here.

Opportunity Loss Matrix (Revised)

Salesman	*Sales territories*				
	I	*II*	*III*	*IV*	*V*
A	16	11	6	21	1
B	0	20	9	16	6
C	13	1	6	11	11
D	26	16	*M*	6	1
E	91	91	91	91	91

Reduced-Cost Table 3

Sales territories				
I	*II*	*III*	*IV*	*V*
15	10	5	20	0
0	20	9	16	6
12	0	5	10	10
25	15	*M*	5	0
0	0	0	0	0

Reduced-Cost Table 4

10	5	0	15	[0]
[0]	20	9	16	11
12	[0]	5	10	15
20	10	*M*	[0]	0
0	0	[0]	0	5

Thus, optimal assignment schedule is:

Salesman	*Territory*	*Sales (Rs lakh)*
A	*V*	90
B	*I*	91
C	*II*	90
D	*IV*	85
	Total	356

22. **Relative Inefficiency Matrix**

	C_1	C_2	C_3	C_4
P_1	10	30	10	0
P_2	50	10	20	0
P_3	50	40	30	10
TA	40	60	40	30

Reduced-Cost Table 1

C_1	C_2	C_3	C_4
10	30	10	0
50	10	20	0
40	30	20	0
10	30	10	0

Reduced-Cost Table 2

[0]	20	0	0
40	[0]	10	0
30	20	10	[0]
0	20	[0]	0

Alternate optimal assignments are:

$P_1 - C_1$, $P_2 - C_2$, $P_3 - C_4$, $TA - C_3$;
$P_1 - C_3$, $P_2 - C_2$, $P_3 - C_4$, $TA - C_1$.

23. (i) **Relative Inefficiency Matrix**

20	20	25	35	10
18	30	44	35	33
10	12	20	0	10
40	41	40	42	35
2	0	1	5	7

Reduced-Cost Table 1

10	10	15	25	0
0	12	26	17	15
10	12	20	0	10
5	6	5	7	0
2	0	1	5	7

Reduced-Cost Table 2

10	10	14	25	0
0	12	25	17	15
10	12	19	0	10
5	6	4	7	0
2	0	0	5	7

Reduced-Cost Table 3

6	6	10	25	[0]
[0]	12	25	21	19
6	8	15	[0]	10
1	2	[0]	7	0
2	[0]	0	9	11

The assignments are:

P : *V*, *Q* : *I*, *R* : *IV*, *S* : *III*, *T* : *II*.

Total runs = 50 + 42 + 60 + 20 + 60 = 232.

(ii) Now we include another batsman *U* in the minimisation matrix by subtracting average runs of the batsman *U* from 60 as before. To balance the resulting problem, a dummy batting position is added. This is shown in the revised Relative Inefficiency Matrix given below. Reduced-Cost Table 4 is derived from this by column reductions.

Relative Inefficiency Matrix (R)

20	20	25	35	10	0
18	30	44	35	33	0
10	12	20	0	10	0
40	41	40	42	35	0
2	0	1	5	7	0
15	8	22	10	11	0

Reduced-Cost Table 4

18	20	24	35	3	0
16	30	43	35	26	0
8	12	19	0	3	0
38	41	39	42	28	0
0	0	0	5	0	0
13	8	21	10	4	0

Reduced-Cost Table 5

15	17	21	32	0	0
13	27	40	32	23	0
8	12	19	0	3	3
35	38	36	39	25	0
0	0	0	5	0	3
10	5	18	7	1	0

Reduced-Cost Table 6

10	12	16	27	0	0
8	22	35	27	23	0
8	12	19	0	8	8
30	33	31	34	25	0
0	0	0	5	5	8
5	0	13	2	1	0

Reduced-Cost Table 7

2	4	8	27	0	0
0	14	27	27	23	0
0	4	11	0	8	8
22	25	23	34	25	0
0	0	0	13	13	16
5	0	13	10	9	8

Optimal assignment is:

Batsman	*Position*	*Runs*
P	*V*	50
Q	*I*	42
R	*IV*	60
S	*Dummy*	—
T	*III*	59
U	*II*	52

Thus, batsman *U* will be included in the team at position *II* and he would replace batsman *S*. Total runs equal 263.

24. **Opportunity Loss Matrix**

	C_1	C_2	C_3	C_4	C_5
G_1	52	54	89	60	65
G_2	94	76	92	85	95
G_3	71	66	80	46	74
G_4	28	8	39	0	37
G_5	110	110	110	110	110

Reduced-Cost Table 1

C_1	C_2	C_3	C_4	C_5
0	2	37	8	13
18	0	16	9	19
25	20	34	0	28
28	8	39	0	37
0	0	0	0	0

Reduced-Cost Table 2

0	2	37	16	13
18	0	16	17	19
17	12	26	0	20
20	0	31	0	29
0	0	0	8	0

Reduced-Cost Table 3

0	18	37	32	13
2	0	0	17	3
1	12	10	0	4
4	0	15	0	13
0	8	0	24	0

The optimal group-city combinations are:
$G_1 : C_1$, $G_2 : C_3$, $G_3 : C_4$ and $G_4 : C_2$ for Total Sales = Rs 242,000.

25. We first calculate the total profit resulting from introducing a product in a particular plant. To illustrate, if product *A* is introduced in plant P_1 then the production and distribution costs aggregate to Rs 32. With a Selling Price of Rs 50 and a sale of 800 units, the total profit will amount to (50 –32) × 800 = Rs 14,400. The profit matrix is given here:

Total Profit Matrix

Product	*Plant*				
	P_1	P_2	P_3	P_4	P_5
A	14,400	16,000	12,800	9,600	12,800
B	24,000	16,000	2,000	15,000	18,000
C	12,800	18,000	11,200	12,000	15,200

Opportunity Loss Matrix

9,600	8,000	11,200	14,400	11,200
0	8,000	22,000	9,000	6,000
11,200	6,000	12,800	12,000	8,800
24,000	24,000	24,000	24,000	24,000
24,000	24,000	24,000	24,000	24,000

Reduced-Cost Table 1

1,600	0	3,200	6,400	3,200
0	8,000	22,000	9,000	6,000
5,200	0	6,800	6,000	2,800
0	0	0	0	0
0	0	0	0	0

Reduced-Cost Table 2

1,600	0	400	3,600	400
0	8,000	400	6,200	3,200
5,200	0	4,000	3,200	0
2,800	2,800	0	0	0
2,800	2,800	0	0	0

Optimal assignments:

Product	*Plant*	*Profit*
A	P_2	16,000
B	P_1	24,000
C	P_5	15,200
	Total	55,200

26. First we calculate expected profit by multiplying the amount of profit obtainable from a sale to a customer by the probability of making the sale. It is given here. Also provided is the Opportunity Loss Matrix, obtained by subtracting each of the values from the largest value in the matrix, 486. Notice the introduction of a dummy customer.

Expected Profit Matrix

	S_1	S_2	S_3	S_4
C_1	350	200	250	400
C_2	225	360	270	315
C_3	162	486	324	108
C_4	0	0	0	0

Opportunity-Loss Matrix

S_1	S_2	S_3	S_4
136	286	236	86
261	126	216	171
324	0	162	378
486	486	486	486

Using the values given in the Opportunity Loss Matrix, Reduced-Cost Table 1 is derived using row reductions. Reduced-Cost Table 2 is based on RCT 1 since number of lines $<n$.

Reduced-Cost Table 1

50	200	150	0
135	0	90	45
324	0	162	378
0	0	0	0

Reduced-Cost Table 2

50	245	150	0
90	0	45	0
279	0	117	333
0	45	0	0

Since the number of lines covering all zeros is smaller than four in Reduced-Cost Table 2 as well, an improved solution is obtained in the form of Reduced-Cost Table 3. Here four lines are needed to cover all zeros. Accordingly, assignments can be made as shown in the table.

Reduced-Cost Table 3

5	245	105	[0]
45	0	[0]	0
234	[0]	72	333
[0]	90	0	45

Optimal assignment is:

Customer	*Salesman*	*Profit (Rs)*
C_1	S_4	400
C_2	S_3	270
C_3	S_2	486
	Total	1,156

27. As a first step, we obtain the matrix of expected responses. The expected responses are obtained by multiplying the number of household expected to interview in each city with the probability of a household contact. After this, the regret matrix is obtained by subtracting each of the values from the largest value. This problem is then solved as a minimisation problem.

Taking 1, 2, 3, and 4 to represent Saturday morning, Saturday evening, Sunday morning, and Sunday evening respectively, the expected responses matrix is given here. Alongside, the regret matrix is provided.

Expected No. of Responses

	C_1	C_2	C_3	C_4
1	48	85	32	128
2	90	56	190	160
3	105	35	80	124
4	15	72	128	180

Regret Matrix

C_1	C_2	C_3	C_4
142	105	158	62
100	134	0	30
85	155	110	66
175	118	62	10

The Hungarian method is now applied to the regret matrix to obtain solution. The row reductions are given in Reduced-Cost Table 1, while column reductions are shown in Reduced-Cost Table 2.

Reduced-Cost Table 1

80	43	96	0
100	134	0	30
19	89	44	0
165	108	52	0

Reduced-Cost Table 2

61	[0]	96	0
81	91	[0]	30
[0]	46	44	0
146	65	52	[0]

It is clear from Reduced-Cost Table 2 that the minimum number of lines covering all zeros matches with the order of the matrix. Accordingly, assignments are made as shown. Thus, optimal assignment is: Saturday morning: City 2; Sunday morning: City 1; Saturday evening: City 3; Sunday evening: City 4. Total expected response = 560.

28. Since the service time is constant, it would not affect the decision of stationing the crew. To begin with, if the entire crew resides at Chennai, then the waiting time at Bangalore for different service line connections may be calculated. These are given below. Similarly, if the crew is assumed to reside at Bangalore then the waiting time at Chennai for different route combinations would be as shown here.

Waiting Time at Bangalore

Route	*1*	*2*	*3*	*4*	*5*
a	17.5	21.0	3.0	6.5	12.0
b	16.0	19.5	1.5	5.0	10.5
c	12.0	15.5	21.5	1.0	6.5
d	4.5	8.0	14.0	17.5	23.0
e	23.0	2.5	8.5	12.0	17.5

Waiting Time at Chennai

Route	*1*	*2*	*3*	*4*	*5*
a	18.5	15.0	9.0	5.5	0.0
b	20.0	16.5	10.5	7.0	1.5
c	0.0	20.5	14.5	11.0	5.5
d	7.5	4.0	22.0	18.5	13.0
e	13.0	9.5	3.5	0.0	18.5

Now, since the crew can be asked to reside at either of the places, minimum waiting times from the above operation can be obtained for different route connections by selecting the corresponding lower value out of the above two above two waiting times, provided that the waiting time is greater than four hours. The resulting waiting time matrix is given below. Applying Hungarian method, row reductions are carried out and results are shown in Reduced-Cost Table 1.

Waiting Time Matrix

Route	*1*	*2*	*3*	*4*	*5*
a	17.5	15.0	9.0	5.5	12.0
b	16.0	16.5	10.5	5.0	10.5
c	12.0	15.5	14.5	11.0	5.5
d	4.5	8.0	14.0	17.5	13.0
e	13.0	9.5	8.5	12.0	17.5

Reduced-Cost Table 1

Route	*1*	*2*	*3*	*4*	*5*
a	12.0	9.5	3.5	0.0	6.5
b	11.0	11.5	5.5	0.0	5.5
c	6.5	10.0	9.0	5.5	0.0
d	0.0	3.5	9.5	13.0	8.5
e	4.5	1.0	0.0	3.5	9.0

Similarly, we apply column reductions to RCT 1 and the resulting values are tabulated in Reduced-Cost Table 2. Here four lines are seen to cover all zeros, against the matrix order five. Accordingly, we obtain revised matrix in the form of Reduced-Cost Table 3. In this matrix, five lines are covering all the zeros. Thus, assignments are made.

Reduced-Cost Table 2

Route	*1*	*2*	*3*	*4*	*5*
a	12.0	8.5	3.5	0.0	6.5
b	11.0	10.5	5.5	0.0	5.5
c	6.5	9.0	9.0	5.5	0.0
d	0.0	2.5	9.5	13.0	8.5
e	4.5	0.0	0.0	3.5	9.0

Reduced-Cost Table 3

Route	*1*	*2*	*3*	*4*	*5*
a	8.5	5.0	[0.0]	~~0.0~~	3.0
b	7.5	7.0	2.0	[0.0]	2.0
c	6.5	9.0	9.0	9.0	[0.0]
d	[0.0]	2.5	9.5	16.5	8.5
e	4.5	[0.0]	~~0.0~~	7.0	9.0

The optimal assignment for the crew is:

Crew	*Residence*
1	Chennai
2	Bangalore
3	Bangalore
4	Chennai
5	Bangalore

Route No.	*Waiting time (hrs)*
1–*d*	4.5
2–*e*	9.5
3–*a*	9.0
4–*b*	5.0
5–*c*	5.5
Total	33.5

CHAPTER 7

1. Let x_i (i = 1, 2, 3, 4) be the variables indicating project A, B, C and D respectively. Each of these can take the value 1 or 0 accordingly as a project is selected or not. Thus,

$$x_i(i = 1, 2, 3, 4) = 1, \text{ if the project is selected}$$
$$= 0 \text{ otherwise}$$

The problem is:

Maximise $Z = 18{,}00{,}000x_1 + 2{,}00{,}000x_2 + 7{,}20{,}000x_3 + 8{,}00{,}000x_4$

Subject to

$$3{,}00{,}000x_1 + 1{,}20{,}000x_2 + 3{,}00{,}000x_3 + 2{,}00{,}000x_4 \le 6{,}50{,}000$$
$$4{,}00{,}000x_1 + 80{,}000x_2 + 2{,}00{,}000x_3 + 4{,}00{,}000x_4 \le 8{,}00{,}000$$
$$4{,}00{,}000x_1 + 2{,}00{,}000x_3 + 4{,}00{,}000x_4 \le 8{,}00{,}000$$
$$3{,}00{,}000x_1 + 40{,}000x_2 + 2{,}00{,}000x_3 + 1{,}00{,}000x_4 \le 5{,}00{,}000$$
$$x_i = 0 \text{ or } 1$$

2. Let x_1, x_2 and x_3 represent the quantities to be produced on machines 1, 2 and 3 respectively, and d_1, d_2 and d_3 indicate whether a machine is be used (1) or not (0). Accordingly, the fixed cost would be $9{,}000d_1 + 6{,}000d_2 + 4{,}500d_3$, while the variable cost would be $11x_1 + 10x_2 + 16x_3$. The IPP is:

Minimise $Z = 9{,}000d_1 + 6{,}000d_2 + 4{,}500d_3 + 11x_1 + 10x_2 + 16x_3$

Subject to

$$x_1 + x_2 + x_3 \ge 5{,}000$$
$$x_1 \le 4{,}000d_1$$
$$x_2 \le 3{,}000d_2$$
$$x_3 \le 1{,}000d_3$$
$$x_1, x_2, x_3 \ge 0;\ d_1, d_2, d_3 = 0, 1$$

3. Let x_1 and x_2 be the number of technicians and apprentices, respectively, employed by the company. The problem is:

Maximise $Z = 8x_1 + 3x_2$ Productivity

Subject to

$6x_1 + 4x_2 \le 25$ Man-hours limit

$120x_1 + 82x_2 \le 240$ Cash-flow limit

$x_1, x_2 \ge 0,$ Integer

4. Let the variables x_1, x_2 .... x_8 represent projects 1,2,...8 respectively. According to given conditions, the constraints are:
   (a) $x_3 + x_8 = 1$
   (b) $x_4 - x_7 \ge 0$
   (c) $x_1 + x_3 + x_5 + x_8 \le 2$
   (d) $x_3 - x_6 \ge 0$
   (e) $2 \le x_1 + x_2 + x_4 + x_5 + x_6 + x_8 \le 4$
   (f) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 5$

5. Let x_1, x_2, x_3 and x_4 represent the products 1, 2, 3 and 4 respectively. Further, $x_j = 1$ if jth product is made and $x_j = 0$ if jth product is bought. With the given information,

Objective function: $Z = (110 \times 2.25)x_1 + 110 \times 3.10\ (1 - x_1) + (110 \times 2.22)x_2 + (110 \times 2.60)\ (1 - x_2) + (110 \times 4.50)x_3 + (110 \times 4.75)\ (1 - x_3) + (110 \times 1.90)x_4 + (110 \times 2.25)\ (1 - x_4)$

Thus, the problem is:

Minimise $Z = 1{,}397 - 93.5x_1 - 41.8x_2 - 27.5x_3 - 38.5x_4$

Subject to

$$0.04x_1 + 0.02x_3 + 0.08x_4 \le 40$$
$$0.02x_1 + 0.01x_2 + 0.06x_3 + 0.04x_4 \le 40$$
$$0.02x_1 + 0.05x_2 + 0.15x_4 \le 40$$
$$0.15x_2 + 0.06x_3 \le 40$$

$0.03x_1 + 0.09x_2 + 0.20x_3 \leq 40$
$0.06x_1 + 0.06x_2 + 0.02x_3 + 0.05x_4 \leq 40$
$x_j = 1$ product is made
$x_j = 0$ product is bought

6. Let x_i (i = 1, 2, 3, 4) be the variables representing projects 1, 2, 3 and 4. Each of these can take a value 1 or 0 accordingly as the project is accepted or not. The problem is,
Maximise $Z = 1250x_1 + 1320x_2 + 620x_3 + 740x_4$
Subject to
$720x_1 + 880x_2 + 300x_3 + 350x_4 \leq 1400$
$560x_1 + 550x_2 + 170x_3 + 210x_4 \leq 800$
$400x_1 + 360x_2 + 110x_3 \leq 440$
$x_1 + x_2 \leq 1$
$x_3 - x_4 \geq 0$
$x_i = 0$ or 1

7. Let x_1 and x_2 be the daily output of the shirts X and Y respectively. The problem may be stated as follows:
Maximise $Z = 10x_1 + 40x_2$
Subject to
$2x_1 + 4x_2 \leq 7$
$5x_1 + 3x_2 \leq 15$
$x_1, x_2 \geq 0$ and integers

We first obtain solution to the LP relaxation of this problem, allowing for fractional values of x_1 and x_2. This is given in tables below.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	2	4*	1	0	7	7/4 ←
S_2	0	5	3	0	1	15	5
C_j		10	40	0	0		
Solution		0	0	7	15		
Δ_j		10	40 ↑	0	0		

Simplex Tableau 2: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
x_2	40	1/2	1	1/4	0	7/4
S_2	0	7/2	0	–3/4	1	39/4
C_j		10	40	0	0	
Solution		0	7/4	0	39/4	
Δ_j		–10	0	–10	0	

Since the optimal does not involve all integer values, a Gomory constraint is introduced by considering the second of the constraints given in the table above. The revised tableau is presented in below while the table following contains an improved solution.

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	40	1/2	1	1/4	0	0	7/4	7/2
S_2	0	7/2	0	–3/4	1	0	39/4	39/14
S_3	0	–1/2*	0	–1/4	0	1	–3/4	3/2 ←
C_j		10	40	0	0	0		
Solution		0	7/4	0	39/4	–3/4		
Δ_j		–10	0	–10	0	0		
		↑						

Simplex Tableau 4: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_2	40	0	1	0	0	1	1
S_2	0	0	0	–5/2	1	7	9/2
x_1	10	1	0	1/2	0	–2	3/2
C_j		10	40	0	0	0	
Solution		3/2	1	0	9/2	0	
Δ_j		0	0	–5	0	–20	

Addition of another Gomory constraint, considering second constraint in Simplex Tableau 4, leads to revised tableau, given here.

Simplex Tableau 5: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
x_2	40	0	1	0	0	1	0	1	—
S_2	0	0	0	–5/2	1	7	0	9/2	—
x_1	10	1	0	1/2	0	–2	0	3/2	3
S_4	0	0	0	–12*	0	0	1	–1/2	1 ←
C_j		10	40	0	0	0	0		
Solution		3/2	1	0	9/2	0	–1/2		
Δ_j		0	0	–5	0	–20	0		

The improved solution, contained in Simplex Tableau 6, is optimal as it involves all integer variables. From the solution, we have $x_1 = 1$, $x_2 = 1$ and $Z = 50$.

Simplex Tableau 6: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i
x_2	40	0	1	0	0	1	0	1
S_2	0	0	0	0	1	7	–5	7
x_1	10	1	0	0	0	–2	1	1
S_1	0	0	0	1	0	0	–2	1
C_j		10	40	0	0	0	0	
Solution		1	1	1	7	0	0	
Δ_j		0	0	0	0	–20	–10	

8. The given solution is represented in Simplex Tableau 1.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
x_2	9	0	1	7/22	1/22	21/2
x_1	7	1	0	–1/22	3/22	27/2
C_j		7	9	0	0	
Solution		27/2	21/2	0	0	$Z = 189$
Δ_j		0	0	–28/11	–15/11	

The solution values are not integers. Revised Simplex Tableau 1 includes a cut in respect of the first row (x_2).

Revised Simplex Tableau 1

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	9	0	1	7/22	1/22	0	21/2	—
x_1	7	1	0	–1/22	3/22	0	27/2	56
S_3	0	0	0	–7/22	–1/22	1	–1/2	11/7 ←
C_j		7	9	0	0	0		
Solutiom		27/2	21/2	0	0	–1/2		
Δ_j		0	0	–28/11	–15/11	0		
				↑				

Revised Simplex Tableau 2

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_2	9	0	1	0	0	1	10
x_1	7	1	0	0	1/7	–1/7	95/7
S_1	0	0	0	1	1/7	–22/7	11/7
C_j		7	9	0	0	0	
Solution		95/7	10	11/7	0	0	$Z = 185$
Δ_j		0	0	0	–1	–8	

Another cut is introduced in respect of second row (x_1) and shown in Revised Simplex Tableau 3.

Revised Simplex Tableau 3

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
x_2	9	0	1	0	0	1	0	10	—
x_1	7	1	0	0	1/7	–1/7	0	95/7	95
S_1	0	0	0	1	1/7	–22/7	0	11/7	11
S_4	0	0	0	0	–1/7	–6/7	1	–4/7	4 ←
C_j		7	9	0	0	0	0		
Solution		95/7	10	11/7	0	0	–4/7		
Δ_j		0	0	0	–1	–1	0		
					↑				

Revised Simplex Tableau 4

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i
x_2	9	0	1	0	0	1	0	10
x_1	7	1	0	0	0	–1	1	13
S_1	0	0	0	1	0	–4	1	1
S_2	0	0	0	0	1	6	–7	4
C_j		7	9	0	0	0	0	
Solution		13	10	1	4	0	0	$Z = 181$
Δ_j		0	0	0	0	–2	–7	

The solution given in Revised Simplex Tableau 4 is optimal solution to the IPP as all the variables have integer solution values. The solution is: $x_1 = 13$ and $x_2 = 10$, with $Z = 181$.

9. Let x_1 and x_2 be number of Molina and Suzie dolls produced per week. The problem is:

Maximise $Z = 6x_1 + 18x_2$

Subject to

$$3x_1 + x_2 \leq 50$$
$$4x_1 + 4x_2 \leq 90$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

With slack variables S_1 and S_2 we first solve the LPP without restriction of integer variables.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	3	1	1	0	50	50
S_2	0	4	4	0	1	90	45/2 ←
C_j		6	18	0	0		
Solution		0	0	50	90	$Z = 0$	
Δ_j		6	18 ↑	0	0		

Simplex Tableau 2: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
S_1	0	2	0	1	–1/4	55/2
x_2	18	1	1	0	1/4	45/2
C_j		6	18	0	0	
Solution		0	45/2	55/2	0	$Z = 405$
Δ_j		–12	0	0	–9/2	

Since the optimal values of the variables are not integers, we introduce a cut in the first row (S_1).

Revised Simplex Tableau 1

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	0	1	–1/4	0	55/2	—
x_2	18	1	1	0	1/4	0	45/2	90
S_3	0	0	0	0	–3/4	1	–1/2	2/3 ←
C_j		6	18	0	0	0		
Solution		0	45/2	55/2	0	–1/2		
Δ_j		–12	0	0	–9/2	0		
					↑			

Revised Simplex Tableau 2

Basis		x_1	x_2	S_1	S_2	S_3	b_i
S_1	0	2	0	1	0	–1/3	83/3
x_2	18	1	1	0	0	1/3	67/3
S_2	0	0	0	0	1	–4/3	2/3
C_j		6	18	0	0	0	
Solution		0	67/3	83/3	2/3	0	Z = 402
Δ_j		–12	0	0	0	–6	

A cut is introduced in row 1 (S_1) and Revised Simplex Tableau 3 is derived.

Revised Simplex Tableau 3

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
S_1	0	2	0	1	0	–1/3	0	83/3	—
x_2	18	1	1	0	0	1/3	0	67/3	67
S_2	0	0	0	0	1	–4/3	0	2/3	—
S_4	0	0	0	0	0	–2/3	1	–2/3	1 ←
C_j		6	18	0	0	0	0		
Solution		0	67/3	87/3	2/3	0	–2/3		
Δ_j		–12	0	0	0	–6	0		
						↑			

Revised Simplex Tableau 4

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i
S_1	0	2	0	1	0	0	–1/2	28
x_2	18	1	1	0	0	0	1/2	22
S_2	0	0	0	0	1	0	–2	2
S_3	0	0	0	0	0	1	–3/2	1
C_j		6	18	0	0	0	0	
Solution		0	22	28	2	1	0	Z = 396
Δ_j		–12	0	0	0	0	–9	

The optimal solution is: $x_1 = 0$ and $x_2 = 22$, with Z = Rs 396.

10. Maximise $Z = 2x_1 + 4x_2 + 3x_3$

Subject to

$$3x_1 + 4x_2 + 2x_3 \le 60$$
$$2x_1 + x_2 + 2x_3 \le 40$$
$$x_1 + 3x_2 + 2x_3 \le 80$$
$$x_1, x_2, x_3 \ge 0$$

First we solve this LP relaxation of the given problem.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	3	4	2	1	0	0	60	15 ←
S_2	0	2	1	2	0	1	0	40	40
S_3	0	1	3	2	0	0	1	80	80/3
C_j		2	4	3	0	0	0		
Solution		0	0	0	60	40	80	$Z = 0$	
Δ_j		2	4 ↑	3	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	4	3/4	1	1/2	1/4	0	0	15	30
S_2	0	5/4	0	3/2	–1/4	1	0	25	50/3←
S_3	0	–5/4	0	1/2	– 3/4	0	1	35	70
C_j		2	4	3	0	0	0		
Solution		0	15	0	0	25	35	$Z = 60$	
Δ_j		–1	0	1 ↑	–1	0	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_2	4	1/3	1	0	1/3	–1/3	0	20/3
x_3	3	5/6	0	1	–1/6	2/3	0	50/3
S_3	0	–5/3	0	0	–2/3	–1/3	1	80/3
C_j		2	4	3	0	0	0	
Solution		0	20/3	50/3	0	0	80/3	$Z = 230/3$
Δ_j		–11/6	0	0	–5/6	–2/3	0	

The optimal solution involves fractional values. So we introduce Gomory's cut. The cut, in respect of the first row is designed and shown in the table. The next table shows improved solution which involves all-integer values and is, therefore, optimal.

Table 1: Gomory's Cut

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
x_2	4	1/3	1	0	1/3	–1/3	0	0	20/3	–
x_3	3	5/6	0	1	–1/6	2/3	0	0	50/3	25
S_3	0	–5/3	0	0	–2/3	–1/3	1	0	80/3	–
S_4	0	–1/3	0	0	–1/3	–2/3*	0	1	–2/3	1
C_j		2	4	3	0	0	0	0		
Solution		0	20/3	50/3	0	0	80/3	–2/3		
Δ_j		–11/6	0	0	–5/6	–2/3	0	0		

Table 2: Optimal Solution (IPP)

Basis		x_1	x_2	x_3	S_1	S_2	S_3	S_4	b_i	
x_2	4	1/6	1	0	1/2	0	0	–1/2	7	
x_3	3	7/6	0	1	–1/2	0	0	1	16	
S_3	0	–11/6	0	0	–1/2	0	1	–1/2	27	
S_2	0	–1/2	0	0	1/2	1	0	–3/2	1	
C_j		2	4	3	0	0	0	0		
Solution		0	7	16	0	1	27	0		$Z = 76$
Δ_j		–13/6	0	0	–1/2	0	0	–1		

The optimal solution, therefore, is : $x_1 = 0$, $x_2 = 7$, $x_3 = 16$ for $Z = 76$.

11. Let x_1 and x_2 be the number of toys of *A* and *B* types, respectively, produced per day. From the given information, the profit function is $25x_1 + 25x_2$. Similarly, machine *X* capacity constraint is $2x_1 + 4x_2 \leq 18$ whereas machine *Y* capacity constraint is $6x_1 + 5x_2 \geq 30$. The IPP can be expressed as follows:

Maximise $\quad Z = 25x_1 + 25x_2$

Subject to

$$2x_1 + 4x_2 \leq 18$$
$$6x_1 + 5x_2 \leq 30$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

We first obtain solution to the problem as an LPP, disregarding that x_1 and x_2 have to be integers.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	2	4	1	0	18	9
S_2	0	6*	5	0	1	30	5 ←
C_j		25	25	0	0		
Solutiion		0	0	18	30		
Δ_j		25	25	0	0		
		↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i	b_i/a_{ij}
S_1	0	0	7/3*	1	–1/3	8	24/7 ←
x_1	25	1	5/6	0	1/6	5	6
C_j		25	25	0	0		
Solution		5	0	8	0		
Δ_j		0	25/6	0	–25/6		
			↑				

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	b_i
x_2	25	0	1	3/7	–1/7	24/7
x_1	25	1	0	–5/14	2/7	15/7
C_j		25	25	0	0	
Solution		15/7	24/7	0	0	
Δ_j		0	0	–25/14	–25/7	

It is evident from Simplex Tableau 3 that the optimal solution to the problem is not in integer values. Therefore, we introduce Gomory's cut. Consider the first constraint given in this tableau, which is given below:

$$0x_1 + 1x_2 + \frac{3}{7}S_1 - \frac{1}{7}S_2 = \frac{24}{7}$$

It may be re-expressed as:

$$0x_1 + 1x_2 + \frac{3}{7}S_1 - S_2 + \frac{6}{7}S_2 = 3 + \frac{3}{7}$$

or $$\frac{3}{7}S_1 + \frac{6}{7}S_2 = \frac{3}{7} + (3 - x_2 + S_2)$$

or $$\frac{3}{7}S_1 + \frac{6}{7}S_2 \geq \frac{3}{7}$$

or $$-\frac{3}{7}S_1 - \frac{6}{7}S_2 + S_3 \leq -\frac{3}{7}$$

This constraint is introduced and the revised tableau is presented in Simplex Tableau 4. For improving the solution given in this table, the incoming variable is S_1 (since it has the least Δ_j/a_{ij} value) while the outgoing variable is S_3.
Tableau 5 contains an improved solution.

Simplex Tableau 4: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_2	25	0	1	3/7	–1/7	0	24/7	8
x_1	25	1	0	–5/14	2/7	0	15/7	—
S_3	0	0	0	–3/7*	–6/7	1	–3/7	1 ←
C_j		25	25	0	0	0		
Solution		15/7	24/7	0	0	–3/7		
Δ_j		0	0	–25/14	0	–25/7		
				↑				

Simplex Tableau 5: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
x_2	25	0	1	0	–1	1	3
x_1	25	1	0	0	1	–5/6	5/2
S_1	0	0	0	1	2	–7/3	1
C_j		25	25	0	0	0	
Solution		5/2	3	1	0	0	
Δ_j		0	0	0	0	–25/6	

Since the solution here also does not involve all integers, another cut is introduced in respect of the second constraint that involves a fraction. The next table contains the revised tableau.

Simplex Tableau 6: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i	b_i/a_{ij}
x_2	25	0	1	0	–1	1	0	3	3
x_1	25	1	0	0	1	–5/6	0	5/2	—
S_1	0	0	0	1	2	–7/3	0	1	—
S_4	0	0	0	0	0	–1/6*	1	–1/2	3 ←
C_j		25	25	0	0	0	0		
Solution		5/2	3	1	0	0	–1/2		
Δ_j		0	0	0	0	–25/6	0		
						↑			

An improved solution is given in Simplex Tableau 7. All the variables in this solution are seen to be integers.

Simplex Tableau 7: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	S_4	b_i
x_2	25	0	1	0	–1	0	6	0
x_1	25	1	0	0	1	0	–5	5
S_1	0	0	0	1	2	0	–14	8
S_3	0	0	0	0}	0	1	–6	3
C_j		25	25	0	0	0	0	
Solution		5	0	8	0	3	0	
Δ_j		0	0	0	0	0	–25	

The optimal integer solution is to produce 5 toys of type *A* and none of type *B*. This would yield a total profit of Rs 125.

12. Let x_1 and x_2 represent the output of products P_1 and P_2 respectively. The problem may be stated as:

Maximise $Z = 30x_1 + 50x_2$ Total profit

Subject to
$2x_1 \leq 50$ Machine M_1
$x_1 + 4x_2 \leq 16$ Machine M_2
$2x_2 \leq 20$ Machine M_3
$x_1, x_2 \geq 0$ and integers.

To solve this problem, we first obtain solution to the LP relaxation of it where x_1 and x_2 are not required to be integers.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	0	1	0	0	50	—
S_2	0	1	4*	0	1	0	16	4 ←
S_3	0	0	2	0	0	1	20	10
C_j		30	50	0	0	0		
Solution		0	0	50	16	20		
Δ_j		30	50	0	0	0		
			↑					

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	2	0	1	0	0	50	25
x_2	50	1/4*	1	0	1/4	0	4	16 ←
S_3	0	–1/2	0	0	–1/2	1	12	–24
C_j		30	50	0	0	0		
Solution		0	4	50	0	12		
Δ_j		35/2 ↑	0	0	–25/2	0		

Simplex Tableau 3: Optimal Solution

Basis		x_1	x_2	S_1	S_2	S_3	b_i
S_1	0	0	–4	1	–1	0	18
x_1	30	1	4	0	1	0	16
S_3	0	0	2	0	0	1	20
C_j		30	50	0	0	0	
Solution		16	0	18	0	20	
Δ_j		0	–70	0	–30	0	

The optimal solution obtained in Simplex Tableau 3, evidently involves all variables having only integer values. Thus, no further improvement is called for. The optimal integer solution to the problem, therefore, is: $x_1 = 16$ and $x_2 = 0$. Total profit = Rs 30 × 16 = Rs 480.

13. The given information can be presented as an IPP as follows:

Let x_1 : the number of scoops of cottage cheese, and
x_2 : the number of scoops of scrambled egg.

Minimise $Z = 2x_1 + 2x_2$ Total cost

Subject to $3x_1 + 2x_2 \geq 12$ Vitamin E

$3x_1 + 8x_2 \geq 24$ Iron

$x_1 \geq 2$ Min. consumption

$x_1, x_2 \geq 0$ and integer

Here since $x_1 \geq 2$, we may replace $x_1 = 2 + x_3$ in the problem and restate it as follows:

Minimise $Z = 2x_3 + 2x_2 + 4$

Subject to $3x_3 + 2x_2 \geq 6$

$3x_3 + 8x_2 \geq 18$

$x_3, x_2 \geq 0$ and integer

To solve this problem, we first consider solution to its LP equivalent. Introducing necessary surplus and artificial variables, we get

Minimise $Z = 2x_3 + 2x_2 + 4 + 0S_1 + 0S_2 + MA_1 + MA_2$

Subject to $3x_3 + 2x_2 - S_1 + A_1 = 6$

$3x_3 + 8x_2 - S_2 + A_2 = 18$

$x_3, x_2, S_1, S_2, A_1, A_2 \leq 0$

Simplex Tableau 1: Non-optimal Solution

Basis		x_3	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	M	3	2	–1	0	1	0	6	3
A_2	M	3	8*	0	–1	0	1	18	9/4 ←
C_j		2	2	0	0	M	M		
Solution		0	0	0	0	6	18		
Δ_j		$2 - 6M$	$2 - 10M$ ↑	M	M	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		x_3	x_2	S_1	S_2	A_1	A_2	b_i	b_i/a_{ij}
A_1	M	9/4*	0	–1	1/4	1	–1/4	3/2	2/3 ←
x_2	2	3/8	1	0	–1/8	0	1/8	9/4	6
C_j		2	2	0	0	M	M		
Solution		0	9/4	0	0	3/2	0		
Δ_j		$\frac{5}{4}-\frac{9M}{4}$	0	M	$\frac{1}{4}-\frac{M}{4}$	0	$-\frac{1}{4}+\frac{M}{4}$		

Simplex Tableau 3: Optimal Solution

Basis		x_3	x_2	S_1	S_2	A_1	A_2	b_i
x_3	2	1	0	–4/9	1/9	4/9	–1/9	2/3
x_2	2	0	1	1/6	–1/6	–1/6	1/6	2
C_j		2	2	0	0	M	M	
Solution		2/3	2	0	0	0	0	
Δ_j		0	0	5/9	1/9	0	0	

The solution given in Simplex Tableau 3 is not an all-integer solution. Hence, a Gomory constraint is added by considering the first constraint as follows:

$$x_1 + 0x_2 - \frac{4}{9}S_1 + \frac{1}{9}S_2 = \frac{2}{3} \qquad \text{(Artificial not to be considered)}$$

or
$$x_1 + 0x_2 - S_1 + \frac{5}{9}S_1 + \frac{1}{9}S_2 = \frac{2}{3}$$

or
$$\frac{5}{9}S_1 + \frac{1}{9}S_2 \geq \frac{2}{3} \quad \text{or} \quad -\frac{5}{9}S_1 - \frac{1}{9}S_2 + S_3 = -\frac{2}{3}$$

The revised tableau is presented in Simplex Tableau 4. The improved solution is given in Tableau 5. A test of optimality indicates the solution to be optimal.

Simplex Tableau 4: Non-optimal Solution

Basis		x_3	x_2	S_1	S_2	S_3	b_i	b_i/a_{ij}
x_3	2	1	0	–4/9	1/9	0	2/3	6
x_2	2	0	1	1/6	–1/6	0	2	—
S_3	0	0	0	–5/9	–1/9	1	–2/3	6 ←
C_j		2	2	0	0	0		
Solution		2/3	2	0	0	–2/3		
Δ_j		0	0	5/9 ↑	1/9	0		

Simplex Tableau 5: Optimal Solution

Basis		x_3	x_2	S_1	S_2	S_3	b_i
x_3	2	1	0	–1	0	1	0
x_2	2	0	1	1	0	–3/2	3
S_2	0	0	0	5	1	–9	6
C_j		2	2	0	0	0	
Solution		0	3	0	6	0	
Δ_j		0	0	0	0	1	

From the optimal solution in Simplex Tableau 5, we have $x_3 = 0$ and $x_2 = 3$. Thus, solution to the given problem is: $x_1 = 2 + 0 = 2$ and $x_2 = 3$. Total cost involved is Rs 2 × 2 + Rs 2 × 3 = Rs 10.

14. A feasible tour is 1–2–3–4–5–1. From the given data, this tour involves a total cost of 15 + 22 + 19 + 19 + 19 = 94. This may be set as upper bound on the solution. We now solve the problem as an assignment problem, by setting for the routes 1–1, 2–2, 3–3, 4–4 and 5–5, an *M* in the cost matrix. Applying HAM, reduced-cost tables are obtained here.

Reduced Cost Table 1

M	0	7	2	3
0	*M*	7	1	2
0	2	*M*	4	0
0	3	4	*M*	3
2	0	4	3	*M*

Reduced Cost Table 2

M	[0]	3	1	3
0	*M*	3	[0]	2
0	2	*M*	3	[0]
[0]	3	0	*M*	3
2	0	[0]	2	*M*

From the assignments in RTC-2, we get two sub-tours: 1–2–4–1 and 3–5–3. The total cost equal to 83 sets lower bound on the solution. We now break the sub-tour 3–5–3.

Make 3–5 unacceptable

RCT-2 is modified by placing an *M* for the route 3–5, and produced as RCT-3. Applying HAM. Reduced Cost Tables 4 and 5 drawn up. Assignments in this provide a tour 1–2–4–5–3–1, with a cost of Rs 86.

Reduced Cost Table 3

M	0	3	1	3
0	*M*	3	0	2
0	2	*M*	3	*M*
0	3	0	*M*	3
2	0	0	2	*M*

Reduced Cost Table 4

M	0	3	1	1
0	*M*	3	0	0
0	2	*M*	3	*M*
0	3	0	*M*	1
2	0	0	2	*M*

Reduced Cost Table 5

M	[0]	3	0	0
[1]	*M*	4	[0]	0
[0]	2	*M*	2	*M*
0	3	0	*M*	[0]
2	0	[0]	1	*M*

Make 5–3 unacceptable

For this, RTC-2 is modified by placing an *M* for the route 5–3 and given as RTC-6. An improvement leads to RCT-7, wherein assignments made also lead to a tour: 1–4–3–5–2–1, involving a cost of Rs 84. Since the cost of this tour is smaller than the one obtained earlier, it represents the optimal solution.

Reduced Cost Table 6

M	0	3	1	3
0	*M*	3	0	2
0	2	*M*	3	0
0	3	0	*M*	3
2	0	*M*	2	*M*

Reduced Cost Table 7

M	0	2	[0]	2
[0]	*M*	3	0	2
0	3	*M*	3	[0]
0	4	[0]	*M*	3
1	[0]	*M*	1	*M*

Optimal tour: 1–4–3–5–2–1 Total cost = Rs 84.

15. A feasible tour is *A–B–C–D–E–A*, which entails a total distance of 17 + 18 + 19 + 18 + 14 = 86 (hundred) km. Thus, we set initial upper bound = 86. To set the power bound, we solve the given problem as an assignment problem. The given matrix is represented below in this context.

Distance-profile

	A	*B*	*C*	*D*	*E*
A	*M*	17	16	18	14
B	17	*M*	18	15	16
C	16	18	*M*	19	17
D	18	15	19	*M*	18
E	14	16	17	18	*M*

The solution is given here.

Reduced-Cost Table 1

	A	*B*	*C*	*D*	*E*
A	*M*	3	2	4	0
B	2	*M*	3	0	1
C	0	2	*M*	3	1
D	3	0	4	*M*	3
E	0	2	3	4	*M*

Reduced-Cost Table 2

	A	*B*	*C*	*D*	*E*
A	*M*	3	0	4	0
B	2	*M*	1	0	1
C	0	2	*M*	3	1
D	3	0	2	*M*	3
E	0	2	1	4	*M*

Reduced-Cost Table 3

	A	*B*	*C*	*D*	*E*
A	*M*	4	0	4	0
B	3	*M*	1	0	1
C	0	2	*M*	2	0
D	3	0	1	*M*	2
E	0	2	0	3	*M*

Assignment : *A–E*, *B–D*, *C–A*, *D–B*, *E – C*
Sub-tours : *A–E–C–A*, *B–D–B*
Total distance : 77 (hundred) km (Lower bound)

Since the optimal solution does not yield a tour and instead provides to sub-tours, we break the sub-tour *B–D–B*.

When B–D is unacceptable:

In the RCT-3, we make the route *B–D* unacceptable and re-solve the problem. An *M* is placed in the cell *B–D* and the solution is obtained as given in RCT-4 and RCT-5.

Reduced-Cost Table 4

	A	*B*	*C*	*D*	*E*
A	*M*	4	0	4	0
B	2	*M*	0	*M*	0
C	0	2	*M*	2	0
D	3	0	1	*M*	2
E	0	2	0	3	*M*

Reduced-Cost Table 5

	A	*B*	*C*	*D*	*E*
A	*M*	4	0	2	0
B	2	*M*	0	*M*	0
C	0	2	*M*	0	0
D	3	0	1	*M*	2
E	0	2	0	1	*M*

Assignments : *A–C*, *B–E*, *C–D*, *D–B*, *E–A*
Tour : *A–C–D–B–E–A*
Total distance : 80 (hundred) km

Since the optimal solution here yields a tour, the upper bound is revised downward at 80.

When D–B is unacceptable:

We place an *M* in the cell *D–B* and solve the problem, beginning with RCT-3. The solution is given in RCT-6 and RCT-7.

Reduced-Cost Table 6

	A	*B*	*C*	*D*	*E*
A	*M*	4	0	4	0
B	3	*M*	1	0	1
C	0	2	*M*	2	0
D	2	*M*	0	*M*	1
E	0	2	0	3	*M*

Reduced-Cost Table 7

	A	*B*	*C*	*D*	*E*
A	*M*	2	0	4	[0]
B	3	*M*	1	[0]	1
C	[0]	0	*M*	2	0
D	2	*M*	[0]	*M*	1
E	0	[0]	0	3	*M*

Assignments : *A–E*, *B–D*, *C–A*, *D–C*, *E–B*
Tour : *A–E–B–D–C–A*
Total distance : 80 (hundred) km

This optimal solution also involves a tour with a total distance of 80 (hundred) km. Thus, the optimal solution to the salesman problem is a tour *A–C–D–B–E–A* or *A–E–B–D–C–A*.

16. Since a feasible sequence is *A–B–C–D–A*, the total set-up cost of 4 + 6 + 7 + 3 = 20 may be set as the upper bound. Now, we solve the given problem as an assignment problem. The given cost matrix is reproduced here with cost elements of each of the cells at the diagonal being set equal to *M*.

Cost Matrix

	A	*B*	*C*	*D*
A	*M*	4	7	3
B	4	*M*	6	3
C	7	6	*M*	7
D	3	3	7	*M*

The solution is given in RCT-1 and RCT-2

Reduced-Cost Table 1

	A	*B*	*C*	*D*
A	*M*	1	4	0
B	1	*M*	3	0
C	1	0	*M*	1
D	0	0	4	*M*

Reduced-Cost Table 2

	A	B	C	D
A	M	1	1	[0]
B	1	M	[0]	0
C	1	[0]	M	1
D	[0]	0	1	M

Assignments : *A–D*, *B–C*, *C–B*, *D–A*
Sub-tours : *A–D–A*, *B–C–B*
Total cost : 6 + 12 = 18 (Lower bound)

Since the optimal solution involves two sub-tours, we now re-solve the problem by breaking one of these: *A–D–A*. For this, we make *A–D* and *D–A* unacceptable, one by one.

When A–D is unacceptable:

Placing an *M* in the cell *A–D* in RCT-2, and solving it, we get the tour *A–C–B–D–A* with a total cost of 19.

Reduced-Cost Table 3

	A	B	C	D
A	M	0	[0]	M
B	1	M	0	[0]
C	1	[0]	M	1
D	[0]	0	1	M

Assignments : *A–C*, *B–D*, *C–B*, *D–A*
Tour : *A–C–B–D–A*
Total cost : 19

The upper bound is revised to 19.

When D–A is unacceptable:

We place an *M* in the cell *D–A* in RCT-2 and solve as a new problem. From the solution obtained to the problem, we get the tour *A–D–B–C–A*, involving a total cost of 19. Accordingly, the optimal solution to the given problem is to set up the jobs either as *A–C–B–D–A* or *A–D–B–C–A*, for a total cost of 19.

Reduced-Cost Table 4

	A	B	C	D
A	M	1	1	[0]
B	0	M	[0]	0
C	[0]	0	M	1
D	M	[0]	1	M

Assignments : *A–D*, *B–C*, *C–A*, *D–B*
Tour : *A–D–B–C–A*
Total cost : 19

17. For the given data, a tour C_1–C_2–C_3–C_4–C_5–C_1 is feasible and involves a total distance of 10 + 12 + 13 + 10 + 12 = 57 hours. Accordingly, we get the initial upper bound = 57 hours. To determine the lower bound, we state and solve the given problem as an assignment problem. The problem is restated on the next page:

Travelling time (in hours)

City	City				
	C_1	C_2	C_3	C_4	C_5
C_1	M	10	13	11	M
C_2	10	M	12	10	12
C_3	14	13	M	13	11
C_4	11	10	14	M	10
C_5	12	11	12	10	M

The solution to the assignment problem is given in RCT-1, RCT-2 and RCT-3.

Reduced-Cost Table 1

City	C_1	C_2	C_3	C_4	C_5
C_1	M	0	3	1	M
C_2	0	M	2	0	2
C_3	3	2	M	2	0
C_4	1	0	4	M	0
C_5	2	1	2	0	M

Reduced-Cost Table 2

City	C_1	C_2	C_3	C_4	C_5
C_1	M	0	1	1	M
C_2	0	M	0	0	2
C_3	3	2	M	2	0
C_4	1	0	2	M	0
C_5	2	1	0	0	M

Reduced-Cost Table 3

City	C_1	C_2	C_3	C_4	C_5
C_1	M	[0]	0	0	M
C_2	0	M	[0]	0	3
C_3	2	2	M	1	[0]
C_4	[0]	0	1	M	0
C_5	2	2	0	[0]	M

The optimal solution obtained yields a tour C_1–C_2–C_3–C_5–C_4–C_1, involving a total distance of 54 hours. Thus, we revise the upper bound to this value and obtain this solution as the optimal travelling plan for the salesman.

18. Let the depot, vendor *A*, vendor *B*, vendor *C*, and vendor *D* be represented as 1, 2, 3, 4 and 5 respectively, Here, a feasible tour is 1–2–3–4–5–1 that involves a distance of 3.5 + 4.0 + 4.5 + 4.0 + 2.0 = 18 km. This is set as upper bound for the solution. We now state the given problem as an assignment problem, assigning as *M* the 'cost' in each of the cells at the diagonal.

Distance profile

	1	2	3	4	5
1	*M*	3.5	3.0	4.0	2.0
2	3.5	*M*	4.0	2.5	3.0
3	3.0	4.0	*M*	4.5	3.5
4	4.0	2.5	4.5	*M*	4.0
5	2.0	3.0	3.5	4.0	*M*

This solution to this problem is contained in RCT-1 through RCT-3.

Reduced-Cost Table 1

	1	2	3	4	5
1	*M*	1.5	1.0	2.0	0.0
2	1.0	*M*	1.5	0.0	0.5
3	0.0	1.0	*M*	1.5	0.5
4	1.5	0.0	2.0	*M*	1.5
5	0.0	1.0	1.5	2.0	*M*

Reduced-Cost Table 2

	1	2	3	4	5
1	*M*	1.5	0.0	2.0	0.0
2	1.0	*M*	0.5	0.0	0.5
3	0.0	1.0	*M*	1.5	0.5
4	1.5	0.0	1.0	*M*	1.5
5	0.0	1.0	0.5	2.0	*M*

Reduced-Cost Table 3

	1	2	3	4	5
1	*M*	1.5	[0.0]	2.5	0.0
2	1.0	*M*	0.0	[0.0]	0.0
3	0.0	0.5	*M*	1.5	[0.0]
4	2.0	[0.0]	1.0	*M*	1.5
5	[0.0]	0.5	0.0	2.0	*M*

Assignments : 1–3, 2–4, 3–5, 4–2, 5–1
Sub-tours : 1–3–5–1, 2–4–2
Total distance : 8.5 + 5.0 = 13.5 km (Lower bound)

The optimal solution involves two sub-tours. We now break the sub-tour 2–4–2, making 2–4 and 4–2 unacceptable one by one.

When 4–2 is unacceptable:

We make 2–4 unacceptable in RCT 3, by replacing the zero by *M* and then solve it. The solution is given on the next page.

Reduced-Cost Table 4

	1	2	3	4	5
1	*M*	1.5	0.0	1.0	0.0
2	1.0	*M*	0.0	*M*	0.0
3	0.0	0.5	*M*	0.0	0.0
4	2.0	0.0	1.0	*M*	1.5
5	0.0	0.5	0.0	0.5	*M*

Assignments : 1–3, 2–5, 3–4, 4–2, 5–1
Tour : 1–3–4–2–5–1
Total distance : 15 km
When 4–2 is unacceptable:

We now make 4–2 unacceptable and obtain the solution. The solution is given in RCT-5 and RCT-6.

Reduced-Cost Table 5

	1	2	3	4	5
1	*M*	1.5	0.0	2.5	0.0
2	1.0	*M*	0.0	0.0	0.0
3	0.0	0.5	*M*	1.5	0.0
4	1.0	*M*	0.0	*M*	0.5
5	0.0	0.5	0.0	2.0	*M*

Reduced-Cost Table 6

	1	2	3	4	5
1	*M*	1.0	0.0	2.5	0.0
2	1.0	*M*	0.0	0.0	0.0
3	0.0	0.0	*M*	1.5	0.0
4	1.0	*M*	0.0	*M*	0.5
5	0.0	0.0	0.0	2.0	*M*

Assignments : 1–5, 2–4, 3–1, 4–3, 5–2
Tour : 1–5–2–4–3–1
Total distance : 15 km

Since each of the two sets of calculation yields a tour, involving a distance of 15 km, the upper bound is revised to 15. Also, it provides optimal solution to the problem. Thus, the optimal schedule is: Depot-vendor *D*-vendor *A*-vendor *C*-vendor *B*-depot, or in the reverse order. It entails a total distance of 15 kilometres.

19. Let the weights of the four factors be x_1, x_2, x_3 and x_4 respectively. Using deviational variables in respect of various goal constraints, we have the following problem:

Minimise $Z = d_1^+ + d_2^- + d_2^+ + d_3^- + d_4^+ + d_5^- + d_6^- + d_7^-$

Subject to
$$4x_1 + 5x_2 + 5x_3 + 5x_4 + d_1^- - d_1^+ = 600 \quad \text{Grade } A$$
$$3x_1 + 4x_2 + 3x_3 + 3x_4 + d_2^- - d_2^+ = 360 \quad \text{Grade } M$$
$$x_1 + 2x_2 + 2x_3 + x_4 + d_3^- - d_3^+ = 120 \quad \text{Grade } Z$$

$$x_2 + x_3 + x_4 + d_4^- - d_4^+ = 120 \quad A\text{–}G$$
$$x_1 + x_3 + x_4 + d_5^- - d_5^+ = 120 \quad G\text{–}M$$
$$x_1 + x_2 + x_3 + x_4 + d_6^- - d_6^+ = 120 \quad M\text{–}S$$
$$x_1 + x_2 + x_4 + d_7^- - d_7^+ = 120 \quad S\text{–}Z$$
all variables ≥ 0

20. Let x_1: number of hours per week to hire *GP*
x_2: number of hours per week to hire a nurse
x_3: number of hours per week to hire an internist

Minimise $Z = P_1 d_1^- + P_2 d_2^+ + P_3 d_3^- + P_4 d_4^-$

Subject to

$$x_2 + d_1^- - d_1^+ = 30 \quad \text{Nurse availability}$$
$$40x_1 + 20x_2 + 150x_3 + d_2^- - d_2^+ = 1200 \quad \text{Budget}$$
$$x_1 + x_3 + d_3^- - d_3^+ = 20 \quad GP \text{ or internist availability}$$
$$x_3 + d_4^- - d_4^+ = 6 \quad \text{Internist availability}$$
all variables ≥ 0

21. Let x_1, x_2 and x_3 be the acerage of corn, wheat and soybeans respectively. With usual deviational variables, the problem is stated below:

Minimise $Z = P_1 d_1^- + P_2 d_2^+ + P_3 d_3^- + P_4 d_1^- + P_5 d_4^- + 3P_6 d_5^- + 4P_6 d_6^- + 2P_6 d_7^-$

Subject to

$$7x_1 + 10x_2 + 8x_3 + d_1^- - d_1^+ = 6000$$
$$10{,}000x_1 + 12{,}000x_2 + 7{,}000x_3 + d_2^- - d_2^+ = 80{,}00{,}000$$
$$3{,}000x_1 + 4{,}000x_2 + 2{,}000x_3 + d_3^- - d_3^+ = 1{,}05{,}00{,}000$$
$$x_1 + x_2 + x_3 + d_4^- = 1{,}000$$
$$x_1 + d_5^- - d_5^+ = 200$$
$$x_2 + d_6^- - d_6^+ = 500$$
$$x_3 + d_7^- - d_7^+ = 300$$
all variables ≥ 0

22. From the given information, the goal programming problem may be stated as follows:

Let x_1, x_2, x_3 and x_4 be the amount invested in Government bounds, blue-chip stocks, speculative stocks, and gold respectively. With appropriate deviational variables, we have

Minimise $Z = 0.32\, d_1^- + 0.16\, d_2^+ + 0.08\, d_3^- + 0.02\, d_4^-$

Subject to

$$x_1 + x_2 + x_3 + x_4 \leq 20{,}00{,}000$$
$$x_4 \leq 0.15\,(x_1 + x_2 + x_3 + x_4)$$
$$0.08x_1 + 0.12x_2 + 0.16x_3 + 0.014x_4 + d_1^- - d_1^+ = 2{,}50{,}000$$
$$x_3 + d_2^- - d_2^+ = 4{,}00{,}000$$
$$x_1 + d_3^- - d_3^+ = 3{,}00{,}000$$
$$0.05x_2 + 0.12x_3 + 0.14x_4 + d_4^- - d_4^+ = 60{,}000$$
$$x_1, x_2, x_3, x_4, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ \geq 0$$

Notes: The provisions given in (a) and (b) represent the constraints. Each of the provisions is a goal as presented above accordingly. Since the investment limitation in speculative stocks is given an importance equal to its return, the coefficient of d_2^+ in the objective function is taken to be 0.16. Further, as per guidelines, the weightage for total return works out to be 0.32. Finally, the weightage for the target of capital gains is taken to be 0.02, which is one-fourth of the return on Government bounds.

23. Let the output of the three products P_1, P_2 and P_3 be x_1, x_2 and x_3 units respectively. With appropriate deviational variables, the problem is:

Minimise $Z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^- + P_4 d_4^-$

Subject to

$$2.5x_1 + 1.0x_2 + 2.0x_3 + d_1^- - d_1^+ = 120$$
$$50x_1 + 80x_2 + 35x_3 + d_2^- - d_2^+ = 8{,}000$$
$$6x_1 + 9x_2 + 4.5x_3 + d_3^- - d_3^+ = 600$$
$$\text{all variables} \geq 0$$

24. Let the output per year of parts 1, 2, 3 and 4 be x_1, x_2, x_3 and x_4 respectively. Further, using appropriate deviational variables, the problem may be stated as follows:

Minimise $Z = P_1 d_1^- + P_1 d_2^+ + P_1 d_3^+ + P_1 d_4^+ + P_2 d_5^- + P_2 d_6^- + P_2 d_7^- + P_2 d_8^- + P_3 d_9^- + P_4 d_{10}^-$

Subject to

$$0.06x_1 + 0.17x_2 + 0.10x_3 + 0.14x_4 + d_1^- - d_1^+ = 600$$
$$0.18x_1 + 0.20x_2 + 0.14x_4 + d_2^- - d_2^+ = 500$$
$$0.07x_1 + 0.20x_2 + 0.08x_3 + 0.12x_4 + d_3^- - d_3^+ = 550$$
$$0.09x_1 + 0.12x_2 + 0.07x_3 + 0.15x_4 + d_4^- - d_4^+ = 450$$
$$x_1 + d_5^- = 2{,}600$$
$$x_2 + d_6^- = 1{,}800$$
$$x_3 + d_7^- = 4{,}100$$
$$x_4 + d_8^- = 1{,}200$$
$$9000x_1 + 10{,}000x_2 + 8{,}000x_3 + 12{,}000x_4 + d_9^- - d_9^+ = 70{,}000{,}000$$
$$2.6x_1 + 1.4x_2 + 2.5x_3 + 3.2x_4 + d_{10}^- - d_{10}^+ = 1{,}50{,}000$$
$$\text{all variables} \geq 0$$

Simplex Tableau 1: Non-optimal Solution

Basis	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	d_5^-	d_5^+	b_i	b_i/a_{ij}
d_1^- 0	2	4	1	–1	0	0	0	0	0	0	0	0	80	20
d_2^- 0	3	3	0	0	1	–1	0	0	0	0	0	0	80	80/3
d_3^- $30P_2$	1	0	0	0	0	0	1	–1	0	0	0	0	10	—
d_4^- $40P_2$	0	1*	0	0	0	0	0	0	1	–1	0	0	10	10 ←
d_5^- P_3	30	40	0	0	0	0	0	0	0	0	1	–1	1,200	30
C_j	0	0	0	P_1	0	P_1	$30P_2$	0	$40P_2$	0	P_3	0		
P_3	–30	–40	0	0	0	0	0	0	0	0	0	1	1,200	
P_2	–30	–40	0	0	0	0	0	30	0	40	0	0	700	
P_1	0	0	0	1	0	1	0	0	0	0	0	0	0	
		↑												

Simplex Tableau 2: Non-optimal Solution

Basis	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	d_5^-	d_5^+	b_i	b_i/a_{ij}
d_1^- 0	2	0	1	–1	0	0	0	0	–4	4	0	0	40	20
d_2^- 0	3	0	0	0	1	–1	0	0	–3	3	0	0	50	50/3
d_3^- $30P_2$	1*	0	0	0	0	0	1	–1	0	0	0	0	10	10 ←
x_2 0	0	1	0	0	0	0	0	0	1	–1	0	0	10	—
d_5^- P_3	30	0	0	0	0	0	0	0	–40	40	1	–1	800	80/3
C_j	0	0	0	P_1	0	P_1	$30P_2$	0	$40P_2$	0	P_3	0		
P_3	–30	0	0	0	0	0	0	0	40	–40	0	1	800	
P_2	–30	0	0	0	0	0	0	30	40	0	0	0	300	
P_1	0	0	0	1	0	1	0	0	0	0	0	0	0	
	↑													

Simplex Tableau 3: Non-optimal Solution

Basis	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	d_5^-	d_5^+	b_i	b_i/a_{ij}
d_1^- 0	0	0	1	–1	0	0	–2	2	–4	4*	0	0	20	5 ←
d_2^- 0	0	0	0	0	1	–1	–3	3	–3	3	0	0	20	20/3
x_1 0	1	0	0	0	0	0	1	–1	0	0	0	0	10	—
x_2 0	0	1	0	0	0	0	0	0	1	–1	0	0	10	—
d_5^- P_3	0	0	0	0	0	0	–30	30	–40	40	1	–1	500	50/4
C_j	0	0	0	P_1	0	P_1	$30P_2$	0	$40P_2$	0	P_3	0		
P_3	0	0	0	0	0	0	30	–30	40	–40	0	1	500	
P_2	0	0	0	0	0	0	30	0	40	0	0	0	0	
P_1	0	0	0	1	0	1	0	0	0	0	0	0	0	
										↑				

Simplex Tableau 4: Non-optimal Solution

Basis	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	d_5^-	d_5^+	b_i	b_i/a_{ij}
d_4^+ 0	0	0	1/4	–1/4	0	0	–1/2	1/2	–1	1	0	0	5	10
d_2^- 0	0	0	–3/4	3/4	1	–1	–3/2	3/2*	0	0	0	0	5	10/3 ←
x_1 0	1	0	0	0	0	0	1	–1	0	0	0	0	10	—
x_2 0	0	1	1/4	–1/4	0	0	–1/2	1/2	0	0	0	0	15	30
d_5^- P_3	0	0	–10	10	0	0	–10	10	0	0	1	–1	300	30
C_j	0	0	0	P_1	0	P_1	$30P_2$	0	$40P_2$	0	P_3	0		
P_3	0	0	10	–10	0	0	10	–10	0	0	0	1	300	
P_2	0	0	0	0	0	0	30	0	40	0	0	0	0	
P_1	0	0	0	1	0	1	0	0	0	0	0	0	0	
								↑						

Simplex Tableau 5: Optimal Solution

Basis	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	d_5^-	d_5^+	b_i	b_i/a_{ij}
d_4^+ 0	0	0	1/2	–1/2	–1/3	1/3	0	0	–1	1	0	0	10/3	
d_3^+ 0	0	0	–1/2	1/2	2/3	–2/3	–1	1	0	0	0	0	10/3	
x_1 0	1	0	–1/2	1/2	2/3	–2/3	0	0	0	0	0	0	40/3	
x_2 0	0	1	1/2	–1/2	–1/3	1/3	0	0	0	0	0	0	40/3	
d_5^- P_3	0	0	–5	5	–20/3	20/3	0	0	0	0	1	–1	800/3	
C_j	0	0	0	P_1	0	P_1	$30P_2$	0	$40P_2$	0	P_3	0		
P_3	0	0	5	–5	20/3	–20/3	0	0	0	0	0	1	800/3	
P_2	0	0	0	0	0	0	30	0	40	0	0	0	0	
P_1	0	0	0	1	0	1	0	0	0	0	0	0	0	

Optimal Solution: $x_1 = 40/3$ $x_2 = 40/3$,

The first and second priority goals have been achieved. There has been under-achievement of the third priority goal with an under-achievement deviational variable d_5^- equal to 800/3.

26. Let x_1 : No. of sports ad slots, and
    x_2 : No. of soap opera ad slots

The goals and constraints of the problem can be stated as follows:

$4x_1 + 3x_2 \geq 20$ Goal 1 (HIM requirement)

$5x_1 + 8x_2 \geq 30$ Goal 2 (MIF requirement)

$2x_1 + 4x_2 \geq 15$ Goal 3 (HIF requirement)

$2x_1 + 3x_2 \leq 12$ Budget constraint

Now, let

d_i^- be the amount by which we numerically fall short of the ith goal, and

d_i^+ be the amount by which we numerically exceed the ith goal.

With penalty rates for falling short of various goals being given, we may state the goal programming problem as follows:

Minimise $Z = 2d_1^- + d_2^- + 0.8d_3^-$

Subject to

$$4x_1 + 3x_2 + d_1^- - d_1^+ = 20$$
$$5x_1 + 8x_2 + d_2^- - d_2^+ = 30$$
$$2x_1 + 4x_2 + d_3^- - d_3^+ = 15$$
$$2x_1 + 3x_2 + S_1 = 12$$

All variables being non-negative.

The solution of the problem is contained in Simplex Tableaus 1 through 4.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	S_1	b_i	b_i/a_{ij}
d_1^-	2	4	3	1	–1	0	0	0	0	0	20	20/3
d_2^-	1	5	8*	0	0	1	–1	0	0	0	30	15/4 ←
d_3^-	4/5	2	4	0	0	0	0	1	–1	0	15	15/4
S_1	0	2	3	0	0	0	0	0	0	1	12	4
C_j		0	0	2	0	1	0	4/5	0	0		
Solution		0	0	20	0	30	0	15	0	12		
Δ_j		$-\frac{73}{3}$	$-\frac{86}{5}$	0	2	0	1	0	4/5	0		
			↑									

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	S_1	b_i	b_i/a_{ij}
d_1^-	2	17/8*	0	1	–1	–3/8	3/8	0	0	0	35/4	70/17 ←
x_2	0	5/8	1	0	0	1/8	–1/8	0	0	0	15/4	6
d_3^-	4/5	–1/2	0	0	0	–1/2	1/2	1	–1	0	0	—
S_1	0	1/8	0	0	0	–3/8	3/8	0	0	1	3/4	6
C_j		0	0	2	0	1	0	4/5	0	0		
Solution		0	15/4	35/4	0	0	0	0	0	3/4		
Δ_j		$-\frac{39}{10}$	0	0	2	$\frac{43}{20}$	$\frac{23}{20}$	0	$\frac{4}{5}$	0		
		↑										

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	S_1	b_i	b_i/a_{ij}
x_1	0	1	0	8/17	–8/17	–3/17	3/17	0	0	0	70/17	70/3
x_2	0	0	1	–5/17	5/17	4/17	–4/17	0	0	0	20/17	—
d_3^-	4/5	0	0	4/17	–4/17	–10/17	10/17	1	–1	0	35/17	7/2
S_1	0	0	0	–1/17	1/17	–6/17	6/17*	0	0	1	4/17	2/3 ←
C_j		0	0	2	0	1	0	4/5	0	0		
Solution		70/17	20/17	0	0	0	0	35/17	0	4/17		
Δ_j		0	0	$\frac{154}{85}$	$\frac{16}{85}$	$\frac{25}{17}$	$\frac{-8}{17}$	0	$\frac{4}{5}$	0		
							↑					

Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	S_1	b_i
x_1	0	1	0	1/2	–1/2	0	0	0	0	–1/2	4
x_2	0	0	1	–1/3	1/3	0	0	0	0	2/3	4/3
d_3^-	4/5	0	0	1/3	–1/3	0	0	1	–1	–4/3	5/3
d_2^+	0	0	0	–1/6	1/6	–1	1	0	0	17/6	2/3
C_j		0	0	2	0	1	0	4/5	0	0	
Solution		4	4/3	0	0	0	2/3	5/3	0	0	
Δ_j		0	0	$\frac{26}{15}$	$\frac{4}{15}$	1	0	0	$\frac{4}{5}$	$\frac{16}{15}$	

From the optimal solution contained in Simplex Tableau 4, it is evident that $x_1 = 4$ and $x_2 = 4/3$. It would leave goal 3 under-achieved by 5/3, implying thereby that the number of high-income females reached would be 40/3 lakh instead of the desired 15 lakh.

27. Let the assembly line 1 run for x_1 hours and line 2 for x_2 hours. The goals may be expressed as follows:

Minimise $Z = P_1 d_1^- + P_2 d_3^+ + P_3(10 d_4^- + 12 d_2^-) + P_4(d_2^+ + d_4^+)$

Solution to

$$10x_1 + 12x_2 + d_1^- - d_1^+ = 200$$
$$x_2 + d_2^- + d_3^- - d_3^+ = 12$$
$$x_1 + d_4^- - d_4^+ = 8$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ \geq 0$$

The explanations follow:

Goal (i): $10x_1 + 12x_2 + d_1^- - d_1^+ = 200$, where d_1^- is the deviational variable representing under-achievement of production level.

Goal (ii): $x_2 + d_2^- - d_2^+ = 8$, where 8 represents the normal working time for line 2 and d_2^+ represents the overtime. Now, since the overtime may be more, less or equal to four hours, we may write

$$d_2^+ + d_3^- - d_3^+ = 4$$

or $$d_2^+ = 4 - d_3^- + d_3^+$$

Accordingly, the equation of working time of line 2 may be expressed as:

$$x_2 + d_2^- - (4 - d_3^- + d_3^+) = 8$$

or $$x_2 + d_2^+ + d_3^- - d_3^+ = 12$$

Goal (iii) and Goal (iv): Working time of assembly line 2 is considered earlier. For assembly line 1, we have

$$x_1 + d_4^- - d_4^+ = 8$$

where d_4^- represents underutilisation and d_4^+ shows overutilisation (overtime)
The solution to the problem is given in Simplex Tableaus 1 through 4.

Simplex Tableau 1: Non-optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	b_i	b_i/a_{ij}
d_1^-	P_1	10	12	1	–1	0	0	0	0	0	0	200	100/12
d_3^-	P_2	0	1*	0	0	1	0	1	–1	0	0	12	12 ←
d_4^-	$10P_3$	1	0	0	0	0	0	0	0	1	–1	8	—
C_j		0	0	P_1	0	$12P_3$	P_4	P_2	0	$10P_3$	P_4		
Δ_j	P_4	0	0	0	0	0	0	0	0	0	0		
	P_3	–10	0	0	0	0	0	0	0	0	10		
	P_2	0	–1	0	0	–1	0	0	1	0	0		
	P_1	–10	–12	0	1	0	0	0	0	0	0		
			↑										

Simplex Tableau 2: Non-optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	b_i	b_i/a_{ij}
d_1^-	P_1	10*	0	1	–1	–1/12	0	–1/12	1/12	0	0	56	28/5 ←
x_2	0	0	1	0	0	1	0	1	–1	0	0	12	—
d_4^-	$10P_3$	1	0	0	0	0	0	0	0	1	–1	8	8
C_j		0	0	P_1	0	$12P_3$	P_4	P_2	0	$10P_3$	P_4		
Δ_j	P_4	0	0	0	0	0	0	0	0	0	0		
	P_3	–10	0	0	0	0	0	0	0	0	10		
	P_2	0	0	0	0	0	0	0	0	0	0		
	P_1	–10	0	0	1	1/12	0	1/12	–1/12	0	0		
		↑											

Simplex Tableau 3: Non-optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	b_i	b_i/a_{ij}
x_1	0	1	0	1/10	–1/10	–1/120	0	–1/120	1/120	0	0	28/5	—
x_2	0	0	1	0	0	0	0	1	–1	0	0	12	—
d_4^-	$10P_3$	0	0	–1/10	1/10*	1/120	0	–1/120	–1/120	1	–1	12/5	24 ←
C_j		0	0	P_1	0	$12P_3$	P_4	P_2	0	$10P_3$	P_4		
Δ_j	P_4	0	0	0	0	0	0	0	0	0	1		
	P_3	0	0	1	–1	143/12	0	1	1/12	0	10		
	P_2	0	0	0	0	0	0	1	0	0	0		
	P_1	0	0	1	0	0	0	0	0	0	0		
					↑								

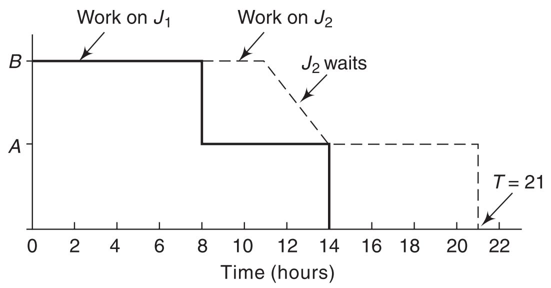
Simplex Tableau 4: Optimal Solution

Basis		x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	b_i
x_1	0	1	0	0	0	0	0	0	0	1	−1	8
x_2	0	0	1	0	0	1	0	1	−1	0	0	12
d_1^+	0	0	0	−1	1	1/12	0	1/12	−1/12	10	−10	24
C_j		0	0	P_1	0	$12P_3$	P_4	P_2	0	$10P_3$	P_4	
Δ_j	P_4	0	0	0	0	0	1	0	0	0	1	
	P_3	0	0	0	0	12	0	0	0	10	0	
	P_2	0	0	0	0	0	0	1	0	0	0	
	P_1	0	0	1	0	0	0	0	0	0	0	

From Simplex Tableau 4, it is evident that the production manager should plan for running the two production lines for eight and twelve hours respectively. It would yield an output of 224 units and meet his goals.

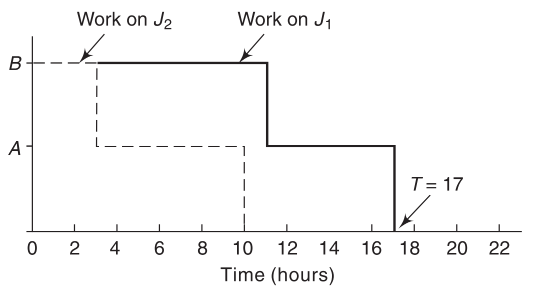
CHAPTER 8

1. Job Sequence: $J_1 - J_2$



(a)

Job Sequence: $J_2 - J_1$

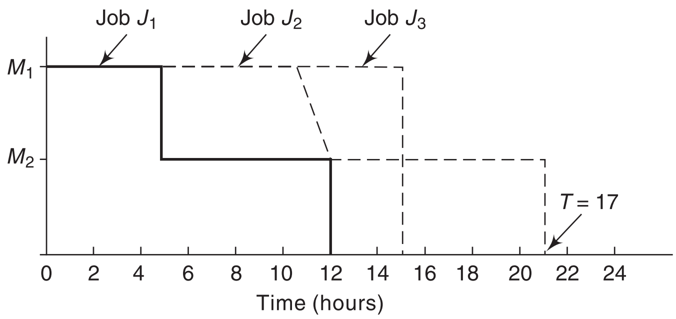


(b)

Gantt Chart

From the problem, the optimal sequence works out to be $J_2 - J_1$ and the total elapsed time is 17 hours.

2. A total of six Gantt Charts need to be prepared here. One of them is drawn here, which represents job sequence J_1–J_2–J_3. The optimal sequence for the problem is J_3–J_1–J_2 which entails a total elapsed time of 20 hours.



3. An optimal sequence of jobs, from the times given for them is as below:

   2, 8, 7, 6, 1, 5, 4, 3

   There are other sequences as well, which are as good as this. The total elapsed time for this sequence is 51, which is shown calculated in the table on next page.

Calculation of Total Elapsed Time

Job	*Machine M_1*		*Machine M_2*	
	In	*Out*	*In*	*Out*
2	0	3	3	11
8	3	6	11	19
7	6	10	19	25
6	10	15	25	30
1	15	22	30	38
5	22	31	38	45
4	31	39	45	49
3	39	45	49	51

4. Using Johnson's Rule as we obtain optimal sequence of tasks, it is observed that there are multiple optimal solutions to the problem. One of these is given below:

   A, C, I, B, H, F, D, E, G

   The minimum total elapsed time is 61, as calculated in the following table.

Calculation of Total Elapsed Time

Task	*Machine M_1*		*Machine M_2*	
	In	*Out*	*In*	*Out*
A	0	2	2	8
C	2	6	8	15
I	6	10	15	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
E	37	43	55	58
G	43	50	58	61

5. In reference to the given times, the optimal sequence of jobs on the two machines is:

   7, 6, 2, 4, 5, 1, 3

   This sequence would result in the total elapsed time equal to 101 hours, with an idle time of 21 hours on machine *B*. The calculations are given in table here.

Calculation of Total Elapsed Time

Job	*Machine A*		*Machine B*		*Idle Time*
	In	*Out*	*In*	*Out*	
7	0	6	6	15	6
6	6	18	18	30	3
2	18	38	38	59	8
4	38	63	63	78	4
5	63	78	78	92	—
1	78	88	92	97	—
3	88	93	97	101	—
				Total	21

6. Using the given information, optimal sequencing of jobs can be obtained as follows:
   Step 1: Schedule *B* in the end
   Step 2: Schedule *D* in the end
   Step 3: Schedule *A* and *G* in the end as *AG* or *GA*
   Step 4: Schedule *F* in the end
   Step 5: Schedule *C* in the first and only place left

   Thus, optimal schedule is: *C*, *F*, *A*, *G*, *D*, *B*; or *C*, *F*, *G*, *A*, *D*, *B*. The calculation of total elapsed time is shown in the following table.

Calculation of Total Elapsed Time

Investment	*Analysis*		*Evaluation*	
	In	*Out*	*In*	*Out*
C	0	10	10	17
F	10	20	20	26
G	20	27	27	32
A	27	35	35	40
D	35	43	43	47
B	43	48	48	51

   The minimum time required for evaluation of the seven projects is, therefore, 51 hours.

7. From the given data, the optimal sequence of jobs is:
   2–4–5–3–6–1

   This sequence has the total elapsed time, T = 85 hours which is shown calculated below.

Calculation of Total Elapsed Time

Job	*Machine A*		*Machine B*	
	In	*Out*	*In*	*Out*
2	0	10	10	25
4	10	22	25	39
5	22	38	39	52
3	38	49	52	62
6	49	69	69	78
1	69	77	78	85

8. Present Schedule:

Calculation of Downtime and Idle Time

Plant	*Crew A*		*Crew B*		*Down Time*	*Idle Time*	
	Start	*Finish*	*Start*	*Finish*		*Crew A*	*Crew B*
P_1	0	6	6	10	10	—	6
P_2	6	12	12	14	8	—	2
P_3	12	16	16	26	14	—	2
P_4	16	22	26	31	15	—	—
P_5	22	27	31	34	12	—	—
P_6	27	35	35	41	14	6	1
				Total	73	6	11

Now, Total Cost = Downtime cost + Crew idle time cost
= 73 × 800 + 6 × 250 + 11 × 430
= Rs 64,630

Optimal schedule: From the given data, optimal sequence is: P_3, P_4, P_6, P_1, P_5, P_2. For this sequence, the total down-time is 71 days, idle time for crew A = 2 days and idle time for crew B = 7 days, as shown calculated below. Accordingly,

Total cost = 71 × 800 + 2 × 250 + 7 × 430
= Rs 60,310

Calculation of Downtime and Idle Time

Plant	*Crew A*		*Crew B*		*Down Time*	*Idle Time*	
	Start	*Finish*	*Start*	*Finish*		*Crew A*	*Crew B*
P_3	0	4	4	14	14	—	4
P_4	4	10	14	19	15	—	—
P_6	10	18	19	25	15	—	—
P_1	18	24	25	29	11	—	—
P_5	24	29	29	32	8	—	—
P_2	29	35	35	37	8	2	3
				Total	71	2	7

9. The optimal order of books processing is:

   4, 1, 3, 2, 5, 6

   This sequence would involve 420 hours of processing in all, which is the minimum. The calculation of total elapsed time is shown in table below.

Calculation of Total Elapsed Time

Book	*Printing time*		*Binding time*	
	In	*Out*	*In*	*Out*
4	0	20	20	80
1	20	50	80	160
3	50	100	160	250
2	100	220	250	350
5	220	310	350	380
6	310	410	410	420

10. In accordance with the algorithm used for sequencing, the optimal sequence of jobs would be:

    J_1, J_5, J_7, J_3, J_4, J_2, J_6

    This optimal sequence is not unique, however.

    The total elapsed time involved is 68 minutes, with 18 minutes idle time for knurling. The calculation is shown in table on next page.

Calculation of Total Elapsed Time

Job	*Turning*		*Knurling*		*Idle time*
	In	*Out*	*In*	*Out*	
J_1	0	3	3	11	3
J_5	3	9	11	17	—
J_7	9	19	19	31	2
J_3	19	31	31	41	—
J_4	31	47	47	57	6
J_2	47	56	57	60	—
J_6	56	67	67	68	7
				Total	18

11. Since the order of processing is *ACB*, we proceed as follows:

    Min $A_i = 5$, Max $C_i = 5$, and Min $B_i = 4$.

    Now, since the condition Min $A_i \geq$ Max C_i is satisfied, we can solve the problem by using an algorithm. First, a consolidation table is prepared by setting $G_i = A_i + C_i$ and $H_i = B_i + C_i$. Thus, we have

Job	G_i	H_i
J_1	15	10
J_2	12	14
J_3	9	11
J_4	16	11
J_5	11.5	11.5
J_6	9	8

From the above, the optimal sequence may be obtained as:

J_3, J_5, J_2, J_4, J_1, J_6.

The total elapsed time for this sequence works out to be 62, as shown in the table.

Calculation of Total Elapsed Time

Job	*Machine A*		*Machine C*		*Machine B*	
	In	*Out*	*In*	*Out*	*In*	*Out*
J_3	0	7	7	9	9	18
J_5	7	17	17	18.5	18.5	28.5
J_2	17	25	25	29	29	39
J_4	25	36	36	41	41	47
J_1	36	48	48	51	51	58
J_6	48	53	53	57	58	62

12. Here Min $A_i = 2$, Max $B_i = 5$ and Min $C_i = 5$. Since the condition Min $C_i \geq$ Max B_i is satisfied, we can solve this problem using the sequencing algorithm. First, a consolidation table is prepared.

Consolidation Table

Job	$G_i = A_i + B_i$	$H_i = B_i + C_i$
1	6	8
2	12	12
3	9	12
4	6	8
5	7	11

An optimal sequence is: 1–4–5–3–2. It involves a total elapsed time T = 42 hours, as shown calculated below.

Calculation of Total Elapsed Time

Job	*Machine A*		*Machine B*		*Machine C*	
	In	*Out*	*In*	*Out*	*In*	*Out*
1	0	3	3	6	6	11
4	3	8	8	9	11	18
5	8	10	10	15	18	24
3	10	17	17	19	24	34
2	17	25	25	29	34	42

13. Here Min A_i = 6, Max B_i = 6 and Min C_i = 4, and, therefore, a necessary condition Min $A_i \geq$ Max B_i is satisfied. To solve the problem, a consolidation table is prepared first.

Consolidation Table

Job	$G_i = A_i + B_i$	$H_i = B_i + C_i$
1	16	15
2	15	9
3	13	9
4	10	9
5	8	10

An optimal sequence is given here: 5–1–2–3–4. It involves a total elapsed time of 51 hours, shown computed below.

Calculation of Total Elapsed Time

Job	*Machine A*		*Machine B*		*Machine C*	
	In	*Out*	*In*	*Out*	*In*	*Out*
5	0	6	6	8	8	16
1	6	16	16	22	22	31
2	16	27	27	31	31	36
3	27	35	35	40	40	44
4	35	42	42	45	45	51

14. If the jobs were performed in the order desired by the manager, it would take 69 hours in all to complete them. The calculation is shown in table below.

Calculation of Total Elapsed Time

Job	*Cutting and planing*		*Chiselling and fitting*		*Finishing and polishing*	
	In	*Out*	*In*	*Out*	*In*	*Out*
1	0	12	12	18	18	25
2	12	22	22	27	27	33
3	22	31	31	37	37	43
4	31	45	45	49	49	54
5	45	52	52	54	54	58
6	52	61	61	65	65	69

We first examine whether this sequence is optimal. Here, the minimum time on cutting and planning is 7 while the maximum time on chiselling and fitting is 6. Thus, optimal sequence can be obtained by using algorithm, since the required condition (7 ≥ 6) is satisfied. For this, consolidation values are calculated first.

Job	*G*	*H*
1	18	13
2	15	11
3	15	12
4	18	9
5	9	6
6	13	8

The optimal sequence of jobs, determined with reference to these times is: 6, 1, 3, 2, 4, 5. The elapsed time T is equal to 67 hours. This is shown calculated in table here. The manager's decision is not the best here.

Calculation of Total Elapsed Time

Job	*Cutting and planning*		*Chiselling and fitting*		*Finishing and polishing*	
	In	*Out*	*In*	*Out*	*In*	*Out*
6	0	9	9	13	13	17
1	9	21	21	27	27	34
3	21	30	30	36	36	42
2	30	40	40	45	45	51
4	40	54	54	58	58	63
5	54	61	61	63	63	67

15. (a) Applying Johnson's Rule, the optimal sequence of jobs would be as follows:

3, 4, 5, 7, 2, 6, 1

(b) For determining the optimal sequence when this process is added, we test if the requisite condition/s is/are satisfied. We have,

Min A_i = 3, Max B_i = 9, and Min C_i = 10.

Since Min $C_i \geq$ Max B_i is satisfied we first prepare consolidation table. Thus,

Consolidation Table

Job	$G_i = A_i + B_i$	$H_i = B_i + C_i$
1	7	12
2	13	18
3	10	18
4	9	18
5	15	21
6	12	15
7	20	19

From the times on G and H, the optimal sequence may be obtained as given here:

1, 4, 3, 6, 2, 5, 7

This sequence would minimise the total time taken to process all the items through three stages. It works out to be 86 units as shown calculated in table on next page.

Calculation of Total Elapsed Time

Job	Cutting		Sewing		Ironing and Packing	
	In	Out	In	Out	In	Out
1	0	5	5	7	7	17
4	5	9	9	14	17	30
3	9	12	14	21	30	41
6	12	19	21	26	41	51
2	19	26	26	32	51	63
5	26	32	32	41	63	75
7	32	44	44	52	75	86

16. Here Min A_i = 8, Max B_i = 7, Max C_i = 7 and Min D_i = 8. Since Min A_i > Max B_I, Max C_i and Min D_i > Max B_I, Max C_i, both the conditions are satisfied so that the optimal sequence can be determined using algorithm, we first prepare the consolidation table for it.

Consolidation Table

Job	$G_i = A_i + B_i + C_i$	$H_i = B_i + C_i + D_i$
J_1	24	20
J_2	17	21
J_3	23	19
J_4	15	20
J_5	23	25
J_6	21	19

From this table, we get the following two optimal sequences:

J_4, J_2, J_5, J_1, J_3, J_6, J_4, J_2, J_5, J_1, J_6, J_3.

The Total elapsed time can be calculated as hwon in table below.

Calculation of Total Elapsed Time (T)

Job	Machine A		Machine B		Machine C		Machine D	
	In	Out	In	Out	In	Out	In	Out
J_4	0	9	9	11	11	15	15	29
J_2	9	17	17	20	20	26	29	41
J_5	17	27	27	33	33	40	41	53
J_1	27	39	39	45	45	51	53	61
J_3	39	52	52	56	56	62	62	71
J_6	52	64	64	71	71	73	73	83

17. From the given information,
Min M_1 = 8, Max M_2 = 8, Max M_3 = 8 and Min M_4 = 14. Since Min M_1 > Max M_2, Max M_3 and Min M_4 > Max M_2, Max M_3, both satisfied we can use algorithm for solving this problem. First we obtain consolidation table as follows:

Consolidation Table

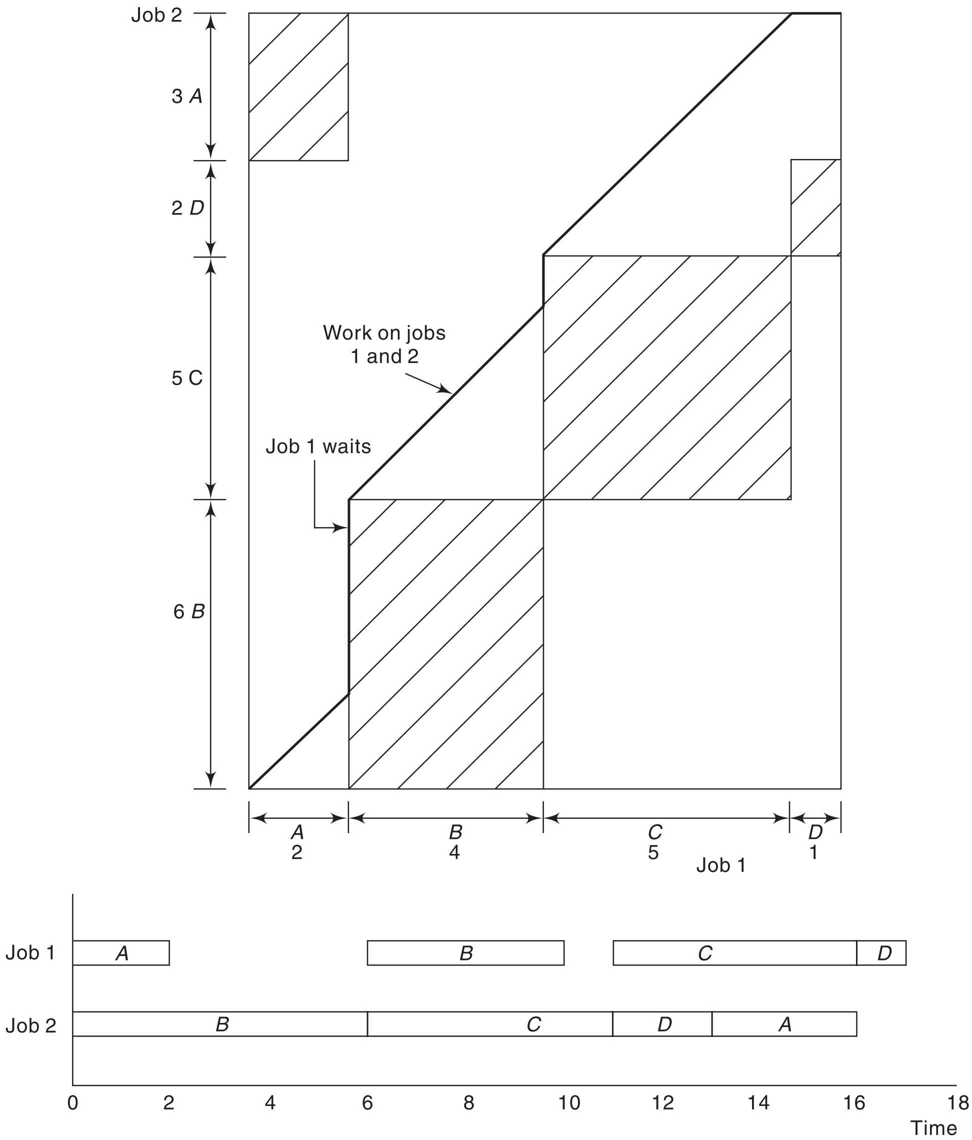
Job	$G = M_1 + M_2 + M_3$	$H = M_2 + M_3 + M_4$
A	28	29
B	26	33
C	21	27
D	19	26

From these values, the optimal sequence is D, C, B, A. The calculation of total elapsed time is shown in table here. It is equal to 82.

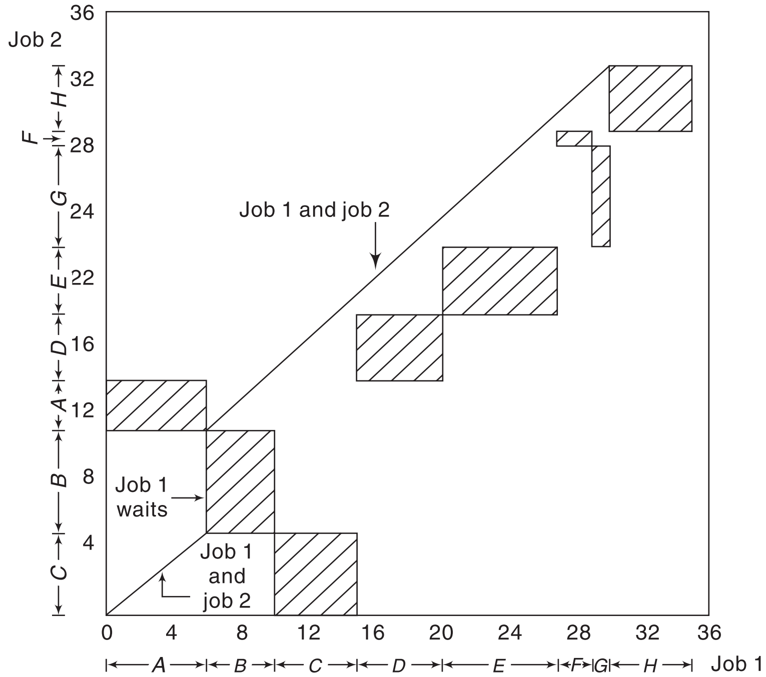
Calculation of Total Elapsed Time

Job	*Machine M_1*		*Machine M_2*		*Machine M_3*		*Machine M_4*	
	In	*Out*	*In*	*Out*	*In*	*Out*	*In*	*Out*
D	0	8	8	13	13	19	19	34
C	8	17	17	24	24	29	34	49
B	17	29	29	35	35	43	49	68
A	29	42	42	50	50	57	68	82

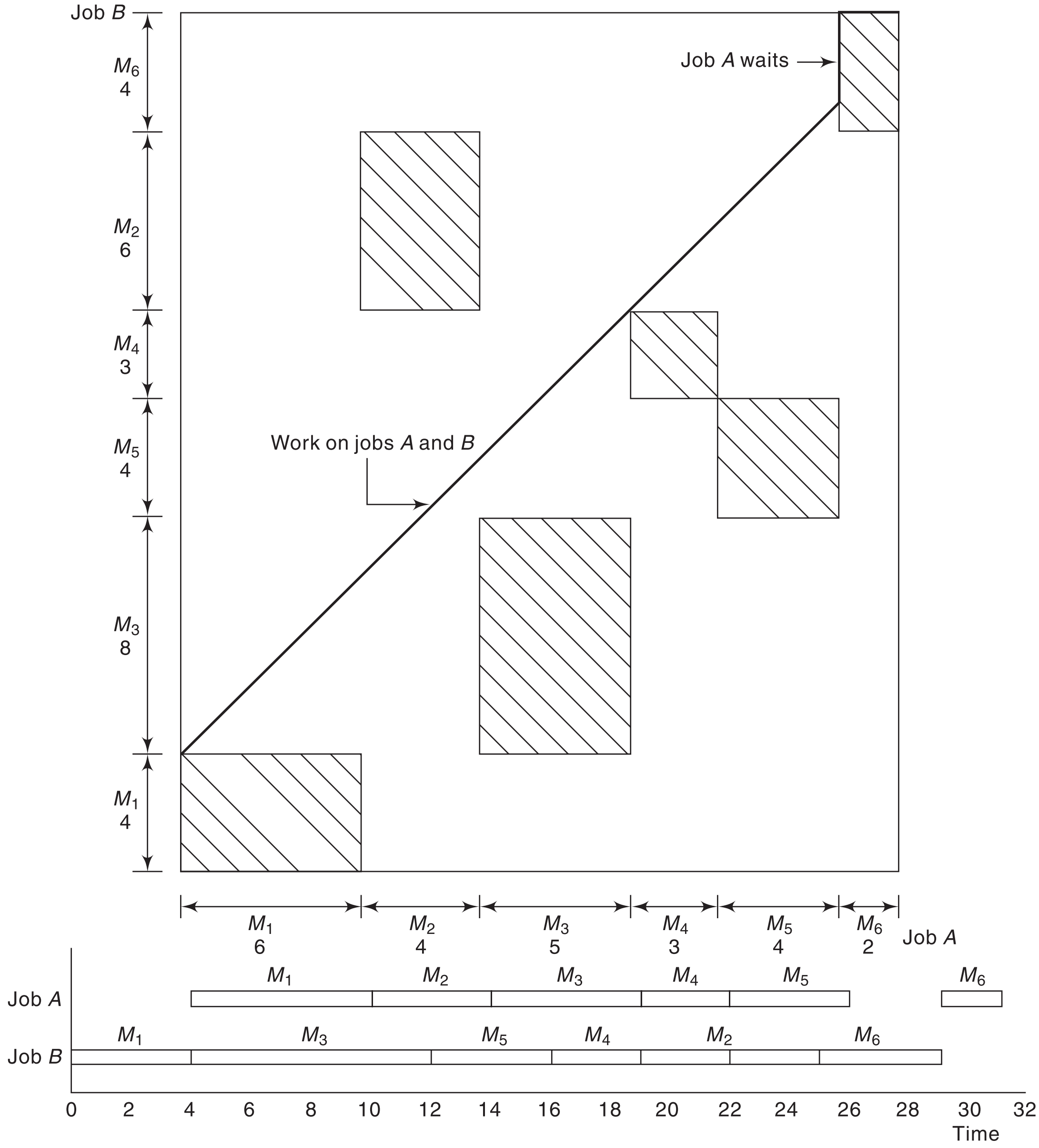
18. The given information is presented graphically. The work on the jobs is to be planned in such a manner that cross-hatched rectangular blocks are to be avoided. The thick line in the graph depicts the work on jobs. It is clear that total time required for the two jobs is 17 hours. The work schedule is as given in lower part of the diagram.



19.


Job 2
Job 1
Job 1 and job 2
Job 1 waits
Job 1 and job 2
0 4 8 12 16 20 24 28 32 36
4 8 12 16 22 24 28 32 36
A B C D E F G H
C B A D E G F H

20. The data given are depicted graphically below. The rectangular blocks show the overlappings, which are to be avoided. The work on two jobs is shown by thick line. The optimal schedule requires job *B* to be worked upon continuously for time 0–29, while job *A* has to wait for the intervals 0–4 and 26–29, in the total interval of 0–31. The two jobs would take total of 31 hours to complete.



CHAPTER 9

1. (a) $$\text{EOQ} = \sqrt{\frac{2AD}{h}}$$

Here, A = Rs 36/order, D = 10,000 units/year, and h = Rs 2/unit/year. Thus,

$$\text{EOQ} = \sqrt{\frac{2 \times 36 \times 10{,}000}{2}}$$

= 600 unit

(b) Assuming 300 working days in a year, the demand rate = 10,000/300 or 100/3 units per day. With this demand rate, the number of days' supply per order would be EOQ/d = 600 ÷ (100/3) or 18 days. Thus, the order quantity is sufficient to last for 18 working days. For a 365-days year, the answer would be 600 × 365/10,000 = 22 days app.

2. Given D = 24,000 units/year, A = Rs 300/order and holding cost h = 24 per cent of Rs 60 = Rs 14.4/unit/year. Accordingly,

$$\text{EOQ} = \sqrt{\frac{2 \times 24{,}000 \times 300}{14.4}}$$

= 1,000 units

No. of orders in a year, $$N = \frac{D}{\text{EOQ}} = \frac{24{,}000}{1{,}000} = 24$$

(i) Optimal interval between placing orders $= \frac{1}{24} \times 360$

= 15 days

(ii) Under the policy of ordering EOQ

Total relevant cost = Ordering cost + Holding cost

$$= \frac{24{,}000}{1{,}000} \times 300 + \frac{1{,}000}{2} \times 14.4$$

= Rs 14,400

When one order is placed every month,

No. of orders in the year = 12, and

Order quantity = 24,000/12 = 2,000 units

Accordingly,

Holding cost $= \frac{2{,}000}{2} \times 14.4$

= Rs 14,400

Ordering cost = 12 × 300

= Rs 3,600

∴ Total cost = 14,400 + 3,600

= Rs 18,000

Thus, extra cost that the factory has to incur

= Rs 18,000 – Rs 14,400

= Rs 3,600

3. (a) It is given here that D = 3,000 units, A = Rs 450/order and h = 10 per cent of Rs 300 = Rs 30.

Accordingly,

(i) $$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$

$$= \sqrt{\frac{2 \times 3{,}000 \times 450}{30}}$$

= 300 units

(ii) When A = Rs 600/order,

$$\text{EOQ} = \sqrt{\frac{2 \times 3{,}000 \times 600}{30}}$$

= 346.4 units

(iii) When h = 7.5 per cent of Rs 300 = Rs 22.5/unit/year

$$\text{EOQ} = \sqrt{\frac{2 \times 3{,}000 \times 450}{22.5}}$$

= 346.4 units

(iv) When A = Rs 600/order and h = Rs 22.5/unit/year

$$\text{EOQ} = \sqrt{\frac{2 \times 3{,}000 \times 600}{22.5}}$$

= 400 units

(v) Total variable cost (Q = 300) = $\frac{3{,}000}{300} \times 450 + \frac{300}{2} \times 30$

= Rs 4,500 + 4,500 = Rs 9,000

Total variable cost (Q = 600) = $\frac{3{,}000}{600} \times 450 + \frac{600}{2} \times 30$

= Rs 2,250 + 9,000 = Rs 11,250

$\therefore$ Increase in TVC as a percentage = $\frac{11{,}250 - 9{,}000}{9{,}000} \times 100$

= 25 per cent

(b) We know

$$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$

When demand increases by 50 per cent, the new demand level = 1.50D. Accordingly,

$$\text{EOQ (new)} = \sqrt{\frac{2 \times 1.50D \times A}{h}}$$

$$= \sqrt{\frac{2DA}{h}}\sqrt{1.50}$$

$$= 1{,}200 \times \sqrt{1.50}$$

= 1470 units

With increase in carrying cost from 25 per cent to 40 per cent, the value of h increases from 0.25C to 0.40C. Thus, new value of h would be 0.40C/0.25C = 1.6 times the original value. Now, since new demand level = 1.50D and new holding cost = 1.60 h, we have

$$\text{EOQ (new)} = \sqrt{\frac{2 \times 1.50D \times A}{1.60h}}$$

$$= \sqrt{\frac{2DA}{h}}\sqrt{\frac{1.50}{1.60}}$$

$$= 1,200\sqrt{\frac{1.50}{1.60}}$$

= 1,162 units

(c) From the given information

$$\text{EOQ }(A) = 100 = \sqrt{\frac{2DA}{h}}$$

$$\text{EOQ }(B) = \sqrt{\frac{2DA}{0.2h}} = \sqrt{\frac{2DA}{h}}\sqrt{\frac{1}{0.2}}$$

$$= 100\sqrt{5} = 223.6 \text{ units}$$

4. (a) $$\text{EOQ} = \sqrt{\frac{2 \times 50,000 \times 200}{0.20}}$$

= Rs 10,000 or 400 units

(b) Revised EOQ = 800 units

5. From the given information, we have D = 1,20,000 units, A = Rs 12/order and h = 20 + 2 + 0.1 = 22.1% of 0.6 = Re 0.1326/unit/year. With these,

(a) $$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$

$$= \sqrt{\frac{2 \times 1,20,000 \times 12}{0.1326}}$$

= 4,660 units approx.

Total annual variable cost,

$$TVC = \sqrt{2DA\,h}$$

$$= \sqrt{2 \times 1,20,000 \times 12 \times 0.1326}$$

= Rs 618

(b) (i) When usage is 25% more, we have

$$TVC = \sqrt{2 \times 1,50,000 \times 12 \times 0.1326}$$

= Rs 691

$$\text{Percent change in } TVC = \frac{618 \times 535}{618} \times 100$$

= 11.81% (increase)

(ii) When usage is 25% less, we have

$$TVC = \sqrt{2 \times 90,000 \times 12 \times 0.1326}$$

= Rs 535 approx.

$$\text{Percent change in } TVC = \frac{618 - 535}{618} \times 100$$

= 13.43% (decrease)

6. Here production rates of the machines are not given. Assuming that the assumptions of classical EOQ model hold, we can decide about the machine to use as follows.

With machine A:

Economic lot size, $\text{ELS} = \sqrt{\dfrac{2AD}{h}}$

Here D = 8,000 units, A = Rs 200/set up and h = 20% of Rs 18 = Rs 3.60. Thus,

$$\text{ELS} = \sqrt{\frac{2 \times 200 \times 8{,}000}{3.60}}$$
$$= 942.81 \text{ units}$$

Total cost = Cost of set up and holding + Cost of production

$$\text{Cost of set up and holding} = \sqrt{2ADh}$$
$$= \sqrt{2 \times 200 \times 8{,}000 \times 3.60}$$
$$= \text{Rs } 3394.11$$
$$\text{Cost of production} = 8{,}000 \times 18$$
$$= \text{Rs } 1{,}44{,}000$$
$$\therefore \quad \text{Total cost} = 3394.11 + 1{,}44{,}000$$
$$= \text{Rs } 1{,}47{,}394.11 \text{ p.a.}$$

With machine B:

$$\text{ELS} = \sqrt{\frac{2 \times 100 \times 8{,}000}{20\% \text{ of } 18.10}}$$
$$= 664.82 \text{ units}$$
$$\text{Cost of set up and holding} = \sqrt{2 \times 100 \times 8{,}000 \times 3.62}$$
$$= \text{Rs } 2{,}406.66$$
$$\text{Cost of production} = 8{,}000 \times 18.10$$
$$= \text{Rs } 1{,}44{,}800$$
$$\therefore \quad \text{Total cost} = 2{,}406.66 + 1{,}44{,}800$$
$$= \text{Rs } 1{,}47{,}206.66 \text{ p.a.}$$

Evidently, machine B should be used since it involves lower cost.

7. Assuming 360 working days, $D = 360 \times 100 = 36{,}000$ and $h = 0.02 \times 360 = 7.2$.

$$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$
$$= \sqrt{\frac{2 \times 36{,}000 \times 100}{7.2}} = 1000 \text{ units}$$
$$\text{Re-order Level} = \text{Lead time} \times \text{Demand rate}$$
$$= 14 \times 100 = 1400 \text{ units}$$

Also,

$$\text{ROL(LT = 7 days)} = 7 \times 100 = 700 \text{ units}$$
$$\text{ROL(LT = 21 days)} = 21 \times 100 = 2100 \text{ units}$$

8. *Item A:*

$$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$
$$= \sqrt{\frac{2 \times 8{,}000 \times 15}{0.06}}$$
$$= 2{,}000 \text{ units}$$
$$\text{Reorder level} = \text{Demand rate} \times \text{Lead time}$$
$$= \frac{8{,}000}{250} \times 10$$
$$= 320 \text{ units}$$

Item B:

With reorder level = 216 units and lead time = 6 days, demand rate d = 216/6 = 36 units. Accordingly, total demand D = 36 × 250 = 9,000 units.

Now,

$$\text{EOQ} = \sqrt{\frac{2 \times 9{,}000 \times 40}{0.18}}$$
$$= 2{,}000 \text{ units}$$

Item C:

If A be the ordering cost per order, we have

$$300 = \sqrt{\frac{2 \times 7{,}500 \times A}{30}}$$

or
$$A = \frac{300 \times 300 \times 30}{2 \times 7{,}500}$$
$$= \text{Rs } 180$$

With D = 7,500 units and number of days = 250, the demand rate d = 7,500/250 = 30 units/day. Further, lead time = 210/30 = 7 days.

9. (i) *From outside supplier:*

With D = 10,000 units/year, A = Rs 10 + 20 = Rs 30/order and h = Rs 2/unit/year, we have

$$\text{EOQ} = \sqrt{\frac{2AD}{h}}$$
$$= \sqrt{\frac{2 \times 30 \times 10{,}000}{2}}$$
$$= 548 \text{ units (approx.)}$$

Total annual cost of ordering EOQ = 548 units,

$$\text{TC} = \frac{D}{Q} \times A + \frac{Q}{2} \times h \; + DC$$
$$= \frac{10{,}000}{548} \times 30 + \frac{548}{2} \times 2 + 10{,}000 \times 12$$
$$= \text{Rs } 121{,}095.45$$

From subsidiary company:

In this case, D = 10,000 units/year, C = Rs 13/unit, A = Rs 10 + 15 = Rs 25/order, h = Rs 2/unit/year, the economic order quantity,

$$\text{EOQ} = \sqrt{\frac{2 \times 25 \times 10{,}000}{2}}$$
$$= 500 \text{ units}$$

Total annual cost corresponding to this order quantity,

$$\text{TC} = \frac{10{,}000}{500} \times 25 + \frac{500}{2} \times 2 + 10{,}000 \times 13$$
$$= \text{Rs } 1{,}31{,}000$$

(ii) It is evident from the total cost calculations that the company should purchase from outside supplier and the order size be 548 units. The minimum total cost in this case would be Rs 1,21,095.45 per annum.

10. The cost relevant for EOQ computation are:

Ordering cost:

Clerical and data processing	: Rs 10.625
Rail transport	: Rs 40.00

Holding cost:

Maintenance cost	: Rs 4.00
Opportunity cost of storage space	: 2 × 0.5 = Re 1.00

Accordingly, D = 40,000 gallons/year, A = Rs 50.625/order and h = Rs 5/unit/year. With these,

$$\text{EOQ} = \sqrt{\frac{2 \times 40{,}000 \times 50.625}{5}}$$

= 900 gallons

The total annual costs are:	Rs
Ordering : 50.625 × 40,000/9	= 2,250
Holding : 5 × 900/2	= 2,250
Rail transport : 40,000 × 0.20	= 8,000
Storage overhead:	= 2,000
Purchase costs : 40,000 × 2	= 80,000
Total	94,500

11. (a) (i) A general expression for the total annual cost of borrowing and holding cash is given by $T(C)$ where,

$$T(C) = \frac{D}{Q} \times C_o + \frac{Q}{2} \times C_h$$

This expression is based on the following assumptions:

1. Cash requirements are uniform during the year.
2. Funds can be obtained in a specified period of time.
3. The interest rates on bonds are constant and are not affected by the size of bond issue.
4. Shortage of funds is not permitted.

(ii) With D = Rs 100 m, C_o = Rs 1,00,000 and C_h = 8 per cent per annum

$$Q = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2 \times 10{,}00{,}00{,}000 \times 1{,}00{,}000}{0.08}}$$

= Rs 1,58,11,388

Number of bond issues to be floated every year,

$$N = \frac{D}{Q}$$

$$= \frac{10{,}00{,}00{,}000}{1{,}58{,}11{,}388}$$

= 6.32 times

(iii) Total cost of floating bonds $= \frac{D}{Q} \times C_o$

$$= \frac{10{,}00{,}00{,}000}{1{,}58{,}11{,}388} \times 100{,}000$$

= Rs 6,32,455

In case of EOQ, annual cost of floating bonds and the opportunity cost are equal. Thus, annual opportunity cost associated with holding cash = Rs 6,32,455.

(iv) With 255 trading days in a year and a lead time of five days, the ‘reorder’ level, R, is given by

R = Demand during lead time + Safety stock

$$= \frac{10,00,00,000}{255} \times 5 + 0$$

= Rs 19,60,784

Thus, when the cash level is Rs 19,60,784, then a bond issue be initiated.

(b) If C_h = 12 per cent per annum, the effects of conducting sensitivity analysis are as follows:

$$Q = \sqrt{\frac{2 \times 10,00,00,000 \times 1,00,000}{0.12}}$$

= Rs 1,29,09,945

$$N = \frac{10,00,00,000}{1,29,09,945}$$

= 7.75

Annual cost of floating bonds

$$= \frac{10,00,00,000}{1,29,09,945} \times 1,00,000$$

= Rs 7,74,596

Also, annual opportunity cost = Rs 7,74,596

Finally, 'reorder' level, i.e., level of cash when a bond issue be initiated, will remain unchanged.

12. $$\text{EOQ} = \sqrt{\frac{2 \times 40,000 \times 80}{0.25 \times 80}}$$

= 566 units

$$\text{Total cost (566)} = 40,000 \times 80 + \frac{40,000}{566} \times 80 + \frac{566}{2} \times 0.25 \times 80$$

= Rs 32,11,314

$$\text{Total cost (2000)} = 40,000 \times 76 + \frac{40,000}{2000} \times 80 + \frac{2000}{2} \times 0.25 \times 76$$

= Rs 30,60,600

Therefore, manager should take advantage of the discount offer.

13. With annual demand = 8 × 200 = 1600 units, ordering cost = Rs 500/order, holding cost = 40% of the unit cost, we have

$$\text{EOQ} = \sqrt{\frac{2 \times 1,600 \times 500}{0.40 \times 400}} = 100 \text{ units}$$

$$\text{TC}(Q = 100) = 1,600 \times 400 + \frac{1,600}{100} \times 500 + \frac{100}{2} \times 0.4 \times 400$$

= Rs 6,56,000

$$\text{TC}(Q = 500) = 1,600 \times 360 + \frac{1,600}{500} \times 500 + \frac{500}{2} \times 0.4 \times 360$$

= Rs 6,13,600

Cost Saving = Rs 6,56,000 – Rs 6,13,600 = Rs 42,400

14. With c = Rs 1.40:

$$\text{EOQ} = \sqrt{\frac{2 \times 10,00,000 \times 28.80}{0.20 \times 1.40}} = 14,343 \text{ (Infeasible)}$$

With c = Rs 1.60:

$$\text{EOQ} = \sqrt{\frac{2 \times 10,00,000 \times 28.80}{0.20 \times 1.60}} = 13,416 \text{ (Feasible)}$$

$$\text{TC}(13,416) = 10,00,000 \times 1.60 + \frac{10,00,000}{13,416} \times 28.80 + \frac{13,416}{2} \times 0.2 \times 1.60$$

= Rs 16,04,293

$$TC(20{,}000) = 10{,}00{,}000 \times 1.40 + \frac{10{,}00{,}000}{20{,}000} \times 28.80 + \frac{20{,}000}{2} \times 0.2 \times 1.40$$
$$= \text{Rs } 14{,}04{,}240$$

Thus, optimal order quantity = 20,000 units

15. With normal price,

$$EOQ = \sqrt{\frac{2 \times 2{,}400 \times 350}{0.24 \times 10.00}} = 837 \text{ units}$$

Even at normal price, the EOQ qualifies for 12.5 per cent discount. Thus, we may recalculate EOQ at c = Rs 8.75 (with a 12.5% discount).

$$EOQ = \sqrt{\frac{2 \times 2{,}400 \times 350}{0.24 \times 8.75}} = 894 \text{ units}$$

$$TC(894) = 2400 \times 8.75 + \frac{2{,}400}{894} \times 350 + \frac{894}{2} \times 8.75 \times 0.24 = \text{Rs } 22{,}878$$

16. With given data,
$D = 50 \times 12 = 600$, $A = 10$, $h = 0.20 \times 6 = 1.20$

$$EOQ = \sqrt{\frac{2DA}{h}}$$
$$= \sqrt{\frac{2 \times 600 \times 10}{1.20}} = 100 \text{ units}$$

$$TC\,(100) = 600 \times 6 + \frac{600}{100} \times 10 + \frac{100}{2} \times 1.20 = \text{Rs } 3{,}720$$

$$TC\,(200) = 600 \times 5.70 + \frac{600}{200} \times 10 + \frac{200}{2} \times 0.20 \times 5.70 = \text{Rs } 3{,}564$$

$$TC\,(1{,}000) = 600 \times 5.40 + \frac{600}{1{,}000} \times 10 + \frac{1000}{2} \times 0.20 \times 5.40 = \text{Rs } 3{,}786$$

Thus, optimal order size = 200 units.

17. With c = Rs 350:

$$EOQ = \sqrt{\frac{2 \times 5{,}000 \times 150}{0.20 \times 350}} = 146 \quad \text{(Infeasible)}$$

With c = Rs 400:

$$EOQ = \sqrt{\frac{2 \times 5{,}000 \times 150}{0.20 \times 400}} = 137 \quad \text{(Infeasible)}$$

With c = Rs 450:

$$EOQ = \sqrt{\frac{2 \times 5{,}000 \times 150}{0.20 \times 450}} = 129 \quad \text{(Infeasible)}$$

With c = Rs 500:

$$EOQ = \sqrt{\frac{2 \times 5{,}000 \times 150}{0.20 \times 500}} = 122 \quad \text{(Feasible)}$$

We have,

$$TC(Q = 122) = 5{,}000 \times 500 + \frac{5{,}000}{122} \times 150 + \frac{122}{2} \times 0.2 \times 500 = \text{Rs } 25{,}12{,}247$$

$$TC(Q = 1{,}000) = 5{,}000 \times 450 + \frac{5{,}000}{1{,}000} \times 150 + \frac{1{,}000}{2} \times 0.2 \times 450 = \text{Rs } 22{,}95{,}750$$

$$TC(Q = 3{,}000) = 5{,}000 \times 400 + \frac{5{,}000}{3{,}000} \times 150 + \frac{3{,}000}{2} \times 0.2 \times 400 = \text{Rs } 21{,}20{,}250$$

$$TC(Q = 5{,}000) = 5{,}000 \times 350 + \frac{5{,}000}{5{,}000} \times 150 + \frac{5{,}000}{2} \times 0.2 \times 350 = \text{Rs } 19{,}25{,}150$$

∴ Optimal order quantity = 5000 units

18. Annual worth of LED read-out circuits, DC = 75,000

Cost per order, A = Rs 45

Carrying charge, i = 25% or 0.25

Since the company is using EOQ purchasing system, the total minimum cost per annum is given by

TC = Material cost + Ordering cost + Carrying cost

$$= DC + \sqrt{2DC\,Ai}$$

$$= 75{,}000 + \sqrt{2 \times 75{,}000 \times 45 \times 0.25}$$

= 75,000 + 1,299 = Rs 76,299

When circuits are bought in equal quantities four times in a year, then we have,

Annual material cost = Rs 75,000 – 1.5% of Rs 75,000

= Rs 75,000 – Rs 1,125 = Rs 73,875

Worth of each order = Rs 73,875/4

= Rs 18,468.75

$$\text{Carrying cost} = \frac{18{,}468.75}{2} \times 0.25$$

= Rs 2,308.59

Ordering cost = 4 × 45

= Rs 180

Total annual cost = 73,875 + 2,308.59 + 180

= Rs 76,363.59

Evidently, since the total cost in the case of discount offer is more than the total cost in case of EOQ policy, the company should reject the discount offer.

To calculate the minimum discount acceptable in order that the present total cost should not be exceeded, we proceed as follows. Let the discount rate be equal to $1 - R$.

With this,

Annual ordering cost = 4 × 45

= Rs 180

$$\text{Annual carrying cost} = \frac{1}{2}\left[\frac{75{,}000}{4} \times R\right] \times 0.25$$

= Rs 2,343.75R

Annual material cost = 75,000 R

From the given information,

180 + 2,343.75 R + 75,000 R = 76,299

or 77,343.75 R = 76,119

or R = 76,119/77,343.75

= 0.984

or $1 - R$ = 0.016

Hence, the minimum discount the company would expect = 0.016 or 1.6%.

19. For the policy followed by the Purchase manager:

First buy

$$TC = 500 \times 14.25 + 400 + \frac{500}{2} \times 0.45 \times 14.25 \times \frac{1}{2} = \text{Rs } 8,327$$

Second buy

$$TC = 250 \times 15 + 400 + \frac{250}{2} \times 0.45 \times 15 \times \frac{1}{4} = \text{Rs } 4361$$

Third buy

$$TC = 250 \times 13.8 + 400 + 250 \times 0.45 \times 13.8 \times \frac{2}{12} + \frac{250}{2} \times 0.45 \times 13.8 \times \frac{1}{4}$$

$$= \text{Rs } 4303$$

$$\text{Total cost} = 8,327 + 4,361 + 4,303 = \text{Rs } 16,991$$

Notes:

(i) The first buy is sufficient for six months. So holding cost for 6 months is provided.

(ii) The second buy would last for 3 months. Accordingly, the holding cost for 3 months be found for this stock.

(iii) The third buy is also good for 3 months. But the purchase is made in the beginning of month 8. So it would be carried for two months until previous stocks last and then its consumption would begin. The holding cost is provided accordingly.

For the policy of EOQ:

$$EOQ = \sqrt{\frac{2 \times 1,000 \times 400}{0.45 \times 15}} = 344 \text{ units}$$

$$TC(EOQ) = 1,000 \times 15 + \frac{1,000}{344} \times 400 + \frac{344}{2} \times 0.45 \times 15 = \text{Rs } 17,324$$

∴ Cost saving by the policy followed = 17,324 – 16,991 = Rs 333

20. (a) (i)

$$EOQ = \sqrt{\frac{2 \times 5,000 \times 200}{0.20 \times 10}} = 1,000 \text{ units}$$

(ii) No. of orders, $n = \frac{5,000}{1,000} = 5$ per annum

(iii) $TC = \sqrt{2 \times 5,000 \times 200 \times 0.2 \times 10}$ = Rs 2,000 (except component cost)

(b) $TC(1800) = 5,000 \times 9.95 + \frac{5,000}{1,800} \times 200 + \frac{1,800}{2} \times 2 = \text{Rs } 52,106$

$TC(2000) = 5,000 \times 9.90 + \frac{5,000}{2,000} \times 200 + \frac{2,000}{2} \times 2 = \text{Rs } 52,000$

$TC(2400) = 5,000 \times 9.85 + \frac{5,000}{2,400} \times 200 + \frac{2,400}{2} \times 2 = \text{Rs } 52,067$

None of the discounts offered results in cost savings. From the cost values observed, he can either buy 1000 units (EOQ) or 2000 units each time.

21. From the given information, annual usage, $D = 20 \times 365 = 7,300$ units, set up cost per lot, A = Rs 50 and holding cost, h = Re 0.3 × 365 = Rs 109.5 units/year. Using these data,

$$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$

$$= \sqrt{\frac{2 \times 7{,}300 \times 50}{109.5}}$$

= 82 units approx.

With this policy of ordering 82 units, the total cost works out to be

$$\text{TC}(82) = 7{,}300 \times 4 + \frac{7{,}300}{82} \times 50 + \frac{82}{2} \times 109.5$$

= Rs 38,141 approx.

When a discount of 10% is accepted:

Unit cost = Rs 3.60 per unit and lot size = 150. Thus,

$$\text{TC}(150) = 7{,}300 \times 3.60 + \frac{7{,}300}{150} \times 50 + \frac{150}{2} \times 109.5$$

= Rs 36,926

Thus, if the discount offer is accepted, there is a net saving of Rs 38,141 – 36,926 = Rs 1,215. In order to determine the range, or the percentage discount in the price of the item for lots of 150 units or more, that will not result in any financial advantage, we proceed as follows.

$$\text{Ordering cost} = \frac{7{,}300}{150} \times 50 = \text{Rs } 2{,}433.33$$

$$\text{Carrying cost} = \frac{150}{2} \times 109.5 = \text{Rs } 8{,}212.50$$

(These remain the same as above.)

Accordingly, the minimum purchase cost of 7,300 items

= Total cost of items without discount – (Ordering cost + Carrying cost) when order quantity is 150

= 38,141 – (2,433.33 + 8,212.50)

= Rs 27,495

From this, the price of one unit of the item works out to be,

C = 27,495/7,300

= Rs 3.766

∴ Range of discount not resulting in any financial advantage will be equal to, or less than, $\frac{0.234 \times 100}{4}$ = 5.85%.

22. Economic Lot Size $= \sqrt{\frac{2 \times 1{,}00{,}000 \times 5{,}000}{0.2 \times 10}}\sqrt{\frac{2{,}00{,}000}{2{,}00{,}000 - 1{,}00{,}000}}$

= 31,623 units

Length of production run $= \frac{31{,}623}{2{,}00{,}000}$ = 0.158 year

23. Economic Lot Size $= \sqrt{\frac{2 \times 2{,}000 \times 300}{1.60}}\sqrt{\frac{8{,}000}{8{,}000 - 2{,}000}}$

= 1,000 units

$$TC(ELS) = 2 \times 300 + \frac{1,000}{8,000} \times \frac{(8,000 - 2,000)}{2} \times 1.60 = \text{Rs } 1,200$$

$$TC(\text{Present Policy}) = 4 \times 300 + \frac{500}{8,000} \times \frac{(8,000 - 2,000)}{2} \times 1.60 = \text{Rs } 1,500$$

∴ Saving in cost by switching to ELS policy = Rs 1,500 – Rs 1,200
= Rs 300

24. (a) $EOQ = \sqrt{\frac{2 \times 24,000 \times 324}{0.1 \times 12}}$ = 3,600 units

(b) Interval between consecutive production runs = $\frac{3,600}{24,000}$
= 0.15 year

(c) Inventory cost = $24,000 \times 12 + \frac{24,000}{3,600} \times 324 + \frac{3,600}{2} \times 0.1 \times 12$ = Rs 2,92,320

25. If h be the holding cost per unit, we have

$$275 = \sqrt{\frac{2 \times 5,000 \times 100}{h}} \sqrt{\frac{50}{50 - 17}}$$

or h = Rs 20

With a 10% increase, revised h = 20 + 10% of 20 = Rs 22

$$\text{Revised ELS} = \sqrt{\frac{2 \times 5,000 \times 100}{22}} \sqrt{\frac{50}{50 - 17}} = 262 \text{ units}$$

26. Here D = 1,92,000 units, A = Rs 1,080/set-up, $h = 0.3 \times 12$ = 3.60/pack/year, d = 1,92,000/240 = 800 packs per day, and p = 20,000/20 = 1,000 packs/day. With these values,

(a) Optimum losts size = $\sqrt{\frac{2DA}{h}\left(\frac{p}{p-d}\right)}$

$$= \sqrt{\frac{2 \times 1,92,000 \times 1,080}{3.60}\left(\frac{1,000}{1,000 - 800}\right)}$$

= 24,000 packs

(b) Optimum number of production runs

$$= \frac{\text{Annual demand}}{\text{Optimum lot size}}$$

$$= \frac{1,92,000}{24,000}$$

= 8

(c) Time interval between successive production runs

$$= \frac{\text{No. of working days}}{\text{No. of runs}}$$

$$= \frac{240}{8}$$

= 30 working days

(d) Total variable cost = $\sqrt{2DAh\left(\frac{p-d}{p}\right)}$

$$= \sqrt{2 \times 1{,}92{,}000 \times 1{,}080 \times 3.60 \times \left(\frac{1{,}000 - 800}{1{,}000}\right)}$$

= Rs 17,280

27. It is given here that production-lot size ELS = 2,600 units, D = 30,000 units/year, A = Rs 135/set-up, p = 200 units/day, d = 100 units/day and h = 28% of the unit cost. Now, if the unit cost be Rs C, we have h = 0.28C. The economic lot size is obtained as follows:

$$\text{ELS} = \sqrt{\frac{2DA}{h}}\sqrt{\frac{p}{p-d}}$$

Substituting given values in this formula, we have

$$2{,}600 = \sqrt{\frac{2 \times 30{,}000 \times 135}{0.28C}}\sqrt{\frac{200}{200-100}}$$

Solving for C, we get

$$C = \frac{2 \times 30{,}000 \times 135 \times 2}{0.28 \times 2{,}600 \times 2{,}600}$$

= Rs 8.56

Thus, company B's cost of producing the item P_7 is Rs 8.56 per unit.

28. D = 20,000, d = 20,000/250 = 80, p = 200 and s = 600, h = 0.025 × 400 = 10

(a) ELS = $\sqrt{\frac{2 \times 20{,}000 \times 600}{10}}\sqrt{\frac{200}{200-80}}$

= 2,000 units

(b) TRC = $\sqrt{\frac{2 \times 20{,}000 \times 600 \times 10 \times 120}{200}}$

= Rs 12,000

(c) N = D/ELS = 20,000/2,000 = 10

(d) t = 2,000/200 = 10 days

(e) Maximum Stock = $\frac{2{,}000}{200}$ (200 – 80)

= 1,200 units

29. *When item is purchased from outside:*

D = 2,500 units/year, A = Rs 12/order, h = 12% of Rs 32 = Rs 3.84/unit/year. Thus,

$$\text{EOQ} = \sqrt{\frac{2 \times 2{,}500 \times 12}{3.84}}$$

= 125 units

Total cost = Cost of items + Holding cost + Ordering cost

$$= 2{,}500 \times 32 + \frac{125}{2} \times 3.84 + \frac{2{,}500}{125} \times 12$$

= 80,000 + 240 + 240 = Rs 80,480 p.a.

When item is produced internally:

D = 2,500 units/year, A = Rs 60/set-up, h = 12% of Rs 30 = Rs 3.60/unit/year, p = 10,000/250 = 40 units/day and d = 10 units/day. With these,

$$\text{Economic-lot size} = \sqrt{\frac{2 \times 2{,}500 \times 60}{3.60}} \sqrt{\frac{40}{40 - 10}}$$

$$= 333.33 \text{ units}$$

$$\text{Total variable cost} = \sqrt{2 \times 2{,}500 \times 60 \times 3.60} \sqrt{\frac{40 - 10}{40}}$$

$$= \text{Rs } 900$$

Total cost = Cost of production + Total variable cost

= 2,500 × 30 + 900 = Rs 75,900

Conclusion: The item should be produced internally.

30. (i) $\text{EOQ} = \sqrt{\frac{2 \times 2{,}500 \times 15}{4}} = 137$ units

Optimal number of orders = $\frac{2{,}500}{137} = 18$

$\text{TC(EOQ)} = 2{,}500 \times 30 + \frac{2{,}500}{137} \times 15 + \frac{137}{2} \times 4 = \text{Rs } 75{,}548$

(ii) $\text{EPLS} = \sqrt{\frac{2 \times 2{,}500 \times 250}{4}} \sqrt{\frac{4{,}800}{4{,}800 - 2{,}500}}$

= 808 units

Length of production run = $\frac{808}{4{,}800} = 0.168$ year

$$\text{TC(ELPS)} = 2{,}500 \times 24 + \frac{2{,}500}{808} \times 250 + \frac{808}{4{,}800} \times \frac{(4{,}800 - 2{,}500)}{2} \times 4$$

= Rs 61,548

(iii) It should manufacture the part internally.

31. According to given data, D = 20 × 12 = 240 units/year, A = Rs 10/set-up, h = Re 0.25 × 12 = Rs 3/unit/year and b = Rs 15/unit/year. With these values,

$$\text{Economic-lot size} = \sqrt{\frac{2DA}{h}} \sqrt{\frac{b + h}{b}}$$

$$= \sqrt{\frac{2 \times 240 \times 10}{3}} \sqrt{\frac{15 + 3}{15}}$$

= 44 units approx.

Time between production runs,

$$T = \frac{\text{EOQ}}{D}$$

$$= \frac{44}{240} = 0.1833 \text{ years or 67 days.}$$

32. (a) Unit price for different order quantities.

Order Size	*Discount*	*Unit Price*
Q < 500	—	Rs 50.0
1000 > Q ≥ 500	4%	Rs 48.0
Q ≥ 1000	5%	Rs 47.5

$$EOQ = \sqrt{\frac{2 \times 2500 \times 50}{0.20 \times 50}} = 158.11$$

$$TC(158.11) = 2500 \times 50 + \frac{2500}{158.11} \times 50 + \frac{158.11}{2} \times 0.20 \times 50 = \text{Rs } 126{,}581$$

$$TC(500) = 2500 \times 48 + \frac{2500}{500} \times 50 + \frac{500}{2} \times 0.20 \times 48 = \text{Rs } 122{,}650$$

$$TC(1000) = 2500 \times 47.5 + \frac{2500}{1000} \times 50 + \frac{1000}{2} \times 0.20 \times 47.5 = \text{Rs } 123{,}625$$

Best order quantity = 500 units

(b) (i) $EBQ = \sqrt{\frac{2 \times 10000 \times 15000}{0.01 \times 15000}} \sqrt{\frac{20000}{20000 - 10000}} = 2{,}000$ units

(ii) $TVC = \sqrt{2 \times 10000 \times 15000 \times 0.01 \times 15000 \times \frac{10000}{20000}} = \text{Rs } 150{,}000$

(iii) No. of production runs = 10000/2000 = 5

(iv) Time required for producing each batch, $t_p = Q/P = \frac{2000}{20000} = 0.1$ year

(v) Maximum inventory level = $t_p\ (P - D)$
= 0.1 (20000 – 10000) = 1000 units

(c) EOQ would be larger in case (ii) because it is obtained by multiplying the EOQ in (i) by $\sqrt{P/(P - D)}$, which is always greater than 1 since P > D.

(d) Given $\sqrt{\frac{2AD}{h}} = 1000$

(i) $\sqrt{\frac{2 \times 1.5 \times AD}{h}} = 1000\sqrt{1.5} = 1224.7 \simeq 1225$ units

(ii) $\sqrt{\frac{2 \times 1.5 \times AD}{1.6h}} = 1000\sqrt{1.5}/\sqrt{1.6} = 968.2 \simeq 968$ units

(e) (i) Let re-order level be x.

$1.645 = \frac{x - 50}{10}$ ∴ $x = 16.45 + 50 = 66.45 \simeq 67$ units

(ii) $2.33 = \frac{x - 50}{10}$ ∴ $x = 23.3 + 50 = 73.3 \simeq 74$ units

Thus, Safety Stock = 74 – 50 = 24 units

(iii) $Z = \dfrac{75-50}{10} = 2.5$ Service Level = 0.5 + 0.4938 = 0.9938 or 99.38%

Area for Z = 2.5 is 0.4938

33. (i) With monthly demand = 90 chairs, annual demand = 12 × 90 = 1080; A = Rs 50, h = Rs 80 and b = Rs 20, we have

$$\text{EOQ} = \sqrt{\frac{2 \times 1080 \times 50}{80}} \sqrt{\frac{20+80}{20}}$$

$= 82.16 \simeq 82$ chairs

(ii) Optimal shortage level = EOQ $\left(\dfrac{h}{b+h}\right)$

$= 82.16 \left(\dfrac{80}{20+80}\right)$

$= 65.73 \simeq 66$

(iii) Total relevant cost $= \sqrt{2 \times 1080 \times 50 \times 80} \sqrt{\dfrac{20}{20+80}}$

= Rs 1314.5

34. (a)(i) $\text{EOQ} = \sqrt{\dfrac{2DA}{h}}$

$= \sqrt{\dfrac{2 \times 8{,}000 \times 50}{5}}$

= 400 units

(ii) Total relevant cost associated with the policy of ordering EOQ,

$\text{TVC} = \sqrt{2DAh}$

$= \sqrt{2 \times 8{,}000 \times 50 \times 5}$

= Rs 2,000

(b) When back-ordering is permitted, we have

(i) $\text{EOQ} = \sqrt{\dfrac{2DA}{h}} \sqrt{\dfrac{b+h}{b}}$

$= \sqrt{\dfrac{2 \times 8{,}000 \times 50}{5}} \sqrt{\dfrac{10+5}{10}}$

= 490 units (approx.)

(ii) Maximum level of inventory, M = EOQ $\left(\dfrac{b}{b+h}\right)$

$= 490 \left(\dfrac{10}{10+5}\right)$

= 327 units

(iii) Optimum number of shortage units, S = EOQ – M

= 490 – 327

= 163 units

(iv) Total relevant cost = $\sqrt{2DAh}\sqrt{\dfrac{b}{h+b}}$

$= \sqrt{2\times 8{,}000\times 50\times 5}\ \sqrt{\dfrac{10}{5+10}}$

= Rs 1,633.

35. EOQ $= \sqrt{\dfrac{2\times 24{,}000\times 90}{3}}\sqrt{\dfrac{2+3}{2}}$

= 1897 units

Re-order level = 24000 $\times\dfrac{1}{24}$ = 1000 units

36. Given, D = 5,000 units, A = Rs 250/order, h = 30% of Rs 100 = Rs 30/unit/year, and b = Rs 10/unit/year.
We have,
When back-ordering is permitted:

(i) EOQ $= \sqrt{\dfrac{2DA}{h}}\sqrt{\dfrac{b+h}{b}}$

$= \sqrt{\dfrac{2\times 5{,}000\times 250}{30}}\sqrt{\dfrac{10+30}{10}}$

= 577.35 units

(ii) Maximum shortage level,

S $= \sqrt{\dfrac{2DAh}{hb+b^2}}$

$= \sqrt{\dfrac{2\times 5{,}000\times 250\times 30}{30\times 10+10\times 10}}$

= 433.01 units

Total variable cost

$= \sqrt{2DAh\left(\dfrac{b}{b+h}\right)}$

$= \sqrt{2\times 5{,}000\times 250\times 30\times\left(\dfrac{10}{10+30}\right)}$

= Rs 4,330.13

When back-ordering is not permitted:

Total variable cost = $\sqrt{2DAh}$

$= \sqrt{2\times 5{,}000\times 250\times 30}$

= Rs 8,660.25

(iii) Additional cost when back-ordering is not permitted

= Rs 8,660.25 – Rs 4,330.13

= Rs 4,330.12

37. *With Back-Orders*

(i) EOQ $= \sqrt{\dfrac{2\times 20{,}000\times 250}{10}}\sqrt{\dfrac{10+30}{30}}$

$= 1,154.7 \simeq 1,155$ units

(ii) Maximum stock level = $1,155 \times \frac{30}{40} \simeq 866$ units

(iii) Maximum shortage level = $1,155 \times \frac{10}{40} \simeq 289$ units

(iv) Total relevant cost = $\sqrt{\frac{2 \times 20,000 \times 250 \times 10 \times 30}{40}}$

= Rs 8,660

Without Back-Order

(i) EOQ = $\sqrt{\frac{2 \times 20,000 \times 250}{10}}$

= 1,000 units

(ii) Maximum stock level = 1,000 units

(iii) Total relevant cost = $\sqrt{2 \times 20,000 \times 250 \times 10}$

= Rs 10,000

38. (a) EOQ = $\sqrt{\frac{2 \times 3000 \times 300}{20}}$

= 300 units

(b) Re-order level = Max. demand rate × Max. lead time

= 15 × 20 = 300 units

(c) Safety stock = ROL – Expected DDLT

= 300 – 10 × 15 = 150 units

39. (i) EOQ = $\sqrt{\frac{2 \times 36,000 \times 25}{0.20 \times 1}}$

= 3,000 units

(ii) No. of orders = 36,000/3,000 = 12 per year

(iii) Re-order Level = Expected DDLT + Safety stock

$= 3000 \times \frac{1}{2} + 3000 = 4,500$ units

Expected DDLT = $d \times LT$ = (36000/12) × (1/2) = 1500 units

Safety stock = 36,000/12 = 3,000 units

(iv) Safety stock = 3,000 units

40. EOQ = $\sqrt{\frac{2 \times 25 \times 25}{0.40}}$

= 56 units

Re-order level = 16 × 25 = 400 units

41. Here average demand = 50 units/day and average lead time = 6 days. Thus, expected demand during lead time (DDLT) = 50 × 6 = 300 units. For consideration of safety stock, we examine DDLT values of 300, 350, 400, and 450 units. Using each of these, we first calculate the expected shortages. The calculations are given in table.

Calculation of Expected Shortage

Option	*ROL*	*SS*	*DDLT*	*Shortage*	*Probability*	*Exp. Value*
1	300	0	300	0	0.68	0
			350	50	0.09	4.5
			400	100	0.07	7.0
			450	150	0.03	4.5
					Total	16.0
2	350	50	350	0	0.09	0
			400	50	0.07	3.5
			450	100	0.03	3.0
					Total	6.5
3	400	100	400	0	0.07	0
			450	50	0.03	1.5
					Total	1.5
4	450	150	450	0	0.03	0

The calculation of shortage cost per annum under each of these conditions is given in table.

Calculation of Shortage Cost

Safety Stock	*Shortage*	*Cost/Unit Short*	*Order Cycles*	*Shortage Cost*
0	16.0	Rs 50	5	Rs 2,500
50	6.5	50	5	1,625
100	1.5	50	5	375
150	0	50	5	0

Finally, to determine the optimum level of safety stock to be kept, the total cost is shown calculated in table.

Calculation of Total Cost

Safety Stock	*Shortage Cost*	*Carrying Cost*	*Total Cost*
0	4,000	0	4,000
50	1,625	500	2,175
100	375	1,000	1,375
150	0	1,500	1,500

Since the total cost corresponding to a safety stock of 100 units is the least, it represents the optimal level.

Accordingly, Reorder level = Expected DDLT + Safety stock

= 300 + 100 = 400 units.

42. Given EOQ = 400 units, ROL = 350 units, LT = 3 weeks, average demand = 100 units/week, standard deviation = 40 units, we have

Expected DDLT = Average demand × Lead time

= 100 × 3 = 300 units

Standard deviation σ (DDLT) = σ (weekly) × $\sqrt{\text{LT}}$

= $40\sqrt{3}$ = 69.28 units

To determine the service level corresponding to the given ROL, we shall obtain the area between 350 (ROL) and 300 (expected DDLT) under the normal curve with μ = 300 and σ = 69.28. This is shown in figure. We have,

$$Z = \frac{X - \mu}{\sigma}$$

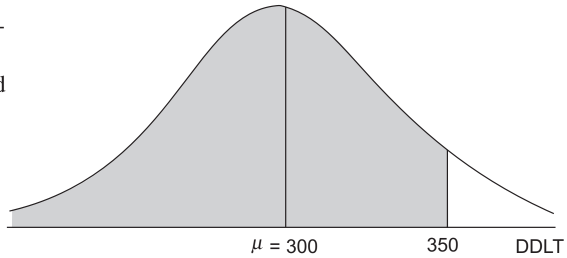
$$= \frac{350 - 300}{69.28}$$

$$= 0.72$$

From the normal-area table (Table B1), area between μ and Z = 0.72 equals 0.2642.

Thus, total area to the left of X = 350 is 0.5 + 0.2642 = 0.7642. Accordingly, service level corresponding to the given reorder level is 76.42%.

To determine the reorder level which would ensure a service level of 97.7%, we have to determine X under the normal curve with μ = 300 and σ = 69.28, to the left of which the area is 0.977. For this area, the Z-value equals 2.0. Thus,



Determination of Service Level

$$2 = \frac{X - 300}{69.28}$$

or $\quad X = 2 \times 69.28 + 300$

$= 439$ units (Approx.)

Therefore, a reorder level of 439 units would imply a service level of 97.7%.

43. From the given information,

expected annual demand D = 90 × 365 = 32,850 units,

ordering cost $\quad A$ = Rs 30/order, and

holding cost $\quad h$ = Re 0.80/unit/year.

From these values, we get

$$\text{EOQ} = \sqrt{\frac{2DA}{h}}$$

$$= \sqrt{\frac{2 \times 32{,}850 \times 30}{0.80}}$$

$$= 1{,}570 \text{ units}$$

Thus, reorder quantity = 1,570 units.

Determination of reorder point: Given,

expected daily demand $\bar{d}$ = 90 units

standard deviation σ_d = 10 units

lead timeLT = 5 days

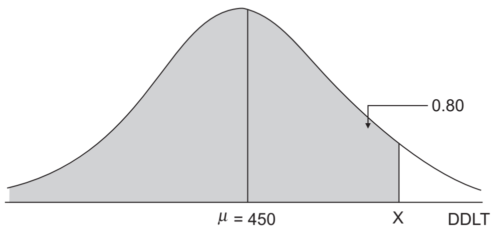
Thus, expected demand during lead time (DDLT) = $\bar{d}$ × LT

= 90 × 5 = 450 units

Standard deviation of DDLT, $\sigma = \sigma_d \sqrt{LT}$

$= 0\sqrt{5}$

= 22.36 units



Determination of Reorder Point

The reorder point is given by X, as shown in the figure, which depicts the distribution of DDLT. The given normal curve has μ = 450 units and σ = 22.36 units, and total area to the left of X is 0.80. To obtain the value of X, we observe that area between μ and X is 0.30. Corresponding to this area, the Z-value is seen to be 0.84 (Table B1). Accordingly,

$$Z = \frac{X - \mu}{\sigma}$$

$$0.84 = \frac{X - 450}{22.36}$$

or $X = 0.84 \times 22.36 + 450$

= 469 units (Approx.)

Thus, reorder point= 469 units.

44. (a) Given D = 30,000 units/year, A = Rs 400/order and h = Rs 600/unit/year, we have

$$EOQ = \sqrt{\frac{2 \times 30{,}000 \times 400}{600}}$$

= 200 units

Thus, option (i) is correct.

(b) With D = 10,000 units, A = Rs 80/set-up, h = Re 0.40/unit/year, we have, the economic lot size,

$$ELS = \sqrt{\frac{2 \times 10{,}000 \times 80}{0.40}}$$

= 2,000 units

Therefore, number of runs = $\frac{10{,}000}{2{,}000}$ = 5.

(c) Stock-out cost = Expected stock-out cost + Safety stock carrying cost

Expected stock-out cost = Average stock-outs × Cost per stock-out

= $np \times 80$

For SS = 10 units,

Total cost = 5 × 0.5 × 80 + 10 × 1.20 = Rs 212

For SS = 20 units,

Total cost = 5 × 0.3 × 80 + 20 × 1.20 = Rs 144

For SS = 40 units,

Total cost = 5 × 0.1 × 80 + 40 × 1.20 = Rs 88

For SS = 60 units,

Total cost = 5 × 0.05 × 80 + 60 × 1.20 = Rs 92

Conclusion: Optimal safety stock = 40 units.

(d) *Determination* of Optimal Safety Stock

SS	*P(stock-out)*	*np*	*Stock-out cost*	*Carrying cost*	*Total cost*
10	0.50	2.50	200	20	220
20	0.40	2.00	160	40	200
30	0.30	1.50	120	60	180
40	0.20	0.80	80	80	160
50	0.10	0.50	40	100	140
60	0.05	0.25	20	120	140

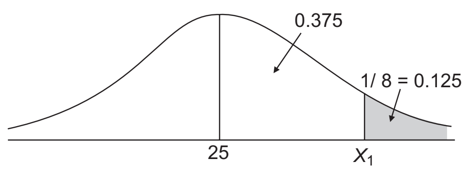
Thus, optimal safety stock level = 50 or 60 units.

45. (a) ROL = $\frac{800}{250} \times 15$ = 48 units

(b) For area between 25 and X_1, Z = 1.15.

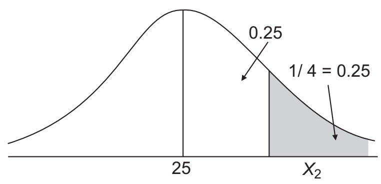
Thus, $1.15 = \frac{X_1 - 25}{4}$

or $X_1 = 1.15 \times 4 + 25 = 29.6 \simeq 30$ units



(ROL) Extra cost due to safety stock

$= 5 \times 5 = \text{Rs } 25$



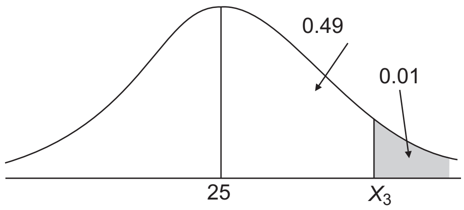
(c) The area between 25 and X_2 is equal to 0.25. Thus, $Z = 0.775$.

$$\therefore \quad 0.675 = \frac{X_2 - 25}{4}$$

or $X_2 = 0.675 \times 4 + 25$

$= 27.7 \simeq 28$ units

This is the required ROL.



(d) The area between 25 and X_3 is equal to 0.49. Corresponding to this area, $Z = 2.33$. Thus,

$$2.33 = \frac{X_3 - 25}{4}$$

or $X_3 = 4 \times 2.33 + 25$

$= 34.32 \simeq 35$ units

The required ROL is, therefore, 35 units.

46. (a) EOQ $= \sqrt{\dfrac{2 \times 3000 \times 30}{8}}$

$= 150$ units

(b) If X be the desired ROL, its value would be such that area included to the right of it (under normal curve with $\mu = 120$ and $\sigma = 20$) would be 0.02. Area between μ and X is therefore, 0.48 and the Z-value corresponding to this is 2.05. Thus,

$$2.05 = \frac{X - 120}{20}$$

$\therefore \quad X = 2 \times 2.05 + 120 = 161$ units

Safety stock = 161 – 120 = 41 units

(c) For $X = 140$, $Z = \dfrac{140 - 120}{20} = 1.0$

From the normal area table, area for $Z = 1.0$ is 0.3413.

$\therefore$ P (stockout in an order cycle) = 0.5 – 0.3413 = 0.1587

No. of order cycles in a year = 3000/150 = 20

Thus, number of times stockouts are expected in a year = $20 \times 0.1587 = 3.174 \simeq 3$.

47. (a) and (b) We are given here

μ (weekly) = 400 units, MAD (weekly) = 250 units, Lead time LT = 2 weeks, and Service level = 95%.

Accordingly,

Standard deviation, σ (weekly) = 1.25 MAD

$= 1.25 \times 250 = 312.5$ units

From these data, the parameters of the distribution of demand during lead time (DDLT) are:

Expected demand, $\mu = \mu \text{ (weekly)} \times LT$

$= 400 \times 2 = 800$ units

Standard deviation, $\sigma = \sigma \text{ (weekly) } \sqrt{LT}$

$= 312.5 \sqrt{2} = 442$ units

The reorder point, X, as shown in figure, corresponding to 95 per cent service level may be determined as follows.

We know,

$$Z = \frac{X - \mu}{\sigma}$$

Here area between μ and X is 0.45, corre sponding to which Z is 1.645.

With μ = 800 and σ = 442, we have

$$1.645 = \frac{X - 800}{442}$$

$\therefore$ X = 1.645 × 442 + 800
= 1,527 units

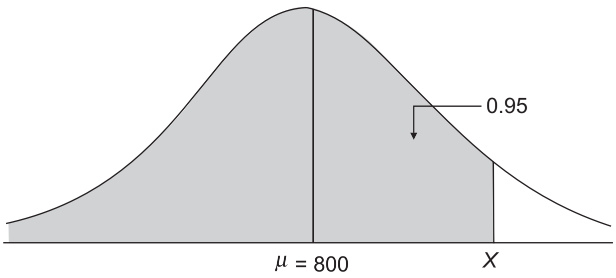


Fig. Determination of Recorder Point

Accordingly, reorder point = 1,527 units, and safety stock = 1,527 – 800 = 727 units.

(c) Annual cost of maintaining safety stock
= Safety stock × Unit holding cost per year
= 727 × 0.01 × 52
= Rs 378.

48. The given lead time distribution is reformulated and is presented in table where calculation of expected lead time is also shown.

Calculation of Expected Lead Time

Lead Time (weeks)	*Frequency*	*Probability p*	*Cumulative Probability*	*X*	*pX*
0–1	4	0.05	0.05	0.5	0.025
1–2	8	0.10	0.15	1.5	0.150
2–3	20	0.25	0.40	2.5	0.625
3–4	24	0.30	0.70	3.5	1.050
4–5	16	0.20	0.90	4.5	0.900
5–6	4	0.05	0.95	5.5	0.275
6–7	4	0.05	1.00	6.5	0.325
Total	80				3.350

From the last column of the table, the expected lead time is seen to be 3.35 weeks. Accordingly, expected demand during lead time = 3.35 × 200 = 670 units. Further, from the *cumulative probability* column, it is evident that 90 per cent service level corresponds to five weeks. Thus, to meet the desired service level, reorder level = 5 × 200 = 1,000 units. Accordingly, safety stock = 1,000 – 670 = 330 units.

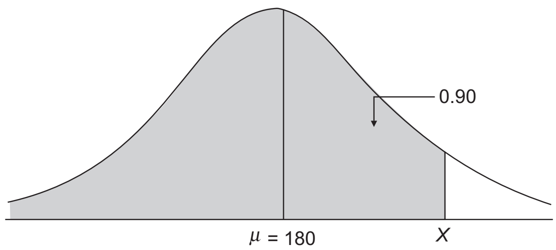
49. From the given data,

Expected demand during lead time,

Exp. DDLT = Expected daily demand × Lead time
= 20 × 9
= 180 units

Also, standard deviation of daily demand,

σ_d = 1.25 MAD
= 1.25 × 5 = 6.25 units



Standard deviation of DDLT = $\sqrt{n}\ \sigma_d$

= $\sqrt{9}$ × 6.25 = 18.75 units

(a) The reorder point corresponding to 50 per cent service level = 180 units.

(b) Service level corresponding to $SS = 0$ would be 50 per cent.

(c) To determine the re-order point as will ensure a 90% service level, we shall determine X to the left of which 90 per cent of the area under the normal curve, with parameters $\mu = 180$ and $\sigma = 18.75$ lies. The Z-value for area = 0.40 is 1.28. Thus,

$$1.28 = \frac{X - 180}{18.75}$$

or $X = 1.28 \times 18.75 + 180$

$= 204$

Thus, ROL = 204 units would ensure 90% service level.

50. We know, MAD = 0.8 σ or 40 = 0.8 σ

$\therefore$ $\sigma = 40/0.8 = 50$ units

(i) With a two weeks' supply, number of orders per year = 26

$\therefore$ Service level = 1 – P (stockout)

= 1 – 1/26 – 0.96

Thus, we have to find X (the safety stock) under the normal curve with $\mu = 0$ and $\sigma = 50$, to the left of which 0.96 of the area lies. Since area between μ and X is 0.46, we have Z-value corresponding to this as 1.75. Now,

$$1.75 = \frac{X - 0}{50}$$

or $X = 50 \times 1.75 = 88$ units app.

(ii) With a four week's supply, service level would be 1 – 1/13 = 0.92 (since there would be 13 order cycles per year). For area = 0.42 (between μ and X), $Z = 1.43$. Thus,

$$1.43 = \frac{X - 0}{50}$$

or X = 1.43 × 50 = 72 units app.

51. Based on given information, the conditional cost matrix is drawn here.

Conditional Cost Matrix (Costs in Rs lac)

Demand (units)	*Prob.*	*Stock (No. of Spares)* 0	1	2	3	4	5
0	0.876	0	2	4	6	8	10
1	0.062	10	2	4	6	8	10
2	0.041	20	12	4	6	8	10
3	0.015	30	22	14	6	8	10
4	0.005	40	32	24	16	8	10
5	0.001	50	42	34	26	18	10
Expected cost		2.14	2.90	4.28	6.07	8.01	10.00

Using the probabilities given, the expected cost for each of the stock levels is calculated. The company should buy no spares, as the expected cost for zero spares is the minimum.

52. The consumption values for various items, obtained as the product of annual consumption and unit prices, are given in descending order, in the following table. In the next column, they are expressed as percentages of the aggregate value. Finally, the percentages are cumulated in the last column.

ABC Classification

Model	Value (Rs)	Value (%)	Cumulative Value (%)	
502	42,000	47.97	47.97	A
506	22,000	25.13	73.10	
509	9,000	10.28	83.38	
504	5,500	6.28	89.66	B
508	4,000	4.57	94.23	
501	3,000	3.43	97.66	
510	800	0.91	98.57	
507	750	0.86	99.43	C
503	300	0.34	99.77	
505	200	0.23	100.00	

Evidently, class *A* items are models 502 and 506, class *B* items are models 509, 504, and 508; while the remaining may be categorized as class *C* items.

53. The values of various items are shown in the second column of the following table. These are re-expressed in percentage form and given in the next column. Finally, the percentage values are shown cumulated in the last columns. As indicated, the values are arranged in the descending order of magnitude. From the values given in the last column of the table, the first four items may be categorized as *A* class items, next three as class *B* items while the remaining as class *C* items.

ABC Classification

Item	Value (Rs)	Value (%)	Cumulative Value (%)	
11	81,650	22.95	22.95	
2	72,000	20.24	43.19	A
5	57,000	16.02	59.21	
1	35,000	9.84	69.05	
12	25,420	7.15	76.19	
6	20,000	5.62	81.81	B
3	15,000	4.22	86.03	
4	13,200	3.71	89.74	
7	12,000	3.37	93.11	
10	11,600	3.26	96.37	C
8	10,500	2.95	99.33	
9	2,400	0.67	100.00	

54. In most practical situations, where large number of items are involved, *A* category items usually constitute 5 to 10 per cent of total items and account for 70 to 85 per cent of total cost (of materials); *B* category items are 10 to 20 per cent in number and value both, while the remaining items fall in the *C* category. To classify the ten items given in three categories, we calculate their annual usage value in the first instance and rank them in the descending order on the basis of the usage value. This is done in table below.

Determination of Usage Value and Rankings

S.No.	*Annual Usage* (units)	*Unit Value* (Rs)	*Annual Usage* (in Rs)	*Ranking*
1	200	40.00	8,000	4
2	100	360.00	36,000	1
3	2,000	0.20	400	9
4	400	20.00	8,000	5
5	6,000	0.04	240	10
6	1,200	0.80	960	8
7	120	100.00	12,000	3
8	2,000	0.70	1,400	6
9	1,000	1.00	1,000	7
10	80	400.00	32,000	2
		Total	1,00,000	

The next step is to accumulate the items in the order of their ranks along with their annual usage values so as to convert the accumulated values into their percentage of grand total. The calculations are given below where it is evident that 20% of the items that constitute 68% of total cost fall in category *A*; 30% of the items are in category *B* while the rest are in category *C*. The *B* category items account for 28% and the *C* category for 4% of the total cost.

Classification of Items

S.No.	*Item No.*	*Annual usage* Rs	Percentage	*Cumulative* Percentage	*Item Nos.* Percentage	Cum	*Category*
1	2	36,000	36.00	36.00	10	10	A
2	10	32,000	32.00	68.00	10	20	
3	7	12,000	12.00	80.00	10	30	
4	1	8,000	8.00	88.00	10	40	B
5	4	8,000	8.00	96.00	10	50	
6	8	1,400	1.40	97.40	10	60	
7	9	1,000	1.00	98.40	10	70	
8	6	960	0.96	99.36	10	80	C
9	3	400	0.40	99.76	10	90	
10	5	240	0.24	100.00	10	100	

CHAPTER 10

1. (a) Arrival rate, $\lambda = 12$ customers/hour and Service rate, $\mu = 30$ customers/hour
   (b) Utilisation parameter, $\rho = \lambda/\mu = 12/30 = 0.40$
   (c) $P(n = 4) = \rho^n(1 - \rho) = 0.4^4(1 - 0.4) = 0.01536$
   (d) $P(n > 4) = \rho^{n+1} = (0.4)^5 = 0.01024$
   (e) $L_q = \rho^2/(1 - \rho) = 0.4^2/(1 - 0.4) = 4/15$ customer
   (f) $L_s = \rho/(1 - \rho) = 0.4/(1 - 0.4) = 2/3$ customer
   (g) $W_q = \lambda/\mu(\mu - \lambda) = 12/30(30 - 12) = 1/45$ hour $= 4/3$ minutes
   (h) $W_s = 1/(\mu - \lambda) = 1/(30 - 12) = 1/18$ hour or $10/3$ minutes
   (i) Let the new arrival rate be λ' which causes W_q to be $3 \times 4/3 = 4$ minutes or $1/15$ hour.

   $$\therefore \quad \frac{1}{15} = \frac{\lambda'}{30(30 - \lambda')}$$

   or $\lambda' = 900/45 = 20$ customers/hour

2. (a) With $\lambda = 3$ letters/hour and $P(n) = e^{-\lambda} \times \dfrac{\lambda^n}{n!}$,

   $$\begin{aligned} P(\text{no more than two letters in an hour}) &= P(0) + P(1) + P(2) \\ &= e^{-3} + e^{-3} \times 3 + e^{-3} \times 3^2/2! \\ &= e^{-3}(1 + 3 + 4.5) \\ &= 0.0498 \times 8.5 = 0.4232 \\ P(\text{at least one letter}) &= 1 - P(0) \\ &= 1 - 0.0498 = 0.9502 \end{aligned}$$

   (b) No. of letters expected to be received in two hours $= 3 \times 2 = 6$.

3. From the given information,
   $\lambda = 36$ customers/hour, $\mu = 60$ customers/hour, and $\rho = \lambda/\mu = 36/60 = 0.60$.
   (a) Probability of arrival of zero through five customers in a 10-minute interval:
   Here $T = 10$ minutes $= 1/6$ hours
   $\therefore$ $m = \lambda T = 36 \times 1/6 = 6$

   Now, $$P(n) = e^{-m} \times \frac{m^n}{n!}$$

   Accordingly,

No. of customers (n)	*Probability*
0	0.00248
1	0.01487
2	0.04462
3	0.08924
4	0.13385
5	0.16062

   (b) Probability (system is idle) $= 1 - \rho$
   $= 1 - 0.6 = 0.4$
   Thus, the system shall be idle 40% of the time.
   (c) Expected free time in an eight-hour period $= 8 \times 0.4 = 3.2$ hr.
   (d) Probability that there shall be exactly n customers in the system, $P_n = \rho^n(1 - \rho)$. For $n = 0, 1, \ldots, 5$, we have

No. of customers (n)	*Probability, P_n*
0	0.4
1	0.24
2	0.144
3	0.0864
4	0.05184
5	0.031104

(e) Expected length of the system, $L_s = \dfrac{\rho}{1-\rho}$

$$= \frac{0.6}{1-0.6} = 1.5 \text{ customers}$$

(f) Expected length of the queue, $L_q = \dfrac{\rho^2}{1-\rho}$

$$= \frac{0.6^2}{1-0.6} = 0.9 \text{ customers}$$

4. (a) μ = 20 customers/hour

P(idle system) = 0.25

∴ P(busy), ρ = 1 − 0.25 = 0.75

Now, $\lambda = \mu\rho = 20 \times 0.75 = 15$ customers/hour

Thus, inter-arrival time = 60/15 = 4 minutes

(b) Given, average service time = 15 minutes/customer, and inter-arrival time = 20 minutes. Accordingly,

average arrival rate, λ = 3 customers/hour,

average service rate, μ = 4 customers/hour, and

ρ = 3/4 = 0.75

Average time a customer waits in a queue

$$W_q = \frac{\rho}{\mu-\lambda}$$

$$= \frac{0.75}{4-3} = 0.75 \text{ hour or 45 minutes}$$

(c) Given λ = 10 customers/hour and μ = 10 customers/hour. Since λ is not less than μ, the system cannot function. The statement is false, therefore.

(d) Here ρ = 0.75 and λ = 60/4 = 15 customers/hour. Thus, $\mu = \lambda/\rho$ = 15/0.75 = 20 customers/hour. The average service time, therefore, is 3 minutes.

(e) λ = 20 customers/hour, μ = 60 × 60/100 = 36 customers/hour, average waiting time of a customer in queue.

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{20}{36(36-20)} = \frac{5}{144} \text{ hour or 2.08 minutes}$$

5. With $\lambda = 10/8 = 1.25$ sets per hour, and $\mu = 2$ sets per hour, we have $\rho = 1.25/2 = 0.625$. Now,

$P(\text{idle}) = 1 - \rho = 1 - 0.625 = 0.375$

$\therefore$ Expected idle time per day $= 0.375 \times 8$
$= 3$ hours

Expected number of units in the system, $L_s = \dfrac{0.625}{1-0.625}$

$= 1.67$

6. Here, $\mu = 60/3 = 20$ customers/hour, and
$\lambda = 12$ customers/hour

(i) Utilisation of the teller, $\rho = 12/20 = 0.6$ or 60%

(ii) Average number in the system, $L_s = \dfrac{\rho}{1-\rho}$

$= \dfrac{0.6}{1-0.6} = 1.5$

(iii) Average waiting time in the line, $W_q = \dfrac{\rho}{\mu - \lambda}$

$= \dfrac{0.6}{20-12} = 0.075$ hour

(iv) Average waiting time in the system, $W_s = \dfrac{1}{\mu - \lambda}$

$= \dfrac{1}{20-12} = 0.125$ hour

7. With $\lambda = 2$ and $\mu = 3$,

(a) $\rho = \lambda/\mu = 2/3$ or 0.67

(b) $L_s = \dfrac{\rho}{1-\rho} = \dfrac{2/3}{1-2/3} = 2$

(c) $W_s = \dfrac{1}{\mu-\lambda} = \dfrac{1}{3-2} = 1$ hour or 60 minutes

$W_q = \dfrac{\rho}{\mu-\lambda} = \dfrac{2}{3\,(3-2)} = \dfrac{2}{3}$ hour or 40 minutes

(d) $P(n > 3) = \rho^4 = \left(\dfrac{2}{3}\right)^4 = 0.1975$

8. Given, average arrival rate, $\lambda = 12$ trucks/hour
average service rate, $\mu = 20$ trucks/hour

(i) Probability that a truck has to wait, $\rho = \dfrac{\lambda}{\mu}$

$= 12/20 = 0.6$

(ii) The waiting time of a truck that waits,

$$W = \frac{1}{\mu - \lambda}$$

$$= \frac{1}{20-12} = \frac{1}{8} \text{ hour or 7.5 minutes}$$

(iii) Since 50% of the total trucks belong to the contractor, the expected waiting time of contractor's truck per day of 24 hours

= No. of truck arrivals per day × Contractor's share × Expected waiting time of a truck

$$= 12 \times 24 \times \frac{50}{100} \times \frac{12}{20(20-12)}$$

$$= 288 \times 0.5 \times \frac{12}{20 \times 8} = 10.8 \text{ hours}$$

9. Here λ = 20 customers/hour and service rate μ = 12 customers/hour. Since $\lambda > \mu$, the system is not workable.

10. From the given information, we have
mean arrival rate, λ = 6 customers/hour
mean service rate, μ = 10 customers/hour
∴ $\rho = \lambda/\mu = 6/10 = 0.6$

(i) Probability that an arriving customer can drive directly to the space in front of the window is given by $P(0) + P(1) + P(2)$. The required probability, therefore, is

$$P = (1 - \rho) + (1 - \rho)\rho + (1 - \rho)\rho^2$$
$$= (1 - \rho)(1 + \rho + \rho^2)$$
$$= (1 - 0.6)(1 + 0.6 + 0.6^2)$$
$$= 0.4 \times 1.96 = 0.784$$

(ii) Probability that an arriving customer will have to wait outside the directed space is given by $1 - [P(0) + P(1) + P(2)]$. It equals $1 - 0.784 = 0.216$.

(iii) Expected waiting time of a customer before getting the service is W_q, calculated as:

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{6}{10(10-6)} = \frac{3}{20} \text{ hour or 9 minutes}$$

11. With λ = 3 customers/hour and μ = 4 customers/hour, we have $\rho = \lambda/\mu = 3/4 = 0.75$.

(a) $\rho = 0.75$, thus he shall be busy 75% of time.

(b) $P(n < 3) = P(0) + P(1) + P(2)$
$= (1 - \rho) + \rho(1 - \rho) + \rho^2(1 - \rho)$
$= (1 - 0.75) + 0.75(1 - 0.75) + 0.75^2(1 - 0.75) = 0.578125$

(c) $L_s = \dfrac{\rho}{1-\rho} = \dfrac{0.75}{1-0.75} = 3$ customers

(d) $L_q = \dfrac{\rho^2}{1-\rho} = \dfrac{0.75^2}{1-0.75} = 2.25$ customers

(e) $W_q = \dfrac{\rho}{\mu-\lambda} = \dfrac{0.75}{4-3} = 0.75$ hour or 45 minutes

(f) $W_s = \dfrac{1}{\mu-\lambda} = \dfrac{1}{4-3} = 1$ hour or 60 minutes

(g) Same as in (f): 60 minutes

(h) $W_q(t) = \rho e^{-t/w_s} = 0.75e^{-1/6} = 0.6348$

(i) $W_s(t) = e^{-t/w_s} = e^{-1/6} = 0.8465$

12. The following are the main assumptions made:
    (i) The arrival of customers follows Poisson probability distribution, with an average arrival rate of λ per hour.
    (ii) The service time has exponential distribution, with the service rate being μ per hour.
    (iii) A customer can book his/her ticket from any of the counters, so that there are as many queues as the number of customers. Thus, it is assumed that the system consists of identical single service stations. If K, M, and N be the number of customers respectively during the peak, normal, and low periods, we have the arrival rates as:

for peak period, $\lambda = 110/K$
for normal period, $\lambda = 60/M$
for low period, $\lambda = 30/N$

The service rate for each of the periods = 12 customers/hour

Peak period:

Since customers are willing to wait for a period of 15 minutes or 0.25 hour, we have

$$0.25 = \frac{110/K}{12\left(12 - \frac{110}{K}\right)} \qquad \left(\text{since } W_q = \frac{\lambda}{\mu(\mu - \lambda)}\right)$$

or $0.25 \times 12(12K - 110) = 110$
or $36K = 110 + 330 = 440$
$\therefore$ $K = 440/36 = 12.22$

Thus, 13 counters should be opened to ensure that the average waiting time does not exceed 15 minutes.

Normal period:

The customers are willing to wait for 10 minutes or 1/6 hour. Accordingly,

$$\frac{1}{6} = \frac{60/M}{12\left(12 - \frac{60}{M}\right)}$$

or $\frac{1}{6} \times 12\left(12 - \frac{60}{M}\right) = \frac{60}{M}$
or $24M - 120 = 60$
or $M = 180/24 = 7.5$

Thus, 8 counters be opened during the normal periods to ensure the required.

Low period:

Customer waiting time permitted for low periods is 5 minutes or 1/12 hour. Thus, during such periods,

$$\frac{1}{12} = \frac{30/N}{12\left(12 - \frac{30}{N}\right)}$$

or $144 - \frac{360}{N} = \frac{360}{N}$
or $144N - 360 = 360$
$\therefore$ $N = 720/144 = 5$

A total of 5 counters, therefore, need to be opened in order to ensure that the customers do not wait for more than 5 minutes during low periods.

13. With $\lambda = 4$ customers/hour and $\mu = 10$ customers/hour, $\rho = \lambda/\mu = 4/10$ or 0.40.
   (a) $P(\text{empty}) = P(0) = 1 - \rho = 1 - 0.40 = 0.60$
   (b) $P(n \geq 1) = 1 - P(0) = 1 - 0.60 = 0.40$
   (c) $L_s = \dfrac{\rho}{1-\rho} = \dfrac{0.4}{1-0.4} = \dfrac{2}{3}$ customers
   (d) $W_s = \dfrac{1}{\mu-\lambda} = \dfrac{1}{10-4} = \dfrac{1}{6}$ hour or 10 minutes
14. With $\lambda = 6$ customers/hour and $\mu = 20$ customers/hour, we have $\rho = \lambda/\mu = 6/20 = 0.3$.
   (a) A customer has to wait when the system is busy. Thus, $P(\text{customer has to wait}) = \rho = 0.3$.
   (b) $P(\text{queue shall be formed}) = 1 - P(0 \text{ or } 1 \text{ customer in system})$
   Now, $P(0) = 0.7$ and $P(1) = 0.3(1 - 0.3) = 0.21$. Thus, the required probability $= 1 - (0.70 + 0.21) = 0.09$.
   (c) Expected waiting time in the queue, $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$$= \frac{6}{20(20-6)} = \frac{3}{140} \text{ hour or 1.29 minutes}$$

   Let the new arrival rate to justify a new clerk be λ'. Accordingly,

$$\frac{4}{60} = \frac{\lambda'}{20(20-\lambda')}$$

or $\quad 80(20 - \lambda') = 60\lambda'$

or $\quad 140\lambda' = 1600$

$\therefore \quad \lambda' = 1600/140 = 11.43$ customers/hour

   Thus, a second clerk is justified when the arrival rate increases to at least 11.43 customers/hour.
15. Existing: $\lambda = 25$, $\mu = 30$

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{30-25} = \frac{1}{5} \text{ hour}$$

   Total cost per day $= 100 + \frac{1}{5} \times 25 \times 8 \times 120 =$ Rs 4,900

   Proposed: $\lambda = 25$, $\mu = 40$

$$W_s = \frac{1}{40-25} = \frac{1}{15} \text{ hour}$$

   Total cost per day $= 100 + 100 + \frac{1}{15} \times 25 \times 8 \times 120 =$ Rs 1,800
16. To determine who of the two mechanics should be employed by the workshop, we calculate and compare total cost for each case as follows:
   Total (daily) cost = Mechanic's charges + Cost of motor downtime
   Cost of motor downtime
   = Expected downtime per motor × Average arrival rate per day × Cost of downtime per motor day
   The calculations are done below:
   Existing mechanic:
   Arrival rate, $\lambda = 5$ motors/day
   Service rate, $\mu = 6$ motors/day
   Expected downtime per motor, $W_s = \dfrac{1}{\mu-\lambda} = \dfrac{1}{6-5} = 1$ day
   Cost of motor downtime $= 1 \times 5 \times 100 =$ Rs 500
   Total (daily) cost = Rs 100 + Rs 500 = Rs 600

Proposed mechanic:
Arrival rate, $\lambda = 5$ motors/day
Service rate, $\mu = 8$ motors/day

Expected downtime per motor, $W_s = \frac{1}{8-5} = 1/3$ day

Cost of motor downtime $= \frac{1}{3} \times 5 \times 100 =$ Rs 500/3 = Rs 167

Total (daily) cost = Rs 200 + Rs 167 = Rs 367

Obviously, the qualified motor mechanic should be employed by the workshop.

17. Arrival rate, $\lambda = 2$ customers/hour
Service rate, $\mu = 3$ customers/hour (I)
$\mu = 4$ customers/hour (II)

	I	II
$L_q = \frac{\rho^2}{1-\rho}$	$\frac{(2/3)^2}{1-2/3} = \frac{4}{3}$	$\frac{(1/2)^2}{1-1/2} = \frac{1}{2}$
$L_s = \frac{\rho}{1-\rho}$	$\frac{2/3}{1-2/3} = 2$	$\frac{1/2}{1-1/2} = 1$
$W_q = \frac{\rho}{\mu-\lambda}$	$\frac{2/3}{3-2} = \frac{2}{3}$ hour	$\frac{1/2}{4-2} = \frac{1}{4}$ hour
$W_s = \frac{1}{\mu-\lambda}$	$\frac{1}{3-2} = 1$ hour	$\frac{1}{4-2} = \frac{1}{2}$ hour

TC(I) = $14 + \frac{2}{3} \times 2 \times 30$ = Rs 54/hour

TC(II) = $20 + \frac{1}{4} \times 2 \times 30$ = Rs 35/hour

18. Assuming that the conditions underlying the Poisson-exponential single server model are satisfied, the choice of mechanic can be done by comparing total cost for the two.

Total cost per day = Repairman's charges + Cost of machine downtime

Cost of machine downtime = $W_s \times \lambda \times$ Cost per machine day

	Mechanic A	*Mechanic B*
Charges per day	Rs 140	Rs 250
Arrival rate, λ	2 machines/day	2 machines/day
Service rate, μ	3 machines/day	4 machines/day
$W_s = \frac{1}{\mu-\lambda}$	$\frac{1}{3-2} = 1$ day	$\frac{1}{4-2} = \frac{1}{2}$ day

Total cost (A) = Rs 140 + 1 × 2 × Rs 800 = Rs 1,740 per day

Total cost (B) = Rs 250 + $\frac{1}{2}$ × 2 × Rs 800 = Rs 1,050 per day

Hence, mechanic B should be engaged.

19. Annual total cost for facility =
Annual capital recovery cost + Annual operating cost + Annual cost of lost equipment time

Annual capital recovery cost = $\dfrac{\text{Total cost of facility}}{\text{Life (in years)}}$

Annual cost of lost-equipment time
= Expected annual lost time (weeks) × Cost of lost production time per week
Expected annual lost time (week) = Expected downtime in the system (W_s) × Expected number of arrivals per annum
($\lambda \times$ No. of weeks)

Facility F_1:

(a) Annual capital recovery cost = $\dfrac{\text{Rs } 1{,}20{,}000}{5}$ = Rs 24,000
(b) Annual operating cost = Rs 40,000
(c) (i) Expected time a down machine spends in the system,

$$W_s = \frac{1}{\mu - \lambda}$$
$$= \frac{1}{40 - 30} = \frac{1}{10} \text{ week}$$

(ii) Expected annual lost time = $\dfrac{1}{10}$ × 30 × 50 = 150 weeks
(iii) Cost of lost production equipment time = 150 × 6 × 100
= Rs 90,000
∴ Total cost = Rs 24,000 + Rs 40,000 + Rs 90,000
= Rs 1,54,000 p.a.

Facility F_2:

(a) Annual capital recovery cost = $\dfrac{\text{Rs } 2{,}00{,}000}{5}$ = Rs 40,000
(b) Annual operating cost = Rs 50,000
(c) (i) Expected time a down machine spends in the system,

$$W_s = \frac{1}{\mu - \lambda}$$
$$= \frac{1}{80 - 30} = \frac{1}{50} \text{ week}$$

(ii) Expected annual lost time = $\dfrac{1}{50}$ × 30 × 50 = 30 weeks
(iii) Cost of lost production equipment time = 30 × 6 × 100
= Rs 18,000
∴ Total cost = Rs 40,000 + Rs 50,000 + Rs 18,000
= Rs 1,08,000 p.a.

Thus, facility F_2 should be preferred to F_1.

20. Mechanic A

$W_s = \dfrac{1}{8 - 6} = \dfrac{1}{2}$ hour

$TC = 20 \times 8 + \dfrac{1}{2} \times 6 \times 8 \times 40$
= 160 + 960 = Rs 1,120 per day

Mechanic B

$W_s = \dfrac{1}{12 - 6} = \dfrac{1}{6}$ hour

$TC = 28 \times 8 + \dfrac{1}{6} \times 6 \times 8 \times 40$
= 224 + 320 = Rs 544 per day

21. (a) From the given information, $\lambda = 1$ generator/year, $\mu = 6$ generators per year, and $M = 4$. The calculation of $P(0)$ = is given here:

i	$M!/(M-i)!$	$(\lambda/\mu)^i$	$\frac{M!}{(M-i)!}\left(\frac{\lambda}{\mu}\right)^i$
0	1	1	1
1	4	1/6	2/3
2	12	1/36	1/3
3	24	1/216	1/9
4	24	1/1296	1/54
		Total	115/54

Now, $P(0) = (115/54)^{-1} = 54/115 = 0.47$

(b) Calculation of different number of defectives is given below:

n	$\left(\frac{\lambda}{\mu}\right)^n\left[\frac{M!}{(M-i)!}\right]\times P(0)$	=	*Probability (n)*
1	(2/3) × (54/115)	=	0.31
2	(1/3) × (54/115)	=	0.16
3	(1/9) × (54/115)	=	0.05
4	(1/54) × (54/115)	=	0.01

(c) Average number of generators waiting repairs,

$$L_q = M - \left(\frac{\lambda+\mu}{\lambda}\right)(1 - P(0))$$

$$= 4 - \frac{1+6}{1}(1 - 0.47) = 0.29$$

(d) Average number of generators out of service,

$$L_s = M - \frac{\mu}{\lambda}(1 - P(0))$$

$$= 4 - \frac{6}{1}(1 - 0.47) = 0.82$$

(e) Average waiting time in the queue,

$$W_q = \frac{1}{\mu}\left[\frac{M}{1-P(0)} + \frac{\lambda+\mu}{\lambda}\right]$$

$$= \frac{1}{6}\left[\frac{4}{1-0.47} + \frac{1+6}{1}\right]$$

= 0.091 year or 1.09 months

(f) Average downtime of a generator,

$$W_s = W_q + \frac{1}{\mu}$$

$$= 0.091 + \frac{1}{6}$$

= 0.258 year or 3.09 months

22. Here λ = 1/4 machines/hour, μ = 4/3 machines/hour and M = 6.

(a) Calculation of $P(0)$ follows:

i	$M!/(M-i)!$	$\frac{M!}{(M-i)!}\left(\frac{\lambda}{\mu}\right)^i$	n	$P(n) = \frac{M!}{(M-i)!}\left(\frac{\lambda}{\mu}\right)^n P(0)$
0	1	1.0000	0	0.217
1	6	1.1250	1	0.244
2	30	1.0547	2	0.229
3	120	0.7910	3	0.171
4	360	0.4449	4	0.096
5	720	0.1669	5	0.036
6	720	0.0313	6	0.007
		4.6138		1.000

$\therefore$ $P(0)$ = Rec 4.6138 = 0.2167

(b) $$W_q = \frac{1}{\mu}\left[\frac{M}{1-P(0)} + \frac{\lambda+\mu}{\lambda}\right] = \frac{1}{4/3}\left[\frac{6}{1-0.2167} + \frac{1/4+4/3}{1/4}\right]$$

= 10.50 hours.

(c) $$L_s = M - \frac{\mu}{\lambda}(1 - P(0)) = 6 - \frac{4}{3}\times\frac{4}{1}(1 - 0.2167)$$

= 1.822 machines

(d) $$L_q = M - \left(\frac{\lambda+\mu}{\lambda}\right)(1 - P(0)) = 6 - \left(\frac{1/4+4/3}{1/4}\right)(1 - 0.2167)$$

= 1.039 machines

23. Here K = 2, λ = 15 customers/hour and μ = 12 customers/hour. Accordingly, ρ = 15/2 × 12 = 0.625. Now,

$$P(0) = \left[\sum_{i=0}^{K-1}\frac{(\lambda/\mu)^i}{i!} + \frac{(\lambda/\mu)^K}{K!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{i=0}^{1}\frac{(15/12)^i}{i!} + \frac{(15/12)^2}{2!(1-0.625)}\right]^{-1}$$

= 0.2308

(i) A customer has to wait if there are two or more customers in the system. Also, P(a customer has to wait)

= 1 – P(0 or 1 customer in system)

We have, $P(0)$ = 0.2308. Now

$$P(n) = \frac{(\lambda/\mu)^n}{n!} \times P(0) \qquad \text{where } n \le K$$

$$\therefore \quad P(1) = \frac{(15/12)^1}{1!} \times 0.2308$$

= 0.2885

Thus, P(a customer has to wait)

$$= 1 - (0.2308 + 0.2885)$$
$$= 0.4807$$

(ii) Waiting time in queue,

$$W_q = \frac{(\lambda/\mu)^K \rho}{\lambda(1-\rho)^2 K!} \times P(0)$$

$$= \frac{(15/12)^2 \times 0.625}{15(1-0.625)^2 \times 2!} \times 0.2308$$

$$= 0.05342 \text{ hour or } 3.21 \text{ minutes}$$

24. (a) Here $K = 5$, $\lambda = 40$ customers/hour and $\mu = 10$ customers/hour. Accordingly,

$$\rho = \frac{\lambda}{K\mu}$$

$$= \frac{40}{5 \times 10} = 0.80$$

This is the traffic intensity.

(b) Probability that none of the doctors is busy is given by $P(0)$. From Table 10.1, for $K = 5$ and $\rho = 0.80$, we find $P(0) = 0.0130$.

(c) Probability of 3 patients in the hospital,

$$P(3) = P(0)\frac{(\lambda/\mu)^n}{n!} \quad \text{for } n \le K$$

$$= 0.0130\,\frac{(40/10)^3}{3!}$$

$$= 0.1386$$

(d) Expected length of queue,

$$L_q = \frac{(\lambda/\mu)^K \rho}{K!(1-\rho)^2} \times P(0)$$

$$= \frac{(40/10)^5 \times 0.80}{5!(1-0.80)^2} \times 0.0130$$

$$= 2.219$$

(e) $L_s = L_q + \dfrac{\lambda}{\mu}$

$$= 2.219 + \frac{40}{10}$$
$$= 6.219$$

(f) Probability that there are eight patients in the hospital,

$$P(8) = \frac{(40/10)^8}{5!5^{8-5}} \times 0.0130$$

$$= 0.0567$$

Here note that

$$P(n) = \frac{(\lambda/\mu)^n}{K!\,K^{n-K}} \times P(0) \qquad \text{when } n > K$$

(g) $W_q = \dfrac{L_q}{\lambda}$

$= \dfrac{2.219}{40}$

= 0.0555 hour or 3.33 minutes

(h) $W_s = W_q + \dfrac{1}{\mu}$

$= 0.0555 + \dfrac{1}{10}$

= 0.1555 hour or 9.33 minutes.

25. For each tool crib, λ = 18 workmen/hour and μ = 24 workmen/hour.
Thus,

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{18}{24(24-18)} = \frac{1}{8} \text{ hour or 7.5 minutes}$$

When tool cribs are combined into one,

λ = 36 workmen/hour and μ = 48 workmen/hour.

$$W_q = \frac{36}{48(48-36)} = \frac{1}{16} \text{ hour or 3.75 minutes}$$

Evidently, waiting time in queue will reduce to one-half of its present value.
(*Note:* It is assumed that service rate would double upon combining the tool cribs into one.)

26. (a) For each typist, λ = 3 letters/hour and μ = 4 letters/hour.

Thus, $$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{3}{4(4-3)} = \frac{3}{4} \text{ hour or 45 minutes}$$

(b) When typists are 'pooled', we have
K = 2, λ = 6 letters/hour and μ = 4 letters/hour. Thus, ρ = 6/2 × 4 = 0.75. For K = 2 and ρ = 0.75, we can estimate $P(0)$ from Table 10.1 as equal to (0.1494 + 0.1364)/2 = 0.1429. Now, we have

$$W_q = \frac{(\lambda/\mu)^K \rho}{K!(1-\rho)^2 \lambda} \times P(0)$$

$$= \frac{(6/4)^2 \times 0.75}{2!(1-0.75)^2 \times 6} \times 0.1429$$

= 0.3215 hour or 19.29 minutes

27. With K = 3, λ = 2 customers/hour, μ = 1.5 customers/hour, $\rho = \lambda/\mu K$ = 2/(1.5 × 3) = 0.44. From Table 10.1, $P(0)$ for K = 3 and ρ = 0.44, equals 0.2580.

(a) Expected time an adjuster would spend with his claimants in a 50-hour week = 50 × 0.44 = 22 hours

(b) Expected time a claimant spends in the office,

$$W_s = \frac{\mu(\lambda/\mu)^K}{(K-1)!(K\mu-\lambda)^2} \times P(0) + \frac{1}{\mu}$$

$$= \frac{1.5(2/1.5)^3}{2!(3 \times 1.5 - 2)^2} \times 0.2580 + \frac{1}{1.5}$$

= 0.0734 + 0.6667 = 0.74 hour or 44.4 minutes

Alternately, $L_q = \dfrac{(\lambda/\mu)^K(\lambda/K\mu)}{K!\left(1-\dfrac{\lambda}{K\mu}\right)^2} \times P(0)$

$$= \frac{(2/1.5)^3\,(2/3\times 1.5)}{3!\left(1-\dfrac{2}{3\times 1.5}\right)^2} \times 0.2580$$

$$= 0.14677$$

$$W_q = L_q/\lambda = 0.14677/2 = 0.0734 \text{ hour}$$

$\therefore$ $W_s = W_q + \dfrac{1}{\mu} = 0.0734 + 0.6667 = 0.74$ hour

28. Here λ = 10 customers/hour, μ = 6 customers/hour, and K = 3. Thus, ρ = 10/3 × 6 = 0.5556. With these parameters,

$$P(0) = 0.17266,\ P(1) = 0.28777 \text{ and } P(2) = 0.23981$$

(a) $L_s = L_q + \dfrac{\lambda}{\mu} = 0.37470 + 10/6 = 2.04137$

Since $L_q = \dfrac{(\lambda/\mu)^K \rho}{K!(1-\rho)^2} \times P(0) = \dfrac{(10/6)^3 \times 0.5556}{3!\,(1-0.5556)^2} \times 0.17266$

$$= 0.37470$$

(b) $W_s = W_q + \dfrac{1}{\mu} = 0.03747 + \dfrac{1}{6} = 0.20414$ hour

(c) $W_q = L_q/\lambda = 0.37470/10 = 0.03747$ hour

(d) Expected number of barbers idle = 3 × $P(0)$ + 2 × $P(1)$ + 1 × $P(2)$
= 3 × 0.17266 + 2 × 0.28777 + 0.23981 = 1.333

29. At present we have,

Counter 1

λ = 10 customers/hour
μ = 15 customers/hour

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{15(15-10)} = \frac{2}{15} \text{ hour or 8 minutes}$$

Counter 2

λ = 12 customers/hour
μ = 15 customers/hour

$$W_q = \frac{12}{15(15-12)} = \frac{4}{15} \text{ hour or 16 minutes}$$

When both counters can be given same service:

Here K = 2, λ = 10 + 12 = 22 customers/hour and μ = 15 customers/hour. Thus, ρ = 22/(2 × 15) = 0.73. Now,

$$W_q = \frac{(\lambda/\mu)^K \rho}{\lambda(1-\rho)^2 K!} \times P(0)$$

From Table 10.1, for K = 2 and ρ = 0.73, the value of $P(0)$ can be interpolated as (0.1628 + 0.1494)/2 = 0.1561. Accordingly,

$$W_q = \frac{(22/15)^2 \times 0.73}{22(1-0.73)^2 \times 2!} \times 0.1561$$

$$= 0.0764 \text{ hour or 4.6 minutes}$$

30. (a) For $K = 2$

$\lambda = 7$ customers/minute

$\mu = 4$ customers/minute

$$\rho = \frac{7}{2 \times 4} = 0.88$$

Using Table 10.1,

$$P(0) = 0.0638$$

$$L_q = \frac{7(7/4)^2(7/8)}{2!\left(1 - \frac{7}{8}\right)^2} \times 0.0638$$

$$= 5.47$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 5.47 + \frac{7}{4}$$

$$= 7.22$$

$$W_q = L_q/\lambda$$

$$= 5.47/7$$

$$= 0.78 \text{ minute}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = 0.78 + \frac{1}{4}$$

$$= 1.03 \text{ minute}$$

For $K = 3$

$\lambda = 7$ customers/minute

$\mu = 4$ customers/minute

$$\rho = \frac{7}{3 \times 4} = 0.58$$

$$P(0) = 0.1576$$

$$L_q = \frac{(7/4)^3(7/12)}{3!\left(1 - \frac{7}{!2}\right)^2} \times 0.1567$$

$$= 0.47$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.47 + \frac{7}{4}$$

$$= 2.22$$

$$W_q = 0.47/7$$

$$= 0.07 \text{ minute}$$

$$W_s = 0.07 + \frac{1}{4}$$

$$= 0.32 \text{ minute}$$

(b) Total cost of providing lanes = Cost of ill-will + Cost of lane operation

Cost of ill-will = Expected number of arrivals per minute × Waiting time in system × Rate per minute

For two lanes:

Cost of ill-will = 4 × 1.03 × 10 = 41.2 paise

Cost of lane operation $= 2 \times \frac{265}{60} = 8.83$ paise

∴ Total cost (per minute) = 41.2 + 8.83
= 50 paise approx.

For three lanes:

Cost of ill-will = 4 × 0.32 × 10 = 12.8 paise

Cost of lane operation $= 3 \times \frac{265}{60} = 13.25$ paise

∴ Total cost (per minute) = 12.8 + 13.25
= 26 paise approx.

Conclusion: Provide three lanes.

31. *At present*

$\lambda = 10$ customers/hour, $\mu = 8$ customers/hour and $K = 2$.

With these inputs, we have

$L_s = 2.05128$, $L_q = 0.80128$, $W_s = 0.20513$ and $W_q = 0.08013$

Total cost of lost time per hour

= No. of arrivals/hour × W_s × Cost per hour

= 10 × 0.20513 × 20 = Rs 41.03

Proposed structure

For each crib,

λ = 5 customers/hour, μ = 8 customers/hour

With these inputs, we have

L_s = 1.66667, L_q = 1.04167, W_s = 0.33333 and W_q = 0.20833

Total cost per hour

= Cost of lost time + Inventory cost – Cost saving due to less walking time

= 5 × 2 × 0.33333 × 20 + 2 – 5 × 2 × 0.1 × 20 = Rs 48.67

Conclusion: It is not advisable to have separate cribs.

32. With K = 4, λ = 20 customers/hour, and μ = 10 customers/hour, we have

$$\rho = \frac{\lambda}{K\mu} = \frac{20}{4 \times 10} = 0.5$$

(a) From Table 10.1, we get $P(0)$ = 0.1304 for K = 4, and ρ = 0.50. Other probabilities are given here:

No. of customers, n	$\frac{(\lambda/\mu)^n}{n!}$ *or* $\frac{(\lambda/\mu)^n}{K!K^{n-K}}$	*P(0)*	*Probability P(n)*
1	2	0.1304	0.2608
2	2	0.1304	0.2608
3	4/3	0.1304	0.1739
4	2/3	0.1304	0.0869
5	1/3	0.1304	0.0435

(b) and (c)

$$L_q = \frac{(\lambda/\mu)^K \rho}{K!(1-\rho)^2} \times P(0)$$

$$= \frac{(20/10)^4 (0.50)}{4!(1-0.5)^2} \times 0.1304$$

= 0.1739 customer

$$L_s = L_q + \frac{\lambda}{\mu}$$

= 0.1739 + 20/10

= 2.1739 customers

(d) Average number of customers being served = $K\rho$

= 4 × 0.5 = 2.0

(e) and (f) Average waiting time in queue, $W_q = L_q/\lambda$

$$= \frac{0.1739}{20} \times 60$$

= 0.52 minute

Average waiting time in the system, $W_s = W_q + \frac{1}{\mu}$

$$= 0.52 + \frac{1}{10} \times 60$$

= 6.52 minutes

33. (a) Here $\lambda = 0.5$ customer/minute, $\mu = 4$ customers/minute, and $\rho = 0.5/4 = 0.125$.

(i) Overall system utilisation, $\rho = 0.125$ or 12.5%

(ii) $L_s = \dfrac{\rho}{1-\rho} = \dfrac{0.125}{1-0.125} = 0.1429$

(iii) $L_q = \dfrac{\rho^2}{1-\rho} = \dfrac{0.125^2}{1-0.125} = 0.0179$

(iv) $W_s = \dfrac{1}{\mu-\lambda} = \dfrac{1}{4-0.5} = 0.2857$ minute

(v) $W_q = \dfrac{\rho}{\mu-\lambda} = \dfrac{0.125}{4.05} = 0.0357$ minute

(vi) $P(0) = 1 - \rho = 0.875$

(vii) P(a customer has to wait) = P(busy) = 0.125

(b)

	Parameter	*Server works twice as fast*	*Two servers*
(i)	ρ	0.0625	0.0625
(ii)	L_s	0.0667	0.1255
(iii)	L_q	0.0042	0.0005
(iv)	W_s	0.1333 mts	0.2510 mts
(v)	W_q	0.0083 mts	0.0010 mts
(vi)	$P(0)$	0.9375	0.8824
(vii)	P(customer to wait)	0.0625 (This is $1 - P(0)$)	0.0074 (This is $1 - P(0 \text{ or } 1)$)

It is evident that with one server working twice as fast, customers spend less time in the system on the average but have to wait for longer time to get service and also have higher probability for having to wait for service.

34. With $\lambda = 60$ customers/hour, $\mu = 40$ customers/hour, $K = 2$, we have $\rho = 60/(40 \times 2) = 0.75$.

(a) Probability that both clerks are idle,

$$P(0) = \left[\sum_{i=0}^{K-1} \frac{(\lambda/\mu)^i}{i!} + \frac{(\lambda/\mu)^K}{K!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{i=0}^{2-1} \frac{(60/40)^i}{i!} + \frac{(60/40)^2}{2!(1-0.75)}\right]^{-1} = 0.1429$$

(b) $P(n) = \dfrac{(\lambda/\mu)^n}{n!} \times P(0)$ when $n \le K$

$P(1) = \dfrac{(60/40)^1}{1!} \times 0.1429 = 0.2143$

(c) $P(n) = \dfrac{(\lambda/\mu)^n}{K!K^{n-K}} \times P(0)$ when $n > K$

$P(5) = \dfrac{(60/40)^5}{2!\,2^{5-2}} \times 0.1429 = 0.0678$

(d) $L_q = \dfrac{(\lambda/\mu)^K \rho}{K!(1-\rho)^2} \times P(0) = \dfrac{(60/40)^2 \times 0.75}{2!(1-0.75)^2} \times 0.1429 = 1.9286$

(e) $L_s = L_q + \dfrac{\lambda}{\mu} = 1.9286 + \dfrac{60}{40} = 3.4286$

(f) $W_q = L_q/\lambda = 1.9286/60$ hour or 1.9286 minutes

(g) $W_s = W_q + \dfrac{1}{\mu} = \dfrac{1.9286}{60} + \dfrac{1}{40} = 0.0571$ hour or 3.4286 minutes.

35. *For a single channel:*
We have,

$$W_s = \frac{1}{\mu - \lambda}$$

With $\mu = 30$ customers/hour and $\lambda = 24$ customers/hour, we have

$$W_s = \frac{1}{30 - 24}$$

$$= \frac{1}{6} \text{ hour or 10 minutes}$$

For 3 channels:
Here $K = 3$, $\mu = 10$ customers/hour, and $\lambda = 24$ customer/hour.
From Table 10.1, for $K = 3$ and $\rho = 0.8$ (where $\rho = 24/3 \times 10$), we get $P(0) = 0.0562$.
Now,

$$W_s = \frac{(\lambda/\mu)^K \rho}{K!(1-\rho)^2 \lambda} \times P(0) + \frac{1}{\mu}$$

$$= \frac{(24/10)^3 \times 0.8}{3!(1-0.8)^2 \times 24} \times 0.0562 + \frac{1}{10}$$

$$= 0.1079 + 0.1$$

$$= 0.2079 \text{ hour or 12.5 minutes approx.}$$

Conclusion: Single channel is better.

36. Given $\lambda = 2$ customers/hour, $\mu = 2.5$ customers/hour, cost of providing service = Rs 4/server/hour, and idle-time cost = Rs 100 per hour. We have,
Total cost per hour
= Cost of providing service per hour + Idle-time cost per hour
Cost of providing service per hour
= No. of servers × Cost of each server
Idle-time cost per hour
= Expected number of customers in system, L_s × Cost per unit per hour
For single server:

$$L_s = \frac{\lambda}{\mu - \lambda}$$

$$= \frac{2}{2.5 - 2}$$

$$= 4 \text{ customers}$$

Cost of providing service
= 1 × 4 = Rs 4

Idle-time cost per hour

= 4 × 100 = Rs 400

∴ Total cost = Rs 4 + Rs 400

= Rs 404 per hour

For two servers:

With $K = 2$, $\lambda = 2$ customers/hour, $\mu = 2.5$ customers/hour, $\rho = \lambda/K\mu$

= 2/2 × 2.5) = 0.4.

From Table 10.1, for $K = 2$ and $\rho = 0.4$, we have $P(0) = 0.4286$.

Now, $$L_s = L_q + \frac{\lambda}{\mu}$$

$$= \frac{(2/2.5)^2 \times 0.4}{2!(1-0.4)^2} \times 0.4286 + 0.8$$

= 0.9524 customer

Total cost per hour

= 2 × 4 + 0.9524 × 100

= Rs 103.24 per hour

For three servers:

With $K = 3$, $\rho = 2/3 \times 2.5 = 0.27$. From Table 10.1, for $K = 3$ and $\rho = 0.27$, $P(0)$, by interpolation, is (0.4564 + 0.4292)/2 = 0.4428.

Now, $$L_s = \frac{(2/2.5)^3 \times 0.27}{3!(1-0.27)^2} \times 0.4428 + 0.8$$

= 0.81914 customer

Total cost per hour

= 3 × 4 + 0.81914 × 100

= Rs 93.91 per hour

For four servers:

With $K = 4$, $\rho = 2/4 \times 2.5 = 0.2$. We have, $\rho = 0.4491$ (Table 10.1).

Accordingly,

$$L_s = \frac{(2/2.5)^4 \times 0.2}{4!\,(1-0.2)^2} \times 0.4491 + 0.8$$

= 0.8024 customer

Total cost per hour

= 4 × 4 + 0.8024 × 100

= Rs 96.24 per hour

Since the total cost has started rising, we do not consider the cases of greater number of servers. From the above calculations, it is evident that the number of servers to provide for minimum cost is three.

CHAPTER 11

1. **Determination of Optimal Replacement Interval**

Year t	*Maintenance Cost MC*	*Cumulative MC*	*Resale Value S*	*C – S*	*Total Cost*	*Average Cost*
1	2,000	2,000	4,000	3,000	5,000	5,000,00
2	2,100	4,100	3,000	4,000	8,100	4,050.00
3	2,300	6,400	2,200	4,800	11,200	3,733.33
4	2,600	9,000	1,600	5,400	14,400	3,600.00
5	3,000	12,000	1,400	5,600	17,600	**3,520.00**
6	3,500	15,500	700	6,300	21,800	3,633.00
7	4,100	19,600	700	6,300	25,900	3,700.00
8	4,600	24,200	700	6,300	30,500	3,812.50

Since the average cost is least, equal to Rs 3,520, corresponding to $t = 5$, replacement should be done every five years.

2. From the calculations given in table, it is found that the average cost is the minimum corresponding to year 6. Accordingly, the equipment should be replaced every six years.

Determination of Optimal Replacement Interval

Year	*Running Cost (R)*	*Resale Value (S)*	*Cumulative, R*	*C – S*	*Total Cost*	*Average Cost*
1	600	3,500	600	1,700	2,300	2,300.00
2	850	2,700	1,450	2,500	3,950	1,975.00
3	1,000	1,800	2,450	3,400	5,850	1,950.00
4	1,250	1,000	3,700	4,200	7,900	1,975.00
5	1,400	850	5,100	4,350	9,450	1,890.00
6	1,475	600	6,575	4,600	11,175	**1,862.50**
7	2,000	425	8,575	4,775	13,350	1,907.14

3. Based on the given data, calculations are shown in table to determine the age at which replacement of the machine be done. The minimum average cost corresponds to year 4. The optimal age of replacement of the machine in question is, therefore, four years.

Determination of Optimal Replacement Interval

Year	*Operating Cost, OC*	*Resale Value (S)*	*Cumulative OC*	*C – S*	*Total Cost*	*Average Cost*
1	1,000	4,000	1,000	4,000	5,000	5,000.00
2	1,500	3,500	2,500	4,500	7,000	3,500.00
3	2,000	3,000	4,500	5,000	9,500	3,166.67
4	2,500	2,500	7,000	5,500	12,500	**3,125.00**
5	3,000	2,000	10,000	6,000	16,000	3,200.00
6	3,500	1,500	13,500	6,500	20,000	3,333.33
7	4,000	1,000	17,500	7,000	24,500	3,500.00

4. **Determination of Optimal Replacement Period**

Year	*Maintenance Cost*	*Cumulative Maintenance Cost*	*Cost-Salvage Value*	*Total Cost*	*Average Cost*
1	200	200	12,000	12,200	12,200
2	500	700	12,000	12,700	6,350
3	800	1,500	12,000	13,500	4,500
4	1,200	2,700	12,000	14,700	3,675
5	1,800	4,500	12,000	16,500	3,300
6	2,500	7,000	12,000	19,000	3,167*
7	3,200	10,200	12,000	22,200	3,171
8	4,000	14,200	12,000	26,200	3,275

Optimal Replacement interval : 6 years

5. **Determination of Optimal Replacement Period**

Year	*Maintenance Cost*	*Cumulative Maintenance Cost*	*Cost-Salvage Value*	*Total Cost*	*Average Cost*
1	100	100	6,000	6,100	6,100.00
2	250	350	6,000	6,350	3,175.00
3	400	750	6,000	6,750	2,250.00
4	600	1,350	6,000	7,350	1,837.50
5	900	2,250	6,000	8,250	1,650.00
6	1,250	3,500	6,000	9,500	**1,583.33**
7	1,800	5,300	6,000	11,300	1,614.29
8	2,500	7,800	6,000	13,800	1,725.00

The average cost of using this machine is lowest in the sixth year. Accordingly, the machine should be replaced at the end of the sixth year.

6. Using the given expressions, the maintenance costs and resale values are as shown in the second and the third columns respectively of the table. From the calculations given in the table, the optimal replacement period is seen to be one year since the average cost is steadily rising.

Determination of Optimal Replacement Interval

Year	*Maintenance Cost, MC*	*Resale Value (S)*	*Cumulative MC*	*C – S*	*Total Cost*	*Average Cost*
1	310	3,500	310	500	810	**810.00**
2	440	3,000	750	1,000	1,750	875.00
3	590	2,500	1,340	1,500	2,840	946.67
4	760	2,000	2,100	2,000	4,100	1,025.00
5	950	1,500	3,050	2,500	5,550	1,110.00
6	1,160	1,000	4,210	3,000	7,210	1,201.67
7	1,390	500	5,600	3,500	9,100	1,300.00

7. The calculation of total cost and average cost to determine optimal policy for replacement of the truck is given in table. The cost and resale values are given in thousands of rupees. The minimum average cost is Rs 1,06,000 each for five and six years. Thus, the truck may be replaced either at the end of five or six years.

Determination of Optimal Replacement Interval

Year	*Maintenance Cost, MC*	*Resale Value (S)*	*Cumulative MC*	*C – S*	*Total Cost*	*Average Cost*
1	36	200	36	100	136	136.00
2	48	150	84	150	234	117.00
3	60	100	144	200	344	114.67
4	72	80	216	220	436	109.00
5	84	70	300	230	530	**106.00**
6	96	60	396	240	636	**106.00**
7	108	50	504	250	754	107.71
8	120	40	624	260	884	110.50

8. *Type A truck:* For type *A* truck, calculations are given in the table.

Determination of Optimal Replacement Interval

Year	*Maintenance Cost, MC*	*Cumulative MC*	*Cost of Truck*	*Total Cost*	*Average Cost*
1	200	200	9,000	9,200	9,200
2	2,200	2,400	9,000	11,400	5,700
3	4,200	6,600	9,000	15,600	**5,200**
4	6,200	12,800	9,000	21,800	5,450
5	8,200	21,000	9,000	30,000	6,000
6	10,200	31,200	9,000	40,200	6,700

Optimal replacement interval: 3 years. Average cost = Rs 5,200.
Type B truck: For type *B* truck, similar calculations are given in table. For this, optimal interval for replacement = 5 years, and the average cost = Rs 4,000.

Determination of Optimal Interval

Year	*Maintenance Cost, MC*	*Cumulative MC*	*C – S*	*Total Cost*	*Average Cost*
1	400	400	10,000	10,400	10,400.00
2	1,200	1,600	10,000	11,600	5,800.00
3	2,000	3,600	10,000	13,600	4,533.33
4	2,800	6,400	10,000	16,400	4,100.00
5	3,600	10,000	10,000	20,000	**4,000.00**
6	4,400	14,400	10,000	24,400	4,066.67
7	5,200	19,600	10,000	29,600	4,228.57
8	6,000	25,600	10,000	35,600	4,450.00

The type A truck should be replaced by type B truck because B's average cost (minimum) is lower that the average cost (minimum) of type A. Yearly cost of running and maintaining type A, which is one-year old is:

Second year : Rs 2,200 + 0 = Rs 2,200

Third year : Rs 4,200 + 0 = Rs 4,200

Since the cost of running type A truck for another year is less than the average cost for type B (= Rs 4,000), it should be used for another year and then replaced.

9. (i)

Determination of Optimal Replacement Period

Years	*Maint. Cost*	*Resale Price*	*Cum. Maint Cost*	*Dep.*	*Total Cost*	*Average Cost*
1	2,600	7,000	2,600	5,000	7,600	7,600
2	3,000	4,500	5,600	7,500	13,100	6,550
3	3,400	3,250	9,000	8,750	17,750	5,917
4	4,000	2,600	13,000	9,400	22,400	5,600
5	4,700	2,400	17,700	9,600	27,300	5,460*
6	5,600	2,400	23,300	9,600	32,900	5,483
7	6,600	2,400	29,900	9,600	39,500	5,643

From the average cost column, it is evident that the optimal replacement interval is 5 years.

(ii) Since the minimum average cost of machine N is lower than that of machine, it is advisable to replace. To determine the time of replacement,

Cost of running and maintaining M in 3rd year = 3,400 + 1,250
= 4,650
in 4th year = 4,000 + 650
= 4,650
in 5th year = 4,700 + 200
= 4,900

The machine M should be used for further 2 years and then be replaced.

10. Here average cost for each type of machine has to be calculated. Given below are (a) maintenance cost totals, (b) depreciation totals, (c) total cost and (d) average costs.

Maintenance cost totals Year of sale

Year of Purchase	*1*	*2*	*3*	*4*	*5*
0	192	436	744	1,128	1,624
1		244	552	936	1,432
2			308	692	1,188
3				384	880
4					496

Depreciation totals

	1	*2*	*3*	*4*	*5*
0	640	1,120	1,560	1,920	2,200
1		840	1,280	1,640	1,920
2			720	1,080	1,360
3				600	880
4					480

Total costs

	1	*2*	*3*	*4*	*5*
0	8	1,556	2,304	3,048	3,824
1		1,084	1,832	2,576	3,352
2			1,028	1,772	2,548
3				984	1,760
4					976

Average costs

	1	*2*	*3*	*4*	*5*
0	832	778	768	**762**	765
1		1,084	916	859	838
2			1,028	886	849
3				984	880
4					976

From the average cost values, it is clear that the optimal policy is to buy a machine now and replace after four years. The cost involved with the policy is Rs 762 per year, which is the least.

11. **Determination of Optimal Replacement Interval**

Year t	*Cost C_t*	*Salvage Value, S_t*	*Operating Cost, O_t*	*$C_t - S_t$*	*Cum. O_t*	*Total Cost*	*Average Cost*
1	200	100	60	100	60	160	160.0
2	210	50	80	160	140	300	150.0
3	220	30	100	190	240	430	**143.3**
4	240	20	120	220	360	580	145.0
5	260	15	150	245	510	755	151.0
6	290	10	180	280	690	970	161.7
7	320	0	230	320	920	1,240	177.1

Since the average cost is the minimum for $t = 3$, replace the machine every three years.

12. *Small trucks:*

The relevant data for each small truck are given in table. Also, cost calculations are given in this table. It is evident that optimal time to replace these trucks is five years.

Determination of Optimal Replacement Interval

Year	*Running Cost, RC*	*Cumulative RC*	*Resale Value, S*	*C – S*	*Total Cost*	*Average Cost*
1	10,000	10,000	30,000	30,000	40,000	40,000.00
2	12,000	22,000	15,000	45,000	67,000	33,500.00
3	14,000	36,000	7,500	52,500	88,500	29,500.00
4	18,000	54,000	3,750	56,250	1,10,250	27,562.50
5	23,000	77,000	2,000	58,000	1,35,000	**27,000.00**
6	28,000	1,05,000	2,000	58,000	1,63,000	27,166.67
7	34,000	1,39,000	2,000	58,000	1,97,000	28,142.86
8	40,000	1,79,000	2,000	58,000	2,37,000	29,625.00

Large trucks:

From the data available about the large trucks, we first calculate the minimum average cost to determine the optimal replacement of such a truck as well as its substitutability in place of the old one. The calculations are given in the following table.

Determination of Optimal Replacement Interval

Year	*Running Cost, RC*	*Cumulative RC*	*Resale Value S*	*C – S*	*Total Cost*	*Average Cost*
1	12,000	12,000	40,000	40,000	52,000	52,000.00
2	15,000	27,000	20,000	60,000	87,000	43,500.00
3	18,000	45,000	10,000	70,000	1,15,000	38,333.33
4	24,000	69,000	5,000	75,000	1,44,000	36,000.00
5	31,000	1,00,000	3,000	77,000	1,77,000	**35,400.00**
6	40,000	1,40,000	3,000	77,000	2,17,000	36,166.67
7	50,000	1,90,000	3,000	77,000	2,67,000	38,142.86
8	60,000	2,50,000	3,000	77,000	3,27,000	40,875.00

Now, since two new trucks are equivalent to three old trucks, (Min) average cost of three old trucks = 3 × 27,000 = Rs 81,000 (Min) average cost of two new trucks = 2 × 35,400 = Rs 70,800.

Clearly, then the old trucks be replaced by new ones. On the timing of replacement, we proceed to compare the costs as follows:

As long as the cost of running old fleet is lower than Rs 70,800, it would be prudent to run the old one. Thus, we have

Trucks		*Replacement one year from now:*	
One year old		12,000 + 15,000	= 27,000
Two years old		2(14,000 + 7,500)	= 43,000
Two years from now:		*Three years from now:* Total	70,000
14,000 + 7,500	= 21,500	18,000 + 3,750	= 21,750
2(18,000 + 3,750)	= 43,500	2(23,000) + 1,750)	= 49,500
Total	65,000	Total	71,250

From the cost calculations, it is clear that the old truck be run for another two years before being replaced.

13. **Determination of Optimal Replacement Period**

Year	*Maintenance Cost*	*Cumulative Maintenance Cost*	*Depreciation*	*Total Cost*	*Average Cost*
1	2,000	2,000	6,000	8,000	8,000
2	2,400	4,400	9,000	13,400	6,700
3	2,800	7,200	10,500	17,700	5,900
4	3,600	10,800	11,200	22,000	5,500
5	4,600	15,400	11,600	27,000	5,400*
6	5,800	21,200	11,600	32,800	5,467

Optimal replacement interval = 5 years.

14. **(i) Determination of Optimal Replacement Period**

Year	*Maintenance Cost*	*Cumulative MC*	*Cost-resale Value*	*Total Cost*	*Average Cost*
1	400	400	7,500	7,900	7900
2	900	1,300	7,500	8,800	4400
3	1,400	2,700	7,500	10,200	3400
4	1,900	4,600	7,500	12,100	3025
5	2,400	7,000	7,500	14,500	2900*
6	2,900	9,900	7,500	17,400	2900*
7	3,400	13,300	7,500	20,800	2971

Optimal interval: 5 or 6 years.

(ii) When future costs are discounted:

Determination of Optimal Replacement Interval

Year	*Maintenance Cost, M_t*	*PVF*	*PV of M_t*	*Cost plus Cumulative M_t*	*Cum. PVF*	*Annualised Cost*
1	400	1.0000	400.0	7900.0	1.0000	7,900
2	900	0.9091	818.2	8718.2	1.9091	4,567
3	1,400	0.8264	1,157.0	9875.2	2.7355	3,610
4	1,900	0.7513	1,427.5	11302.7	3.4868	3,242
5	2,400	0.6830	1,639.2	12941.9	4.1698	3,104
6	2,900	0.6209	1,800.7	14742.6	4.7907	3,077
7	3,400	0.5645	1,919.2	16661.8	5.3552	3,111
8	3,900	0.5132	2,001.3	18662.8	5.8684	3,180

Optimal replacement interval = 6 years.

15. For M_1:

Determination of Optimal Replacement Interval

Year	*Maintenance Cost, MC*	*PV Factor*	*PV of MC*	*Cum. PV of MC + Cost*	*Cum. PV Factor*	*Average*
1	800	1.0000	800.0	5,800.0	1.0000	5,800.0
2	800	0.9091	727.3	6,527.3	1.9091	3,419.0
3	800	0.8264	661.1	7,188.4	2.7355	2,627.8
4	800	0.7513	601.0	7,789.4	3.4868	2,234.0
5	800	0.6830	546.4	8,335.8	4.1698	1,999.1
6	1,000	0.6209	620.9	8,956.7	4.7907	1,869.6
7	1,200	0.5645	677.4	9,634.1	5.3552	1,799.0
8	1,400	0.5132	718.5	10,352.6	3.8684	1,764.1
9	1,600	0.4665	746.4	11,099.0	6.3349	**1,752.0**
10	1,800	0.4241	743.4	11,862.4	6.7590	1,755.1
11	2,000	0.3855	771.0	12,633.4	7.1445	1,768.3
12	2,200	0.3505	771.1	13,404.5	7.4950	1,788.5

For M_2:

Determination of Optimal Replacement Interval

Year	*Maintenance Cost, MC*	*PV Factor*	*MV of MC*	*Cum. PV of MC + Cost*	*Cum. PV Factor*	*Average*
1	1,200	1.0000	1,200.0	3,700.0	1.0000	3,700.0
2	1,200	0.9091	1,090.9	4,790.9	1.9091	2,509.5
3	1,200	0.8264	991.7	5,782.6	2.7355	2,113.9
4	1,200	0.7513	901.6	6,684.2	3.4868	1,917.0
5	1,200	0.6830	819.6	7,503.8	4,1698	1,799.5
6	1,200	0.6209	745.1	8,248.8	4,7907	1,721.8
7	1,400	0.5645	790.3	9,039.1	5,3552	1687.9
8	1,600	0.5132	821.1	9,860.3	5.8684	**1,680.2**
9	1,800	0.4665	839.7	10,700.0	6.3349	1,689.1
10	2,000	0.4241	848.2	11,548.2	6.7590	1,708.6
11	2,200	0.3855	848.1	12,396.3	7.1445	1,735.1
12	2,400	0.3505	841.2	13,237.5	7.4950	1,766.2

(i) Optimal replacement period of M_1: nine years; of M_2: eight years.
(ii) Machine M_2 is better.

16. We first calculate the expected life of bulbs as follows:

Life (X)	*Probability (p)*	*pX*
1	0.10	0.10
2	0.30	0.60
3	0.45	1.35
4	0.10	0.40
5	0.05	0.25
		Expected value = 2.70

Thus, expected life of bulbs = 2.70 weeks.
Now,
Expected cost of replacement per week

$$= \frac{\text{No. of bulbs}}{\text{Expected life of bulbs}} \times \text{Cost per replacement}$$

$$= \frac{2{,}000}{2.70} \times 2$$

= Rs 1,481.5

17. *Individual replacement policy:*
Step 1: Obtain expected life of bulbs. This is shown below:

Life (months)	*Mid-value (X)*	*Probability (p)*	*pX*
0-1	0.5	0.10	0.050
1-2	1.5	0.15	0.225
2-3	2.5	0.25	0.625
3-4	3.5	0.30	1.050
4-5	4.5	0.20	0.900
			Expected value = 2.850

Step 2: Calculate the cost per month.
With the expected life of the bulbs equal to 2.85 months, the average number of bulbs to replace every month, if replacements are to be made only as soon bulbs fail

$$= \frac{\text{No. of bulbs}}{\text{Expected life of bulbs}}$$

$$= \frac{500 \times 6}{2.85} = 1{,}053$$

Thus, total cost per month = 1,053 × 3 = Rs 3,159
Periodic replacement policy:
Step 1: To evaluate group replacement policy, we first calculate the expected number of failures to be replaced every month, for next of the five months.

$N_0 = 3{,}000$
$N_1 = N_0 \times p_1$
$= 3{,}000 \times 0.10 \qquad = 300$

$N_2 = N_0 \times p_2 + N_1 \times p_1$
$= 3{,}000 \times 0.15 + 300 \times 0.10 \qquad = 480$
$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$
$= 3{,}000 \times 0.25 + 300 \times 0.15 + 480 \times 0.10 \qquad = 843$
$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$
$= 3{,}000 \times 0.30 + 300 \times 0.25 + 480 \times 0.15 + 843 \times 0.10 \qquad = 1{,}131$
$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$
$= 3{,}000 \times 0.20 + 300 \times 0.30 + 480 \times 0.25 + 843 \times 0.15 + 1{,}131 \times 0.10 \qquad = 1{,}050$

Step 2: Calculate the cost per month associated with alternative policies. This is given below.

Determination of Optimal GR Policy

GR every	*Replacements*		*Cost of Replacements*			*Average*
	Individual	*Group*	*Individual*	*Group*	*Total*	*Cost*
1 month	300	3,000	900	3,000	3,900	3,900
2 months	780	3,000	2,340	3,000	5,340	2,670
3 months	1,623	3,000	4,869	3,000	7,869	**2,623**
4 months	2,754	3,000	8,262	3,000	11,262	2,816
5 months	3,804	3,000	11,412	3,000	14,412	2,882

From these calculations, it may be concluded that:

(i) The optimal group replacement interval is three months, with an average monthly cost of Rs 2,623.

(ii) The group and individual replacements policy is better than the policy of only individual replacements as it involves a lower cost.

18. As a first step, we obtain expected number of individual replacements during various months.

$N_0 = 200 \qquad$ *Cumulative*
$N_1 = N_0 \times p_1 = 200 \times 0.1 \qquad = 20 \qquad = 20$
$N_2 = N_0 \times p_2 + N_1 p_1$
$= 200 \times 0.2 + 20 \times 0.1 \qquad = 42 \qquad = 62$
$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$
$= 200 \times 0.2 + 20 \times 0.2 + 42 \times 0.1 \qquad = 48.2 \qquad 110.2$
$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$
$= 200 \times 0.3 + 20 \times 0.2 + 42 \times 0.2 + 48.2 \times 0.1 \qquad = 77.22 \qquad 187.42$
$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$
$= 200 \times 0.2 = 20 \times 0.3 + 42 \times 0.2 + 48.2 \times 0.2 + 77.22 \times 0.1 \qquad = 71.76 \qquad 259.18$
$N_6 = N_1 \times p_5 + N_2 \times p_4 + N_3 \times p_3 + N_4 \times p_2 + N_5 \times p_1$
$= 20 \times 0.2 + 42 \times 0.3 + 48.2 \times 0.2 + 77.22 \times 0.2 + 71.76 \times 0.1 = 48.86 \qquad 308.04$
$N_7 = N_2 \times p_5 + N_3 \times p_4 + N_4 \times p_3 + N_5 \times p_2 + N_6 \times p_1$
$= 42.4 \times 0.2 + 48.2 \times 0.3 + 77.22 \times 0.2 + 71.76 \times 0.2$
$+ 48.86 \times 0.1 \qquad = 57.54 \qquad 365.58$
$N_8 = N_3 \times p_5 + N_4 \times p_4 + N_5 \times p_3 + N_6 \times p_2 + N_7 \times p_1$
$= 48.2 \times 0.2 + 77.22 \times 0.3 + 71.76 \times 0.2 + 48.86 \times 0.2 + 57.54$
$\times 0.1 \qquad = 62.68 \qquad 428.26$

The cost, in respect of various alternative policies, is shown calculated here. From the table, it is clear that optimal policy is to replace every three months and the average cost is Rs 4,605.33 p.m.

Determination of Optimal Replacement Interval

Group Replacement (Months)	*No. of Individual Replacements*	*Cost of Replacements*			*Average Cost*
		Group	*Individual*	*Total*	
1	20	5,000	1,600.0	6,600.0	6,600.00
2	62	5,000	4,960.0	9,960.0	4,980.00
3	110.2	5,000	8,816.0	13,816.0	**4,605.33**
4	187.42	5,000	14,993.6	19,993.6	4,998.40
5	259.18	5,000	20,734.4	25,734.4	5,146.88
6	308.04	5,000	24,643.2	29,643.2	4,940.53
7	365.58	5,000	29,246.4	34,246.4	4,892.34
8	428.27	5,000	34,261.5	39,261.5	4,907.69

19. (a) For policy of complete individual replacements, we first calculate expected life of the components.

Life (months)	*Mid-value (X)*	*Probability (p)*	*pX*
0-1	0.5	0.12	0.060
1-2	1.5	0.16	0.240
2-3	2.5	0.22	0.550
3-4	3.5	0.25	0.875
4-5	4.5	0.15	0.675
5-6	5.5	0.10	0.550
			Expected life = 2.950

Number of replacements expected per month

$$= \frac{\text{No. of components}}{\text{Expected life of components}}$$

$$= \frac{100}{2.95}$$

$$= 33.9$$

Cost per month = 33.9 × 4 = Rs 135.6

For periodic replacement for the entire group, we first estimate the number of replacements needed each month. This is done here.

$N_0 = 100$

$N_1 = N_0 \times p_1$

$= 100 \times 0.12 \qquad = 12.00$

$N_2 = N_0 \times p_2 + N_1 \times p_1$

$= 100 \times 0.16 + 12 \times 0.12 \qquad = 17.44$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$

$= 100 \times 0.22 + 12 \times 0.16 + 17.44 \times 0.12 \qquad = 26.00$

$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$

$= 100 \times 0.25 + 12 \times 0.22 + 17.44 \times 0.16 + 26.0 \times 0.12 \qquad = 33.56$

$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$

$= 100 \times 0.15 + 12 \times 0.25 + 17.44 \times 0.22 + 26.0 \times 0.16 + 33.56 \times 0.12 \qquad = 30.00$

$N_6 = N_0 \times p_6 + N_1 \times p_5 + N_2 \times p_4 + N_3 \times p_3 + N_4 \times p_2 + N_5 \times p_1$

$= 100 \times 0.10 + 12 \times 0.15 + 17.44 \times 0.25 + 26.0 \times 0.22 + 33.56 \times 0.16$

$+ 30.00 \times 0.12 \qquad = 30.85$

Now we can calculate the cost involved with various alternative policies of group replacement. It is shown in table below.

Determination of Optimal Replacement Policy

GR; Months	*Replacements*		*Cost of Replacements*			*Average Cost*
	Individual	*Group*	*Individual*	*Group*	*Total*	
1	12.00	100	48.00	150	198.00	198.00
2	29.44	100	117.76	150	267.76	133.88
3	55.44	100	221.76	150	371.76	**123.92**
4	89.00	100	356.00	150	506.00	126.50
5	119.00	100	476.00	150	626.00	125.20
6	149.85	100	599.40	150	749.40	124.90

From the above table, it is evident that the optimal period of group replacements is three months since this policy involves the least average cost. Also, this policy is superior to the policy of individual replacements only.

(b) The calculations are repeated and shown below, when the cost of replacement is Rs 2 instead of Rs 1.50, for group replacement policies.

Determination of Optimal Replacement Policy

GR; Months	*Replacements*		*Cost of Replacements*			*Average Cost*
	Individual	*Group*	*Individual*	*Group*	*Total*	
1	12.00	100	48.00	200	248.00	248.00
2	29.44	100	117.76	200	317.76	158.88
3	55.44	100	221.76	200	421.76	140.59
4	89.00	100	356.00	200	556.00	139.00
5	119.00	100	476.00	200	676.00	135.20
6	149.85	100	599.40	200	799.40	**133.20**

Evidently, even when the cost of replacement is Rs 12 in case of group replacement, the policy of group replacement is better than the policy of individual replacements. However, from the calculation, it is clear that the optimal interval between group replacements is now six months.

20. (a) The average life of bulbs is calculated here:

Life (in months) X	*Proportion of bulbs failing, p*	*pX*
1	0.08	0.08
2	0.12	0.24
3	0.20	0.60
4	0.30	1.20
5	0.20	1.00
6	0.10	0.60
		Expected value = 3.72

Thus, expected life of bulbs = 3.72 months.

(b) Expected cost of the policy of individual replacements:

= Expected number of replacements per month × Cost of individual replacement

Further, expected number of replacements per month

$$= \frac{\text{No. of bulbs}}{\text{Expected life of bulbs}}$$

$$\text{Expected cost} = \frac{1{,}000}{3.72} \times 5 = \text{Rs } 1{,}344.09 \text{ per month}$$

Group replacement policy:

The expected number of replacements in each of the three months is first calculated. This is shown below:

$N_0 = 1{,}000$

$N_1 = N_0 \times p_1$

$= 1{,}000 \times 0.08 \quad = 80.0$

$N_2 = N_0 \times p_2 + N_1 \times p_1$

$= 1{,}000 \times 0.12 + 80 \times 0.08 \quad = 126.4$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$

$= 1{,}000 \times 0.20 + 80 \times 0.12 + 126.4 \times 0.08 \quad = 219.712$

The cost of various policies is calculated as given below.

Calculation of Average Cost

Replacement: Every	*Individual Replacements*	*Group Replacements*	*Cost*		*Total*	*Average*
			IR	*GR*		
One month	80	1,000	400	2,000	2,400	2,400
Two months	206.4	1,000	1,032	2,000	3,032	1,516
Three months	426.112	1,000	2,130.56	2,000	4,130.56	1,377

It is clear from the table that each of the average cost values is greatest than Rs 1,344.09 calculated earlier. Accordingly, the policy of individual replacements is superior to each of the group replacement policies considered.

21. For *individual replacements only policy:*

Calculation of Expected Life

Life (months)	*X*	*p*	*pX*
0-1	0.5	0.10	0.05
1-2	1.5	0.20	0.30
2-3	2.5	0.20	0.50
3-4	3.5	0.30	1.05
4-5	4.5	0.20	0.90
			Expected value = 2.80

Expected number of failures per month = 2,000/2.80 = 714.29

Expected cost of replacements per month = 714.29 × 7 = Rs 5,000

For *group and individual replacements policy:*

Calculation of Expected No. of Failures

Month	*No. of Failures*	
1	2,000 × 0.10	= 200
2	2,000 × 0.20 + 200 × 0.10	= 420
3	2,000 × 0.20 + 200 × 0.20 + 420 × 0.10	= 482
4	2,000 × 0.30 + 200 × 0.20 + 420 × 0.20 + 482 × 0.10	= 772.2
5	2,000 × 0.20 + 200 × 0.30 + 420 × 0.20 + 482 × 0.20 + 772.2 × 0.10	= 717.6

Cumulative replacements in various months are: Month 1: 200; Month 2: 620; Month 3: 1102; Month 4: 1874.2 and Month 5: 2591.8.

Calculation of Expected Cost

Group Replacement Every	*No. of Replacements*		*Cost of Replacements*		*Total Cost*	*Average Cost*
	Group	*Ind.*	*Group*	*Ind.*		
1 Month	2,000	200	6,000	1,400	7,400	7,400
2 Months	2,000	620	6,000	4,340	10,340	5,170
3 Months	2,000	1,102	6,000	7,714	13,714	4,571
4 Months	2,000	1874.20	6,000	13,119	19,119	4,780
5 Months	2,000	2,591.8	6,000	18,143	24,143	4,829

Optimal interval between group replacements = 3 months

Cost (expected) per month = Rs 4,571

Group replacement policy is better than individual replacements only policy.

22. First, we calculate expected breakdown interval:

Month (X)	*Probability (p)*	*pX*
1	0.10	0.10
2	0.05	0.10
3	0.10	0.30
4	0.15	0.60
5	0.20	1.00
6	0.25	1.50
7	0.10	0.70
8	0.05	0.40
	Expected value	4.70

(a) Expected downtime cost per month

$$= \frac{\text{No. of machines}}{\text{Expected time until breakdown}} \times \text{Cost of repairing a breakdown}$$

$$= \frac{40}{4.7} \times 1{,}000$$

= Rs 8,510

(b) The expected number of breakdowns every month are shown calculated below:

$N_0 = 40$

$N_1 = N_0 \times p_1$

$= 40 \times 0.10 = 4.00$

$N_2 = N_0 \times p_2 + N_1 \times p_1$

$= 40 \times 0.05 + 4 \times 0.10 = 2.40$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$

$= 40 \times 0.10 + 4 \times 0.05 + 2.4 \times 0.10 = 4.44$

$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$

$= 40 \times 0.15 + 4 \times 0.10 + 2.4 \times 0.05 + 4.44 \times 0.10 = 6.96$

$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$

$= 40 \times 0.20 + 4 \times 0.15 + 2.4 \times 0.10 + 4.44 \times 0.05 + 6.964 \times 0.10 = 9.76$

$N_6 = N_0 \times p_6 + N_1 \times p_5 + N_2 \times p_4 + N_3 \times p_3 + N_4 \times p_2 + N_5 + p_1$

$= 40 \times 0.25 + 4 \times 0.20 + 2.4 \times 0.15 + 4.44 \times 0.10 + 6.964 \times 0.10 + 9.76 \times 0.05 = 12.93$

$N_7 = N_0 \times p_7 + N_1 \times p_6 + N_2 \times p_5 + N_3 \times p_4 + N_4 \times p_3 + N_5 \times p_2 + N_6 \times p_1$

$= 40 \times 0.10 + 4 \times 0.25 + 2.4 \times 0.20 + 4.44 \times 0.15 + 6.964 \times 0.10 + 9.76 \times 0.05$
$+ 12.93 \times 0.10 = 8.62$

$N_8 = N_0 \times p_8 + N_1 \times p_7 + N_2 \times p_6 + N_3 \times p_5 + N_4 \times p_4 + N_5 \times p_3 + N_6 \times p_2 + N_7 \times p_1$

$= 40 \times 0.05 + 4 \times 0.10 + 2.4 \times 0.25 + 4.44 \times 0.20 + 6.964 \times 0.15 + 9.76 \times 0.10$
$+ 12.93 \times 0.05 + 9.32 \times 0.10 = 7.42$

Next, the cost associated with each of the alternative policies of servicing time may be calculated to determine the optimal policy as shown in the table.

Determination of Optimal Servicing Policy

GS: Months	*Servicing*		*Cost of Servicing*			*Average Cost*
	Individual	*Group*	*Individual*	*Group*	*Total*	
1	4.00	40	4,000	12,000	16,000	16,000
2	6.40	40	6,400	12,000	18,400	9,200
3	10.84	40	10,840	12,000	22,840	7,613
4	17.80	40	17,800	12,000	29,800	**7,450**
5	27.56	40	27,560	12,000	39,560	7,912
6	40.49	40	40,490	12,000	52,490	8,748
7	49.11	40	49,114	12,000	61,114	8,730
8	56.53	40	56,531	12,000	68,531	8,566

From the table, it is clear that the minimum average cost, Rs 7,450, corresponds to four-monthly policy. Accordingly, the optimal interval between group servicing of machines is four months.

23. From the given data, we have the life distribution of the component and the expected life as shown here:

Life (months)	*Mid-value (X)*	*Probability (p)*	*pX*
0-1	0.5	0.05	0.025
1-2	1.5	0.20	0.300
2-3	2.5	0.20	0.500
3-4	3.5	0.25	0.875
4-5	4.5	0.15	0.675
5-6	5.5	0.15	0.825
		Expected value = 3.200	

Now,

$$\text{Average number of replacements per month} = \frac{\text{No. of components}}{\text{Expected life of components}}$$

$$= \frac{1{,}600}{3.2} = 500$$

For the policy of individual replacements, the total cost per month,

TC = (Replacement cost + Disruption cost) × Average number of replacements per month

= (1 + 9) × 500 = Rs 5,000

Group replacement policy:

For this, we first calculate the expected number of replacements month-after-month as given below:

$N_0 = 1{,}600$

$N_1 = N_0 \times p_1$

$= 1{,}600 \times 0.05 = 80$

$N_2 = N_0 \times p_2 + N_1 \times p_1$

$= 1{,}600 \times 0.20 + 80 \times 0.05 = 324$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$

$= 1{,}600 \times 0.20 + 80 \times 0.20 + 324 \times 0.05 = 352$

$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$

$= 1{,}600 \times 0.25 + 80 \times 0.20 + 324 \times 0.20 + 352 \times 0.05 = 498$

$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$

$= 1{,}600 \times 0.15 + 80 \times 0.25 + 324 \times 0.20 + 352 \times 0.20 + 498 \times 0.05 = 420$

$N_6 = N_0 \times p_6 + N_1 \times p_5 + N_2 \times p_4 + N_3 \times p_3 + N_4 \times p_2 + N_5 \times p_1$

$= 1{,}600 \times 0.15 + 80 \times 0.15 + 324 \times 0.25 + 352 \times 0.20 + 498 \times 0.20 + 420 \times 0.05 = 524$

The cost involved with different policies of group replacement is shown calculated here.

Calculation of Total and Average Cost

Group Replacements Every:	*Replacements*		*Cost of Replacements*			*Average Cost*
	Individual	*Group*	*Individual*	*Group*	*Total*	
One month	80	1,600	800	11,200	12,000	12,000
Two months	404	1,600	4,040	11,200	15,240	7,620
Three months	756	1,600	7,560	11,200	18,760	6,253
Four months	1,254	1,600	12,540	11,200	23,740	5,935
Five months	1,674	1,600	16,740	11,200	27,940	5,588
Six months	2,198	1,600	21,980	11,200	33,180	5,530

Since the cost involved with individual replacement policy is lower than each of the alternative policies of group replacement considered, it is not desirable to switch over from the existing policy.

24. (a) To calculate the cost for this policy, we first obtain average life of the bulbs as follows:

Life (Quarters)	*X*	*Probability p*	*pX*
0-1	0.5	0.1	0.05
1-2	1.5	0.3	0.45
2-3	2.5	0.6	1.50
		Expected value = 2.00	

$$\text{Expected cost per quarter} = \frac{\text{No. of bulbs}}{\text{Expected life of bulbs}} \times \text{Cost per bulb}$$

$$= \frac{50{,}000}{2} \times 6.4$$

$$= \text{Rs } 1{,}60{,}000$$

(b) For this policy, we first obtain the expected number of replacements every quarter, as given below:

$N_0 = 50{,}000$

$N_1 = N_0 \times p_1 = 50{,}000 \times 0.1 \quad = 5{,}000$

$N_2 = N_0 \times p_2 + N_1 \times p_1 = 50{,}000 \times 0.3 + 5{,}000 \times 0.1 \quad = 15{,}500$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1 = 50{,}000 \times 0.6 + 5{,}000 \times 0.3 + 15{,}500 \times 0.1 \quad = 33{,}050$

Now, we calculate the cost for the three alternative policies.

Determination of Optimal Replacement Policy

Replacements Every:	*No. of Replacements*		*Cost of Replacements*			*Average Cost*
	Group	*Individual*	*Group*	*Individual*	*Total*	
One Quarter	50,000	5,000	1,20,000	32,000	1,52,000	1,52,000
Two Quarters	50,000	20,500	1,20,000	1,31,200	2,51,200	**1,25,600**
Three Quarters	50,000	53,550	1,20,000	3,42,720	4,62,720	1,54,240

From the table, it is evident that the optimal policy is to replace all the bulbs every two quarters.

(c) The difference in cost is likely to be due to relatively large effort required in individual replacements, on a per replacement basis.

25. Various steps involved are given here:

Step I: Obtain probability distribution of component lives:

End of week	*No. of components servicing*	*No. of components failed*		*Prob. of failure*
		till week-end	*during week*	
1	455	45	45	0.09
2	375	125	80	0.16
3	250	250	125	0.25
4	75	425	175	0.35
5	15	485	60	0.12
6	0	500	15	0.03

Step 2: Determine the number of individual replacements every week:

Cumulative

$N_0 = 500$

$N_1 = N_0 \times p_1$

$= 500 \times 0.09 \quad = 45 \quad 45.00$

$N_2 = N_0 \times p_2 + N_1 \times p_1$

$= 500 \times 0.16 + 45 \times 0.09 \quad = 84.05 \quad 129.05$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$

$= 500 \times 0.25 + 45 \times 0.16 + 84.05 \times 0.09 \quad = 139.76 \quad 268.81$

$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 p_2 + N_3 \times p_1$

$= 500 \times 0.35 + 45 \times 0.25 + 84.05 \times 0.16 + 139.76 \times 0.09 \quad = 212.28 \quad 481.09$

$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$

$= 500 \times 0.12 + 45 \times 0.35 + 84.05 \times 0.25 + 139.76 \times 0.16$
$+ 212.28 \times 0.09 \quad = 138.23 \quad 619.32$

$N_6 = N_0 \times p_6 + N_1 \times p_5 + N_2 \times p_4 + N_3 \times p_3 + N_4 \times p_2 + N_5 \times p_1$
$= 500 \times 0.03 + 45 \times 0.12 + 84.05 \times 0.35 + 139.76 \times 0.25 + 212.28$
$\times 0.16 + 138.23 \times 0.09 \quad = 131.16 \quad 750.48$

Step 3: Obtain cost of individual replacement policy:

Life (weeks)	*X*	*Probability p*	*pX*
0-1	0.5	0.09	0.045
1-2	1.5	0.16	0.240
2-3	2.5	0.25	0.625
3-4	3.5	0.35	1.225
4-5	4.5	0.12	0.540
5-6	5.5	0.03	0.165
		Expected life = 2.840	

Average number of replacements per week

$$= \frac{\text{No. of components}}{\text{Average life}}$$

$$= \frac{500}{2.840} = 176.06$$

∴ Cost of replacements per week = 176.06 × 20
= Rs 3,521.20

Step 4: Obtain the cost of group replacement policy:

Determination of Optimal Replacement Interval

Replacement Interval (weeks)	*Individual Replacements to date (IR)*	*Cost of IR*	*Cost of GR*	*Total Cost*	*Average Cost*
1	45.00	900.0	3,500.0	4,400.0	4,400.0
2	129.05	2,581.0	3,500.0	6,081.0	3,040.5
3	268.81	5,376.2	3,500.0	8,876.2	**2,958.7**
4	481.09	9,621.8	3,500.0	13,121.8	3,280.5
5	619.32	12,386.4	3,500.0	15,886.4	3,177.3
6	750.48	15,009.6	3,500.0	18,509.6	3,084.9

The average cost per week (for optimal policy) is lower in respect of group replacement policy in comparison with that for individual replacement policy. Hence, the former is better. Optimal period between group replacements is equal to three weeks.

26. Using the given information, we first calculate the expected life of the item in question, as given below:

Life (months)	*X*	*Probability p*	*pX*
0-1	0.5	0.1	0.05
1-2	1.5	0.1	0.15
2-3	2.5	0.2	0.50
3-4	3.5	0.3	1.05
4-5	4.5	0.3	1.35
		Expected value = 3.10	

Thus, expected life of the item = 3.1 months.

Now, average number of failures per month = $\dfrac{2,000}{3.1}$

= 645 approx.

$\therefore$ Cost (per month) of replacing the unit only on failure = 645 × 800 = Rs 5,16,000.

For periodic replacement policies, we first compute the expected number of replacements per month. This is done below:

$$N_0 = 2000$$
$$N_1 = N_0 \times p_1 = 2000 \times 0.1 = 200$$
$$N_2 = N_0 \times p_2 + N_1 \times p_1 = 2000 \times 0.1 + 200 \times 0.1 = 220$$
$$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1 = 2000 \times 0.2 + 200 \times 0.1 + 200 \times 0.1 = 442$$
$$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1 = 2000 \times 0.3 + 200 \times 0.2 + 220 \times 0.1 + 442 \times 0.1 \simeq 706$$
$$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1 = 2000 \times 0.3 + 200 \times 0.3 + 220 \times 0.2 + 442 \times 0.1 + 706 \times 0.1 \simeq 819$$

The cost for various alternative policies is shown calculated here:

Calculation of Cost: Alternative Policies

Replacement Every:	*Individual Replacements*	*Cost of Replacements*			*Average Costs*
		Individual	*Group*	*Total*	
One month	200	1,60,000	13,00,000	14,60,000	14,60,000
Two months	420	3,36,000	13,00,000	16,36,000	8,18,000
Three months	862	6,89,600	13,00,000	19,89,600	6,63,200
Four months	1,568	12,54,400	13,00,000	25,54,400	**6,38,600**
Five months	2,387	19,09,600	13,00,000	32,09,000	6,41,920

In these, the optimal policy is to replace the group every four months, since it involves the lowest average cost of Rs 6,38,600. However, since the cost of the policy of replacement of items as and when they fail is lower than this, it is prudent to follow that policy only. Group replacement policy should not be adopted.

27. Here, total number of items = 1,000. From the given information, the life distribution may be obtained as given below. Also shown is the calculation of the expected life of the item.

Life (Weeks)	*Mid-value (X)*	*Prob. of failure (p)*	*pX*
0-1	0.5	10/100 = 0.10	0.050
1-2	1.5	15/100 = 0.15	0.225
2-3	2.5	35/100 = 0.35	0.875
3-4	3.5	25/100 = 0.25	0.875
4-5	4.5	15/100 = 0.15	0.675
		Expected value = 2.700	

For 'failure replacements' policy:

Expected number of replacements per week

$$= \frac{\text{No. of items}}{\text{Expected life}}$$

$$= \frac{1,000}{2.7} \simeq 370$$

Expected cost per week = 370 × 300 = Rs 1,11,000

Common preventive replacement policy:

For this policy, we first calculate the expected number of replacements required in each of the five weeks. The calculations are given below:

$N_0 = 1000$

$N_1 = N_0 \times p_1$

$= 1000 \times 0.10 \qquad = 100$

$N_2 = N_0 \times p_2 + N_1 \times p_1$

$= 100 \times 0.15 + 100 \times 0.10 \qquad = 160$

$N_3 = N_0 \times p_3 + N_1 \times p_2 + N_2 \times p_1$

$= 1000 \times 0.35 + 100 \times 0.15 + 160 \times 0.10 \qquad = 381$

$N_4 = N_0 \times p_4 + N_1 \times p_3 + N_2 \times p_2 + N_3 \times p_1$

$= 1000 \times 0.25 + 100 \times 0.35 + 160 \times 0.15 + 381 \times 0.10 \qquad = 347$

$N_5 = N_0 \times p_5 + N_1 \times p_4 + N_2 \times p_3 + N_3 \times p_2 + N_4 \times p_1$

$= 1000 \times 0.15 + 100 \times 0.25 + 160 \times 0.35 + 381 \times 0.15 + 347 \times 0.10 \qquad = 323$

The cost associated with various alternative policies is shown below:

Determination of Optimal Replacement Policy

GR: Weeks	*Replacements*		*Cost of Replacements*			*Average Cost*
	Individual	*Group*	*Individual*	*Group*	*Total*	
1	100	1,000	30,000	1,00,000	1,30,000	1,30,000
2	260	1,000	78,000	1,00,000	1,78,000	89,000
3	641	1,000	1,92,300	1,00,000	2,92,300	97,430
4	988	1,000	2,96,400	1,00,000	3,96,400	99,100
5	1,311	1,000	3,93,300	1,00,000	4,93,300	98,660

The least cost policy as is evident from the table, is to replace all items every two weeks. The average cost is Rs 89,000 per week.

The group replacement policy is superior to the individual failure replacement policy due to lower cost involved.

28. (a) Number of new rentals required in each of the next four years:

N_0 : 160

N_1 : $160 \times 0.25 \qquad = 40$

N_2 : $160 \times 0.40 + 40 \times 0.25 \qquad = 74$

N_3 : $160 \times 0.25 + 40 \times 0.40 + 74 \times 0.25 \qquad = 75$

N_4 : $160 \times 0.10 + 40 \times 0.25 + 74 \times 0.40 + 75 \times 0.25 \qquad = 74$

Average length of hire period $= 1 \times 0.25 + 2 \times 0.40 + 3 \times 0.25 + 4 \times 0.10 = 2.2$ years

Average number of rentals per year $= \frac{160}{2.2} = 72.7 \simeq 73.$

(b) New average length of the hire period

$= 0.1 \times 1 + 0.2 \times 2 + 0.4 \times 3 + 0.15 \times 4 + 0.15 \times 5 = 3.05$ years

Average number of new rentals per year $= \frac{160}{3.05}$

$= 52.5$

Thus, saving in administrative cost is $(72.7 - 52.5) \times 40 = 20.2 \times 40 =$ Rs 808. This is the upper limit on the costs for advertising compaign.

29. Here, group servicing policy is in operation, with a time interval of seven months between successive group servicings. To see whether it is optimal, we first estimate the number of repairs needed in different months, along with group servicing at fixed intervals. The calculations are given below:

$N_0 = 100$

$N_5 = N_0 \times p_5 = 100 \times 0.10 = 10$

$N_6 = N_0 \times p_6 + N_5 \times p_5 = 100 \times 0.3 + 10 \times 0.1 = 31$

$N_7 = N_0 \times p_7 + N_5 \times p_6 + N_6 \times p_5 = 100 \times 0.4 + 10 \times 0.3 + 31 \times 0.1 = 46.1$

$N_8 = N_0 \times p_8 + N_5 \times p_7 + N_6 \times p_6 + N_7 \times p_5 = 100 \times 0.10 + 10 \times 0.4 + 31 \times 0.3 + 46.1 \times 0.1 = 27.9$

$N_9 = N_0 \times p_9 + N_5 \times p_8 + N_6 \times p_7 + N_7 \times p_6 + N_8 \times p_5 = 100 \times 0.10 + 10 \times 0.1 + 31 \times 0.4 + 46.1 \times 0.3 + 27.9 \times 0.1 = 40$

Determination of Optimal Servicing Policy

Group Service Months	*Repairs/Servicing*		*Cost of Servicing*			*Average Cost*
	Individual	*Group*	*Individual*	*Group*	*Total*	
5	10	100	4,000	20,000	24,000	**4,800**
6	41	100	16,400	20,000	36,400	6,067
7	87.1	100	34,840	20,000	54,840	7,834
8	115	100	46,000	20,000	66,000	8,250
9	155	100	62,000	20,000	82,000	9,111

From the above table it is evident that the minimum average cost, Rs 4,800 p.m., results when group servicing is done every five months. This is the optimal policy, and not the current one.

30. (a) Let k be the number of employees recruited each year. If 1,000 employees be recruited each year for the last 39 years, there would be λ_x now in service at each age, x. Hence, the total number in service will be

$$\sum_{x=21}^{59} \lambda_x = 7,277$$

By proportion, if k employees are recruited each year, the total number in the system will be 7,277 (k/1,000). Since a total of 700 employees are required, the number of employees to be recruited each year,

$$k = \frac{1,000 \times 700}{7,277} = 96$$

(b) Suppose that promotion from clerk to officer takes place at age x_1. We always have in the system (96/1,000) λ_x employees of age x. The number of clerks in the system is thus 0.096 $\Sigma\lambda_x$ where summation extends from $x = 21$ to $x = x_1 - 1$. We required 400 clerks.

Hence, the condition on x_1 is

$$0.096 \sum_{x=21}^{x_1 - 1} \lambda_x = 400$$

or

$$\sum_{x=21}^{x_1 - 1} \lambda_x = \frac{400}{0.096} = 4,167$$

From the λ_k column of the life table, we find that $x_1 = 31$.

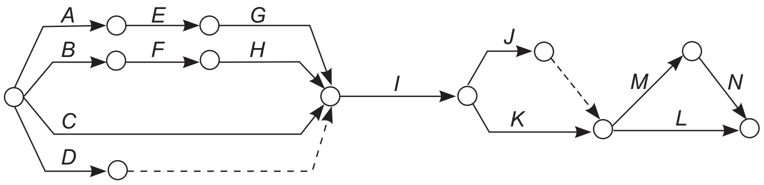
Similarly, if promotion from officer to manager takes place at age x_2, we obtain the condition

$$\sum_{x=31}^{x_2-1} \lambda_x = \frac{250}{0.096} = 2{,}604$$

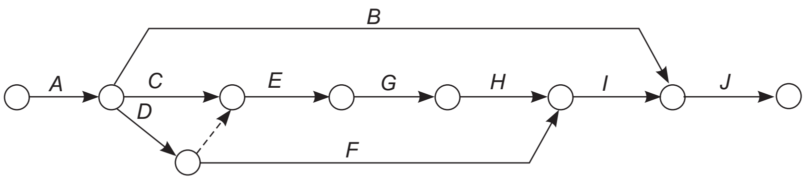
From the λ_x column of the life table, we observe that $x_2 = 52$.

Thus, clerk to officer promotion should take place at the age of 31 and from officer to manager at the age of 52 years.

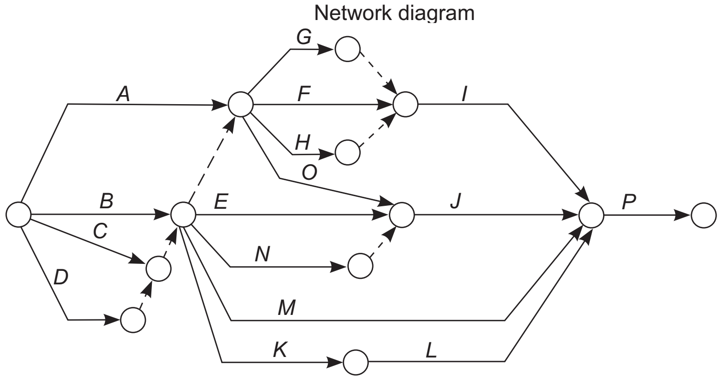
CHAPTER 12

1. 

Network diagram

2. 

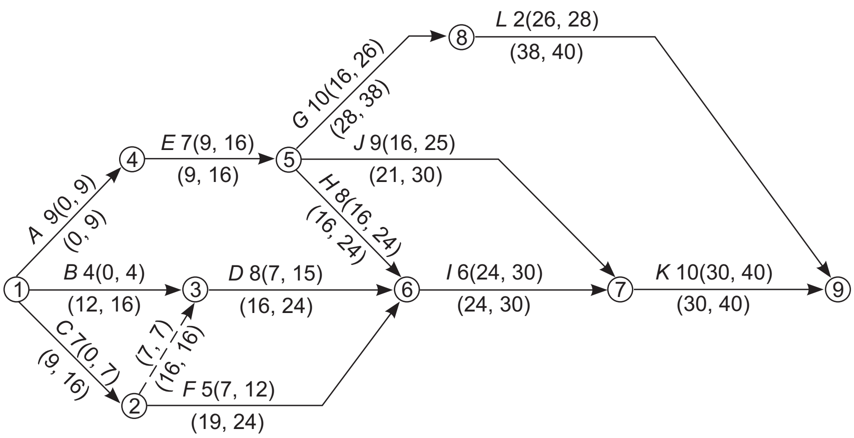
Network diagram

3. 

Network diagram

4.

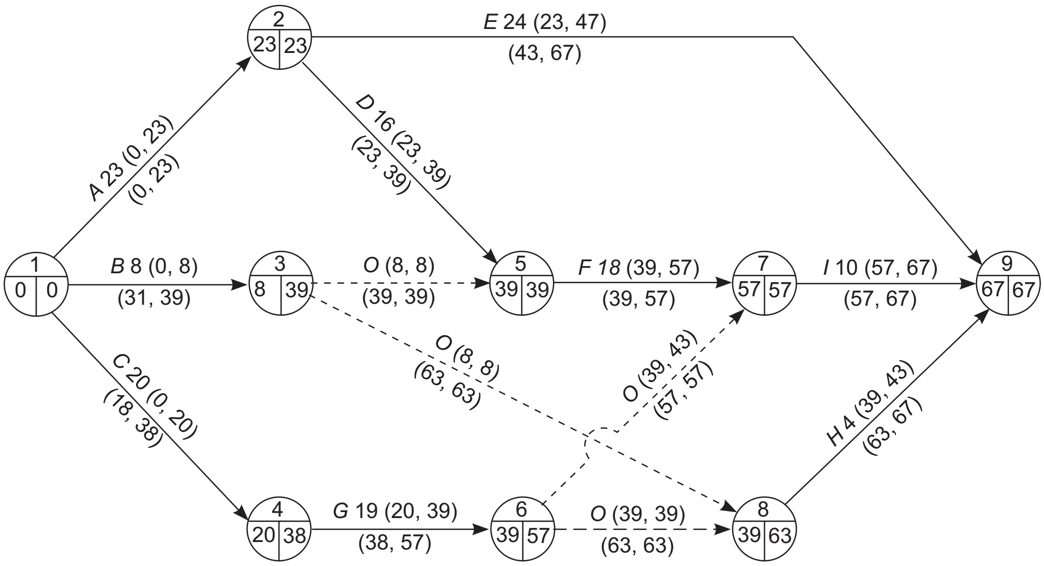
Activity :	K	L	M	N	O	P	Q	R	S
Imm. Predecessor(s) :	–	–	–	K	M	K, L	N, O	L, O	R

5. 

Network diagram

The critical activities of the project are *A*, *E*, *H*, *I* and *K*, while the project duration is 40 days. The earliest occurrence times (start and finish) for various activities are given on the top side of their respective arrows while the latest occurrence times (start and finish) are given on the bottom side.

6.



Calculation of Floats

Activity	*Node*	*ES*	*EF*	*LS*	*LF*	*Floats*		
						Total	*Free*	*Independent*
A	1-2	0	23	0	23	0	0	0
B	1-3	0	8	31	39	31	31	0
C	1-4	0	20	18	38	18	0	0
D	2-5	23	39	23	39	0	0	0
E	2-9	23	47	43	67	20	20	20
F	5-7	39	57	39	57	0	0	0
G	4-6	20	39	38	57	18	0	0
H	8-9	39	43	63	67	24	24	0
I	7-9	57	67	57	67	0	0	0

7. (i) The identification of redundant relationships is given in table below. Activities are listed both row and column-wise in the first instance. Each row is considered and the predecessors given for the activity in question are marked by an 'X'. After this, each of the predecessors of every activity is considered individually and its own predecessors are marked by circles. Finally, the spots with circled cross-marks are indicative of the redundancy as shown thereafter.

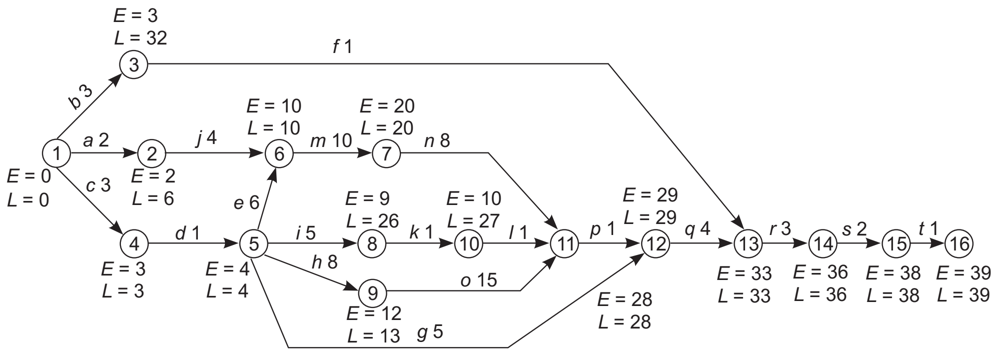
Redundancy in Precedence Relationships

Activity	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*
a																			
b																			
c																			
d			×																
e			○	×															
f		×																	
g			⊗	×															
h			⊗	×															
i			⊗	×															
j	×																		
k			○	⊗					×										
l			○	⊗					○		×								
m	○		⊗	⊗	×					×									
n	○		○	○	⊗					⊗			×						
o			○	⊗				×											
p				○	○			○	○	○	⊗	×	○	×	×				
q			○	○			×				○	○		○	○	×			
r		○				×	○									○	×		
s						○											○	×	
t																		○	×

Accordingly, the precedence relationships in the given project may be stated as follows:

Activity	*Immediate Predecessors*	*Activity*	*Immediate Predecessors*
a	—	*k*	*i*
b	—	*l*	*k*
c	—	*m*	*e, j*
d	*c*	*n*	*m*
e	*d*	*o*	*h*
f	*b*	*p*	*l, n, o*
g	*d*	*q*	*g, p*
h	*d*	*r*	*f, q*
i	*d*	*s*	*r*
j	*a*	*t*	*s*

(ii) The arrow diagram in given in the figure.

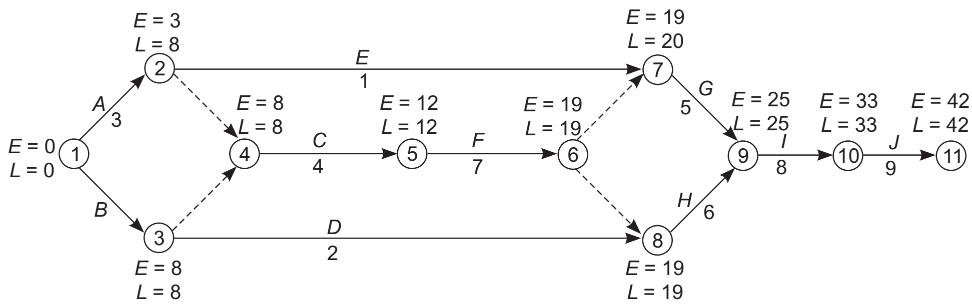


Arrow diagram

(iii) The total float for various activities is given here:

Activity	*ES*	*LS*	*TS*
a	0	4	4
b	0	29	29
c	0	0	0
d	3	3	0
e	4	4	0
f	3	32	29
g	4	24	20
h	4	5	1
i	4	21	17
j	2	6	4
k	9	26	17
l	10	27	17
m	10	10	0
n	20	20	0
o	12	13	1
p	28	28	0
q	29	29	0
r	33	33	0
s	36	36	0
t	38	38	0

8.(a)



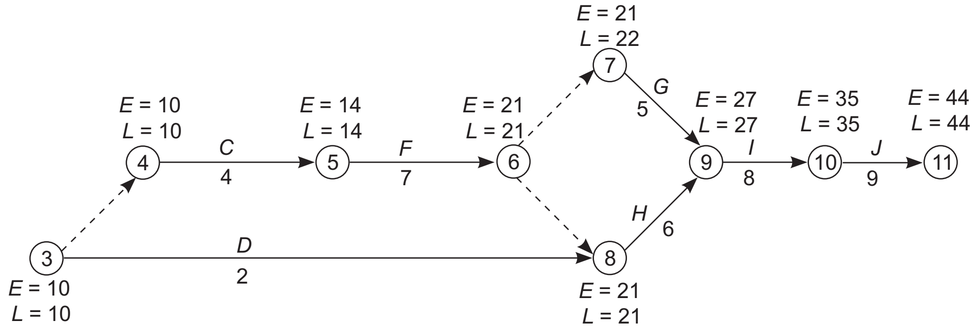
Index: *E*: Earliest time of event
L: Latest time of event

Network Diagram

The critical part of the network as shown in the figure is 1-3-4-5-6-8-9-10-11. Thus, activities *B*, *C*, *F*, *H*, *I*, *J* of the project are critical. The project duration is 42 weeks.

It is given that at the end of week 10, activities *A*, *B* and *E* are completed while other have not begun. Thus, the *ES* for the activities *C* and *D* is revised to 10.

The revised schedule is displayed here:



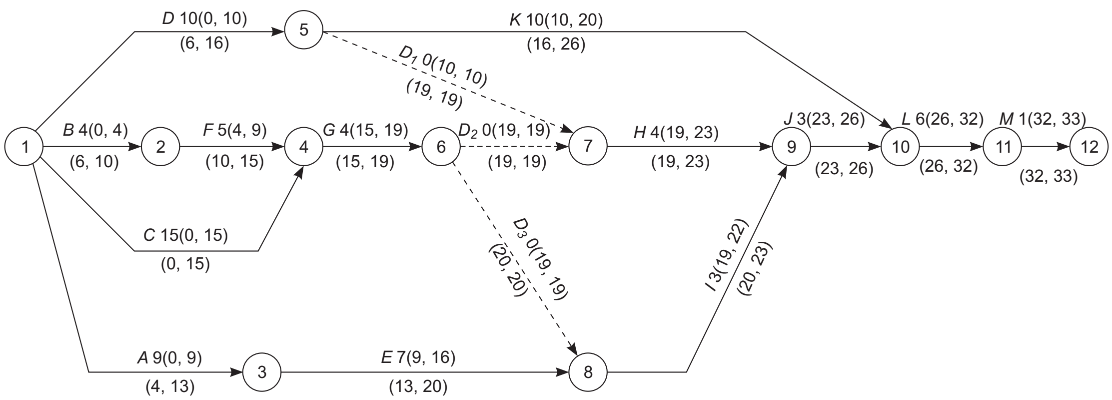
Index: *E*: Earliest time of event
L: Latest time of event

Revised Network Diagram

(i) As is evident from the revised network diagram, if no managerial action is taken at all, the project will be delayed by 2 weeks and shall be completed in 44 weeks.

(ii) In order to get the project completed by the end of 42 weeks, the activities on the critical path (revised) 3-4-5-6-8-9-10-11 should be crashed by 2 weeks.

9.

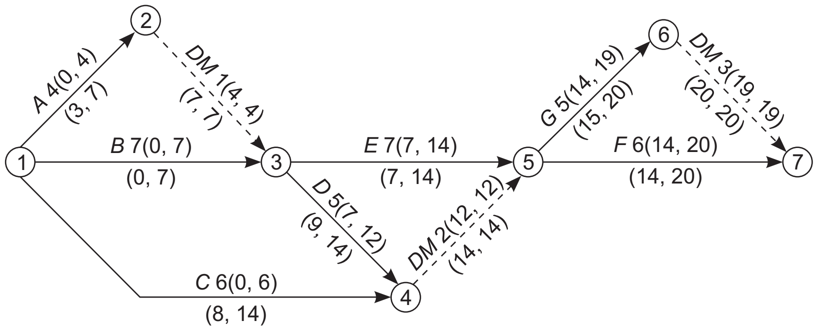


Activity	*ES*	*EF*	*LS*	*LF*	*FLOATS*		
					Total	*Free*	*Indep.*
A	0	9	4	13	4	0	0
B	0	4	6	10	6	0	0
C	0	15	0	15	0	0	0
D	0	10	6	16	6	0	0
E	9	16	13	20	4	3	0
F	4	9	10	15	6	6	0

(*Contd.*)

G	15	19	15	19	0	0	0
H	19	23	19	23	0	0	0
I	19	22	20	23	1	1	0
J	23	26	23	26	0	0	0
K	10	20	16	26	6	6	0
L	26	32	26	32	0	0	0
M	32	33	32	33	0	0	0

10. (i) Network is shown below.



Network

From the network diagram, it is clear that the project duration is 20 days.

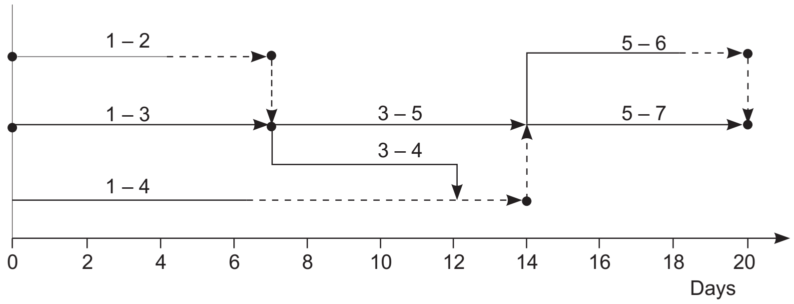
(ii) The earliest and the latest scheduling times may be used to calculate total floats for various activities. We have, Total float = Latest start – Earliest start. The values are shown in table.

Calculation of Total Float

Activity	*Nodes*	*Earliest*		*Latest*		*Total*
		Start	*Finish*	*Start*	*Finish*	*Float*
A	1-2	0	4	3	7	3
B	1-3	0	7	0	7	0
C	1-4	0	6	8	14	8
*DM*1	2-3	4	4	7	7	3
D	3-4	7	12	9	14	2
E	3-5	7	14	7	14	0
*DM*2	4-5	12	12	14	14	2
F	5-7	14	20	14	20	0
G	5-6	14	19	15	20	1
*DM*3	6-7	19	19	20	20	1

*Note: DM*1, *DM*2, and *DM*3 denote dummy activities.

(iii) The network is reproduced below, shown on time scale. It is drawn on the assumption that each activity starts at the earliest.



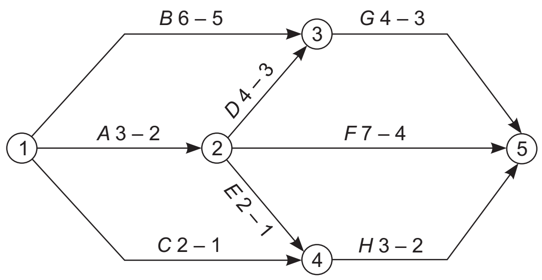
Time-scaled Network

11.

Part (a)					Part (b)				
Activity	*ES*	*EF*	*LS*	*LF*	*Activity*	*ES*	*EF*	*LS*	*LF*
1-2	0	7	0	7	1-2	0	7	0	7
1-3	0	3	1	4	1-3	0	3	7	10
2-5	7	12	7	12	2-4	7	12	7	12
3-4	3	5	4	6	3-5	3	5	10	12
4-5	5	5	12	12	4-5	12	12	12	12
4-7	5	13	6	14	4-6	12	18	12	18
5-6	12	18	12	18	5-7	12	20	12	20
6-8	18	24	18	24	6-8	18	24	18	24
7-8	13	23	14	24	7-8	20	30	20	30

Project duration increases from 24 to 30.

12. (a)



Path	*Normal Length*	*Crash Length*
1-3-5	10	8
1-2-3-5	11	8
1-2-5	10	6
1-2-4-5	8	5
1-4-5	5	3

(b) Normal duration = 11 days, Crash duration = 8 days.

(c) Crashing the Projects:

Crashing cost per day of the activities of the project is:

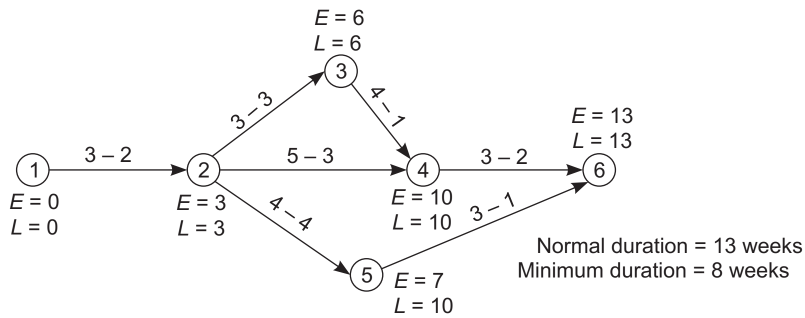
	A	B	C	D	E	F	G	H
Rs	70	60	80	50	80	40	80	50

Crashing the Project

Crashing	*Critical Path(s)*	*Decision*	*Cost*
I	1-2-3-5	2-3	50
II	1-3-5		
	1-2-3-5	2-5 and 3-5	120
	1-2-5		
III	1-3-5	1-2 and 1-3	130
	1-2-3-5		
	1-2-5		

Minimum duration = 8 days. Associated cost = Normal cost + Crashing cost
= Rs 1,300 + Rs (50 + 120 + 130) = Rs 1,600

13.



From the given data, we have

Activity	*Cost of Reduction/Week*
1-2	150
2-3	—
2-4	50
2-5	—
3-4	30
4-6	40
5-6	25

Crashing Table

Crashing	*Critical Path(s)*	*Decision*	*Cost*
I	1-2-3-4-6	3-4	30
II	1-2-3-4-6	3-4	30
III	1-2-3-4-6		
	1-2-4-6	4-6	40
IV	1-2-3-4-6	3-4	
	1-2-4-6	2-4	105
	1-2-5-6	5-6	
V	1-2-3-4-6		
	1-2-4-6	1-2	150
	1-2-5-6		

The cost table follows.

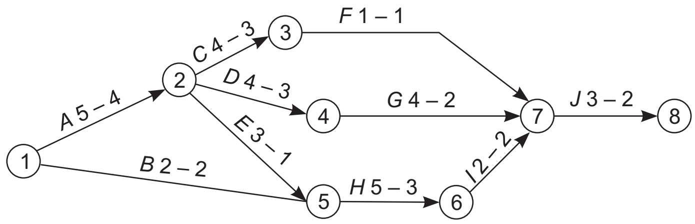
Cost Table

Duration	*Normal Cost*	*Crashing Cost*	*Indirect Cost*	*Total Cost*
13	945	0	1,300	2,245
12	945	30	1,200	2,175
11	945	60	1,100	2,105
10	945	100	1,000	2,045
9	945	205	900	2,050
8	945	355	800	2,100

Calculation of Float

Activity	:	1-2	2-3	2-4	2-5	3-4	4-6	5-6
ES	:	0	3	3	3	6	10	7
LS	:	0	3	5	6	6	10	10
Total Float	:	0	0	2	3	0	0	3

14. Network:



Path	*Normal*	*Shortest*
1-2-3-7-8	13	10
1-2-4-7-8	16	11
1-2-5-6-7-8	18*	12*
1-5-6-7-8	12	9

Normal duration = 18
Shortest duration = 12

Cost Slope

Activity :	*A*(1-2)	*B*(1-5)	*C*(2-3)	*D*(2-5)	*E*(2-5)	*F*(3-7)	*G*(4-7)
Slope :	2000	—	2000	5000	2500	—	2000
Activity :	*H*(5-6)	*I*(6-7)	*J*(7-8)				
Slope :	1500	—	2500				

Crashing Table

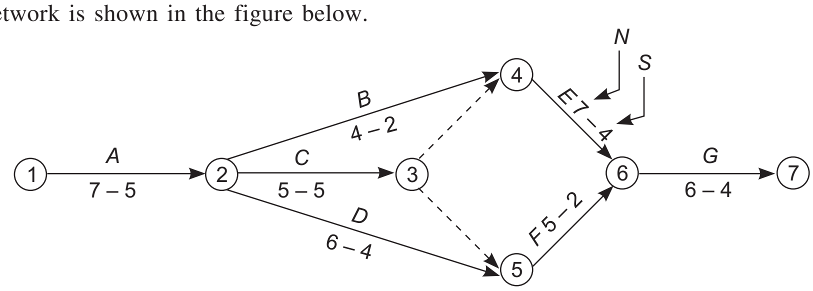
Crashing	*Critical Path(s)*	*Decision*	*Cost*
I	1-2-5-6-7-8	5-6	1500
II	1-2-5-6-7-8	5-6	1500
III	{1-2-4-7-8 1-2-5-6-7-8	1-2	2000
IV	{1-2-4-7-8 1-2-5-6-7-8	7-8	2500
V	{1-2-4-7-8 1-2-5-6-7-8	4-7 and 2-5	4500
VI	{1-2-4-7-8 1-2-5-6-7-8	4-7 and 2-5	4500

The cost table is given here.

Cost Table

Duration	*Normal Cost*	*Crashing Cost*	*Indirect Cost*	*Total Cost*
18	85,000	0	72,000	1,57,000
17	85,000	1,500	68,000	1,54,500
16	85,000	3,000	64,000	1,52,000
15	85,000	5,000	60,000	1,50,000
14	85,000	7,500	56,000	1,48,500
13	85,000	12,000	52,000	1,49,000
12	85,000	16,500	48,000	1,49,500

15. (i) The project network is shown in the figure below.



Project Network

Various paths and their lengths using normal (*N*) and crash (*S*) times are given here:

Path	*N*	*S*
1-2-4-6-7	24	15
1-2-3-4-6-7	25*	18*
1-2-3-5-6-7	23	16
1-2-5-6-7	24	15

Thus, normal duration of the project is 25 days and shortest duration is 18 days.

(iii) Crashing of the project requires us to calculate the cost of reduction per day for various activities. This is given below.

Calculation of Cost Slopes

Activity	*Duration (Days)*		*Cost (Rs)*		*Cost of Reduction per Day (Rs)*
	Normal	*Crash*	*Crash*	*Normal*	
A(1-2)	7	5	900	500	200
B(2-4)	4	2	600	400	100
C(2-3)	5	5	500	500	—
D(2-5)	6	4	1,000	800	100
E(4-6)	7	4	1,000	700	100
F(5-6)	5	2	1,400	800	200
G(6-7)	6	4	1,600	800	400

Now we attempt to reduce duration of the project to 21 days.

I Crashing: Critical path: 1-2-3-4-6-7. Activity 4-6 on this path has the least cost slope. Reducing 4-6 by one day increases the cost by Rs 100. After this, the revised length of various paths is:

1-2-4-6-7	23
1-2-3-4-6-7	24
1-2-3-5-6-7	23
1-2-5-6-7	24

II Crashing: To reduce the two critical paths simultaneously by one day each, we have two options with equal cost of Rs 200: either 1-2 or 4-6 and 2-5. Reducing 1-2 by a day causes the cost to rise by Rs 200 and reduction of the length of each path by 1 day. Now the project duration is 23 days.

III Crashing: Crash the activity 1-2 by one day more. This increases cost by another Rs 200 and reduces the length of each path and project duration by a day.

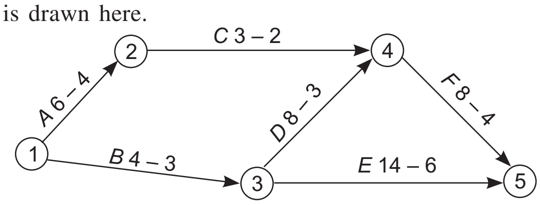
IV Crashing: Since 1-2 cannot be crashed any further, reduces 4-6 and 2-5 by a day each. The cost increases by Rs 200 and project duration reduces to the desired 21 days. At this stage, the lengths of various paths are:

1-2-4-6-7: 20, 1-2-3-4-6-7: 21.
1-2-3-5-6-7: 21, and 1-2-5-6-7: 21.

Now additional cost for reducing project duration = 100 + 200 + 200 + 200 = Rs 700.
Normal cost of completing the project in 25 days = Rs 4,500.

$\therefore$ Percentage increase in cost = $\frac{700}{4,500} \times 100 = 15.5.$

16. As a first step, we draw the project network and determine the normal and shortest duration of the project. The network is drawn here.



Network

The different paths and their lengths using normal (*N*) and crash (*S*) times are given as follows:

Path	*N*	*S*
1-2-4-5	17	10*
1-3-4-5	20*	10*
1-3-5	18	9

Thus, normal duration of the project is 20 weeks and the shortest duration 10 weeks. Now, for crashing we first have to obtain cost slope for every activity as follows:

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal duration} - \text{Crash duration}}$$

The calculations are given in the following table.

Calculation of Cost Slopes

Activity	*Time (in weeks)*		*Cost (in Rs)*		*Cost Slope*
	Normal	*Crash*	*Normal*	*Crash*	*(Rs/week)*
A(1-2)	6	4	10,000	14,000	2,000
B(1-3)	4	3	5,000	8,000	3,000
C(2-4)	3	2	4,000	5,000	1,000
D(3-4)	8	3	1,000	6,000	1,000
E(3-5)	14	6	9,000	13,000	500
F(4-5)	8	4	7,000	8,000	250

The crashing is shown here.

Crashing the Project

Crashing	*Critical Path(s)*	*Decision*	*Cost*	*Duration*
I	1-3-4-5	4-5	250	19
II	1-3-4-5	4-5	250	18
III	1-3-4-5, 1-3-5	4-5, 3-5	750	17
IV	1-3-4-5, 1-3-5	4-5, 3-5	750	16
V	1-3-4-5, 1-3-5	3-4, 3-5	1,500	15
VI	1-3-4-5, 1-3-5	3-4, 3-5	1,500	14
VII	1-3-4-5, 1-3-5	3-4, 3-5	1,500	13
VIII	all	2-4, 3-4, 3-5	2,500	12
IX	all	1-2, 3-4, 3-5	3,500	11
X	all	1-2, 1-3	5,000	10

The length of various paths after each crashing is given here:

Path	*Normal length*	*Length after crashing*									
		I	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*	*X*
1-2-4-5	17	16	15	14	13	13	13	13	12	11	10
1-3-4-5	20	19	18	17	16	15	14	13	12	11	10
1-3-5	18	18	18	17	16	15	14	13	12	11	10

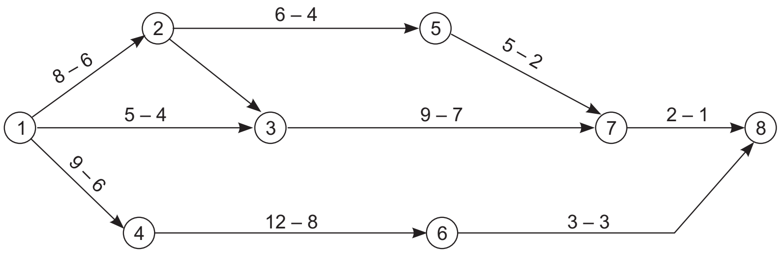
The varying project durations and the cost associated therewith are shown in the following table.

Determination of Total Cost

Duration (Weeks)	*Normal Cost*	*Crashing Cost*	*Overhead*	*Penalty*	*Total Cost*
20	36,000	0	20,000	16,000	72,000
19	36,000	250	19,000	14,000	69,250
18	36,000	500	18,000	12,000	66,500
17	36,000	1,250	17,000	10,000	64,250
16	36,000	2,000	16,000	8,000	62,000
15	36,000	3,500	15,000	6,000	60,500
14	36,000	5,000	14,000	4,000	59,000
13	36,000	6,500	13,000	2,000	57,500
12	36,000	9,000	12,000	0	57,000
11	36,000	12,500	11,000	0	59,500
10	36,000	17,500	10,000	0	63,500

From the table, it is evident that the optimal project duration is 12 weeks. All activities are critical after this crashing.

17.



Path	*N*	*S*
1-2-5-7-8	21	13
1-2-3-7-8	22	17
1-3-7-8	16	12
1-4-6-8	24	17

Thus, normal duration of the project is 24 days while the minimum duration is 17 days.

Crashing	*Critical Path(s)*	*Decision*	*Cost*
I	1-4-6-8	1-4	100
II	1-4-6-8	1-4	100
III	1-4-6-8 1-2-3-7-8	1-4, 1-2	300
IV	1-4-6-8 1-2-3-7-8	4-6, 1-2	600
V	1-4-6-8 1-2-3-7-8	4-6, 7-8	700
VI	1-4-6-8 1-2-3-7-8	4-6, 3-7	1,200
VII	1-4-6-8 1-2-3-7-8 1-2-5-7-8	4-6 3-7 4-6	1,450

Lengths of Paths after Crashings

Path	*Crashing*							
	0	1	2	3	4	5	6	7
1-2-5-7-8	21	21	21	20	19	18	18	17
1-2-3-7-8	22	22	22	21	20	19	18	17
1-3-7-8	16	16	16	16	16	15	14	13
1-4-6-8	24	23	22	21	20	19	18	17

Time/Cost Table

Duration (Days)	*Normal Cost*	*Crashing Cost*	*Overhead*	*Penalty*	*Total Cost*
24	18,400	0	12,000	6,000	36,400
23	18,400	100	11,500	4,000	33,900
22	18,400	200	11,000	2,000	31,600
21	18,400	500	10,500	0	29,400
20	18,400	1,100	10,000	0	29,500
19	18,400	1,800	9,500	0	29,700
18	18,400	3,000	9,000	0	30,400
17	18,400	4,450	8,500	0	31,350

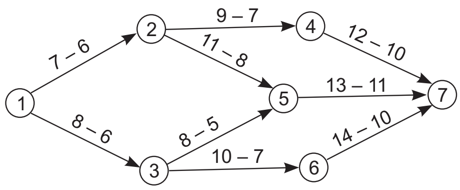
From the table,

(a) Normal duration: 24 days, Cost: Rs 36,400

(b) Minimum duration: 17 days, Cost: Rs. 31,350

(c) Minimum Cost: Rs 29,400, Time: 21 days

18. Network:



Normal duration = 32 days

Shortest duration = 25 days

Activity :	1-2	1-3	2-4	2-5	3-5	3-6	4-7	5-7	6-7
Cost slope :	50	50	50	67	83	67	50	50	75

Path	*Length after successive Crashings*							
1-2-4-7	28	28	27	27	27	27	26	25
1-2-5-7	31	31	30	29	28	27	26	25
1-3-5-7	29	28	27	26	25	25	25	25
1-3-6-7	32	31	30	29	28	27	26	25

Crashing Table

Crashing	*Critical Path(s)*	*Decision*	*Cost*
I	1-3-6-7	1-3	50
II	1-2-5-7 1-3-6-7	1-2, 1-3	100
III	1-2-5-7 1-3-6-7	5-7, 3-6	117
IV	1-2-5-7 1-3-6-7	5-7, 3-6	117
V	1-2-5-7 1-3-6-7	2-5, 3-6	134
VI	1-2-4-7 1-2-5-7 1-3-6-7	2-4, 2-5, and 6-7	192
VII	1-2-4-7 1-2-5-7 1-3-6-7	2-4, 2-5, and 6-7	192

Cost Table

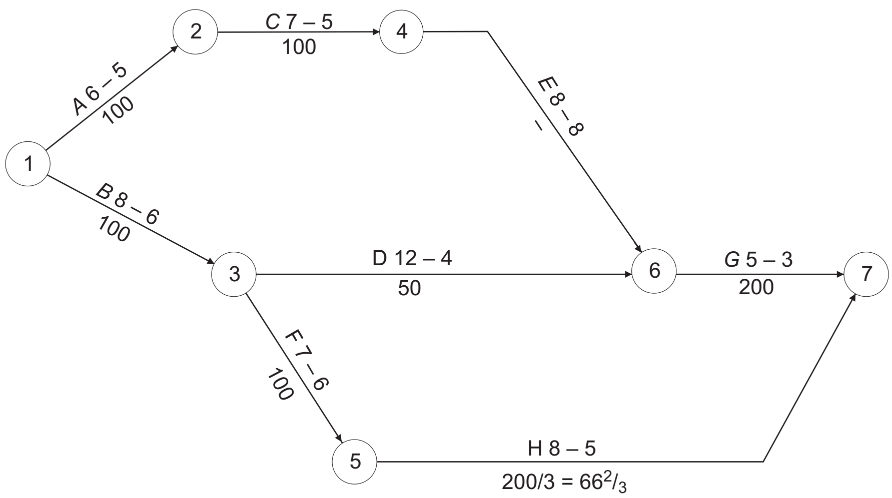
Duration	*Normal Cost*	*Crashing Cost*	*Total Cost*
32	10,400	0	10,400
31	10,400	50	10,450
30	10,400	150	10,550
29	10,400	267	10,667
28	10,400	384	10,784
27	10,400	518	10,918
26	10,400	710	11,110
25	10,400	902	11,302

To complete the project in 25 days, the cost involved is Rs 11,302 and the activities which need to crash are: 1-2: 1 day; 1-3: 2 days; 2-4: 2 days; 2-5: 3 days; 3-5: none; 3-6: 3 days; 4-7: none; 5-7: 2 days; and 6-7: 2 days.

The project cannot be completed in 22 days. When the duration is 25 days, all paths are seen to be critical, and therefore, need to be crashed. But, evidently, not all of them can be reduced any further. Hence, no further crashing can be possible.

19. **Calculation of Cost of Reduction**

Activity	*Time*		*Cost*		*Cost of Reduction per day*
	Normal	*Crash*	*Normal*	*Crash*	
A	6	5	300	400	100
B	8	6	400	600	100
C	7	5	400	600	100
D	12	4	1,000	1,400	50
E	8	8	800	800	–
F	7	6	400	500	100
G	5	3	1,000	1,400	200
H	8	5	500	700	200/3



Path	*N*	*(S)*	*I*	*II*	*III*	*IV*	*V*
		Length after Crashing					
1-2-4-6-7	26	(21)	25	24	23	22	21
1-3-4-6-7	25	(13)	25	24	23	22	21
1-3-5-7	23	(17)	23	23	23	22	21

Crashing	*Critical Path(s)*	*Decision*	*Cost*
I	1-2-4-6-7	1-2	100
II	1-2-4-6-7	2-4	150
	1-3-6-7	3-6	
III	1-2-4-6-7	2-4	150
	1-3-6-7	3-6	
IV	1-2-4-6-7	6-7, 5-7 (Rs 267)	
	1-3-6-7	1-3, 6-7 and	250
	1-3-5-7	Reverse 3-6 (Rs 250)	
V	1-2-4-6-7	1-3, 6-7	
	1-3-6-7	and Reverse	250
	1-3-5-7	3-6	

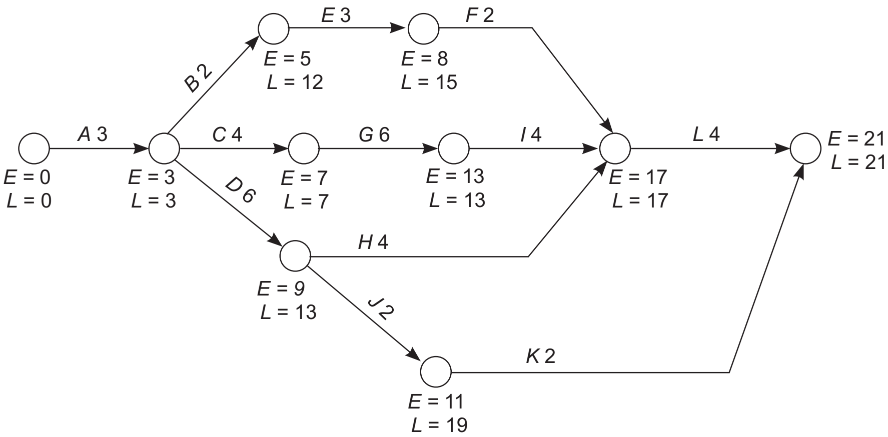
Time and Cost Table

Duration	*Normal Cost*	*Crashing Cost*	*Total Cost*
26	4,800	0	4,800
25	4,800	100	4,900
24	4,800	250	50,50
23	4,800	400	5,200
22	4,800	650	5,450
21	4,800	900	5,700

Activities to crash: *A* : 1 day; *B* : 2 days; *C* : 2 days and *G* : 2 days

Minimum crashing cost to complete project in 21 days = Rs 900

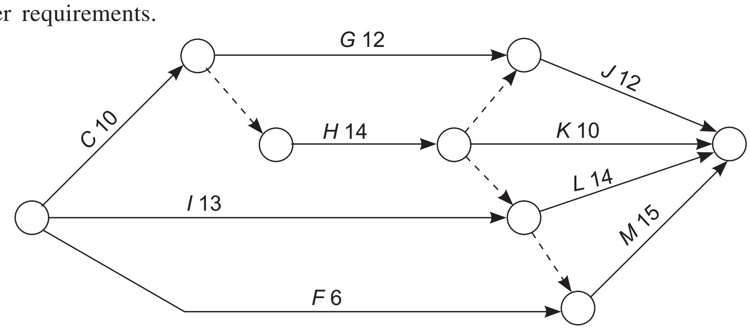
20.



Critical Path : ACGIL
Duration : 21 days

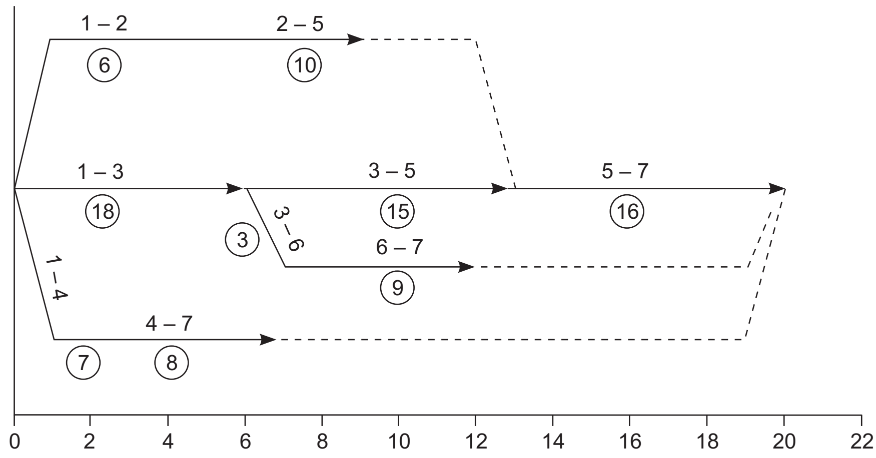
Activity	*t*	*ES*	*EF*	*LS*	*LF*
A	3	0	3	0	3
B	2	3	5	10	12
C	4	3	7	3	7
D	6	3	9	7	13
E	3	5	8	12	15
F	2	8	10	15	17
G	6	7	13	7	13
H	4	9	13	13	17
I	4	13	17	13	17
J	2	9	11	17	19
K	2	11	13	19	21
L	4	17	21	17	21

21. For the given information, the network is shown below. Also indicated alongside are activity durations and worker requirements.



Network

The network is redrawn on a time scale on the assumption that each activity is scheduled at its earliest start. This is shown in the figure below. Requirement of labour over time is also given below.



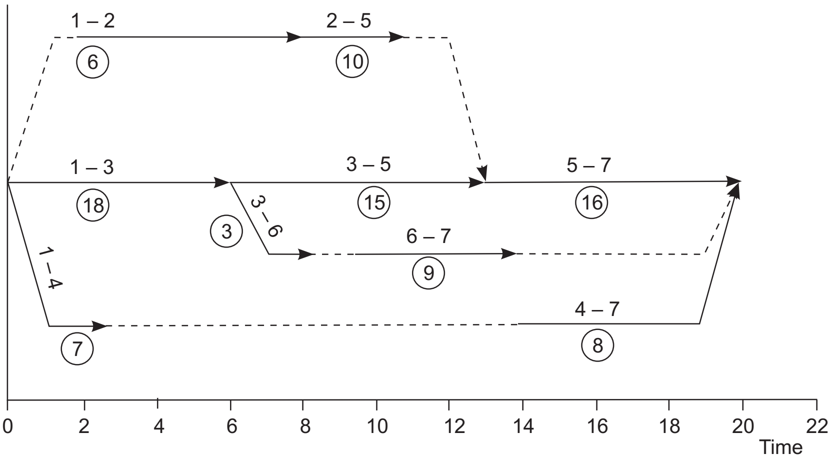
Time-Scaled Network

Days	1, 2	3-6	7	8	9	10-12	13	14-20
Workers needed	31	32	36	28	34	24	15	16

The large variations in the demand of labour can be reduced by means of:

Step 1 : shift 4-7 by 8 days.
Step 2 : shift 6-7 by 2 days.
Step 3 : shift 2-5 by 2 days.
Step 4 : shift 1-2 by 2 days.

The resulting schedule is shown below.



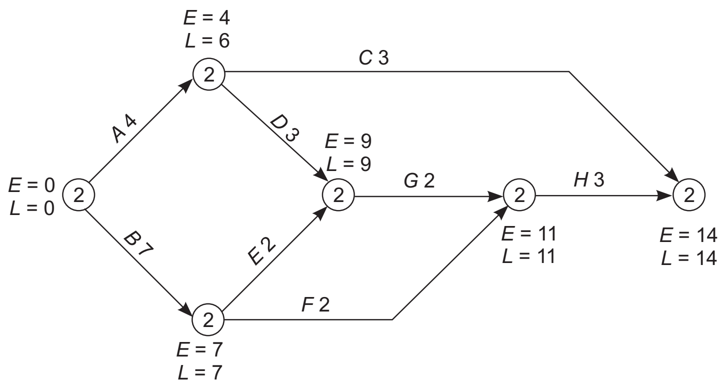
Time-scaled Network (Revised)

The labour requirement is given below:

Days	1, 2	3-8	9-11	12-13	14-15	16-20
Workers needed	25	24	25	24	25	24

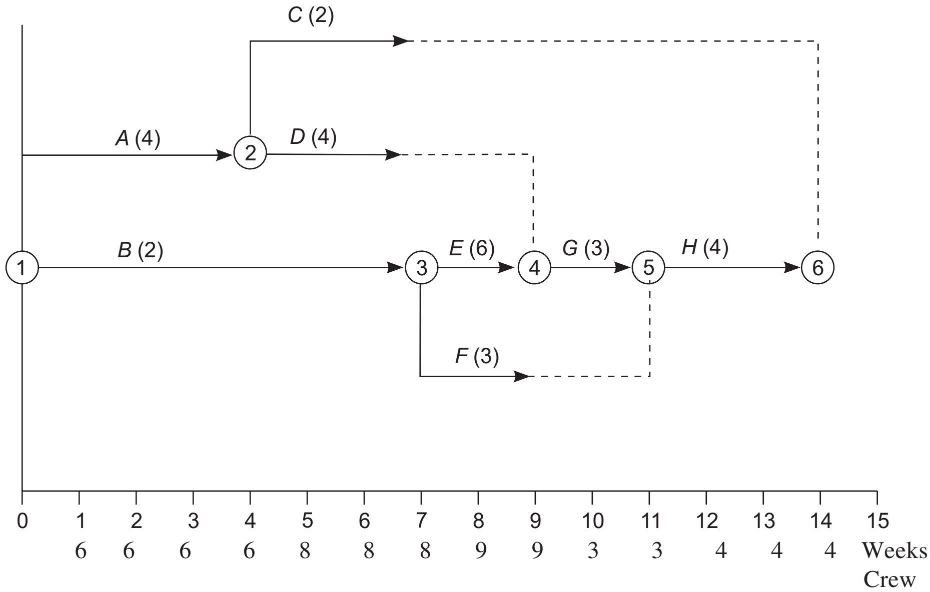
From these, a requirement of 25 workers is worked out. Else, if overtime may be permissible, 24 workers should be sufficient.

22. As a first step, we represent the given relationships through an arrow diagram.



Network

From the figure, it is evident that the project duration is 14 weeks, which is what is needed. This network is redrawn on a time scale. It is drawn on the assumption that every activity is started at the earliest. In the lower part of the diagram, the crew required on each day of the project is shown.



Time-scaled Network and Crew Requirement

In an attempt to even out the crew requirement in various weeks, we may reschedule the activities. The dotted lines in the diagram indicate the float available on various activities. This is done below:

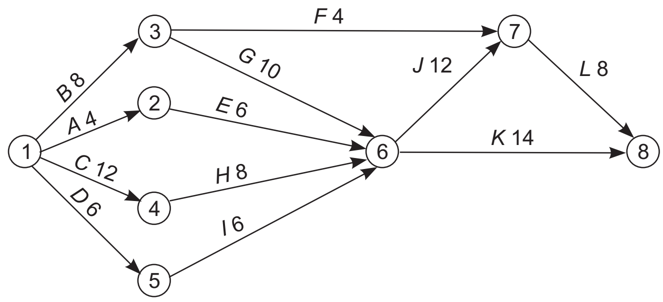
Step 1: Shift activity *C* by 7 weeks.

Step 2: Shift activity *F* by 2 weeks.

Step 1 would cause crew requirement reduced by 2 in weeks 5, 6 and 7, and increase in weeks 12, 13 and 14. Similarly, step 2 would result in a shift of 3 crew members from weeks 8 and 9 to weeks10 and 11. The crew requirements are indicated below. It is clear that after both the steps, the crew demand would be set equal to 6 for the entire span of 14 weeks.

Week :	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Crew requirement :														
Original	6	6	6	6	8	8	8	9	9	3	3	4	4	4
Step 1	6	6	6	6	6	6	6	9	9	3	3	6	6	6
Step 2	6	6	6	6	6	6	6	6	6	6	6	6	6	6

23. The project network is shown below.



Network

The various paths along with their lengths are given here:

1-3-7-8	20;	1-4-6-7-8	40;
1-3-6-7-8	38;	1-4-6-8	34;
1-3-6-8	32;	1-5-6-7-8	32; and
1-2-6-7-8	30;	1-5-6-8	26
1-2-6-8	24;		

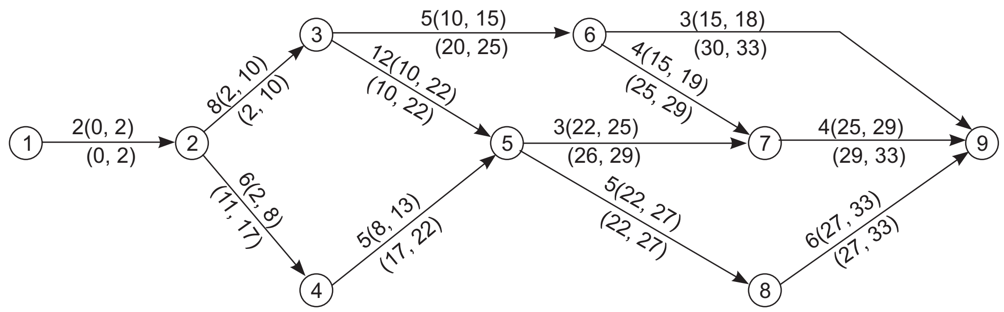
(a) Activity *J* is critical one while *K* is non-critical. Shifting resources will reduce the project duration by 2 weeks (the longest non-critical path being of 38-week duration). Hence, a net saving of Rs 1,000 would result.

(b) *H* is a critical activity. A reduction of 3 weeks would cause the project duration equal to 38 weeks. Hence, no change in cost would take place as the additional cost and cost-saving would match.

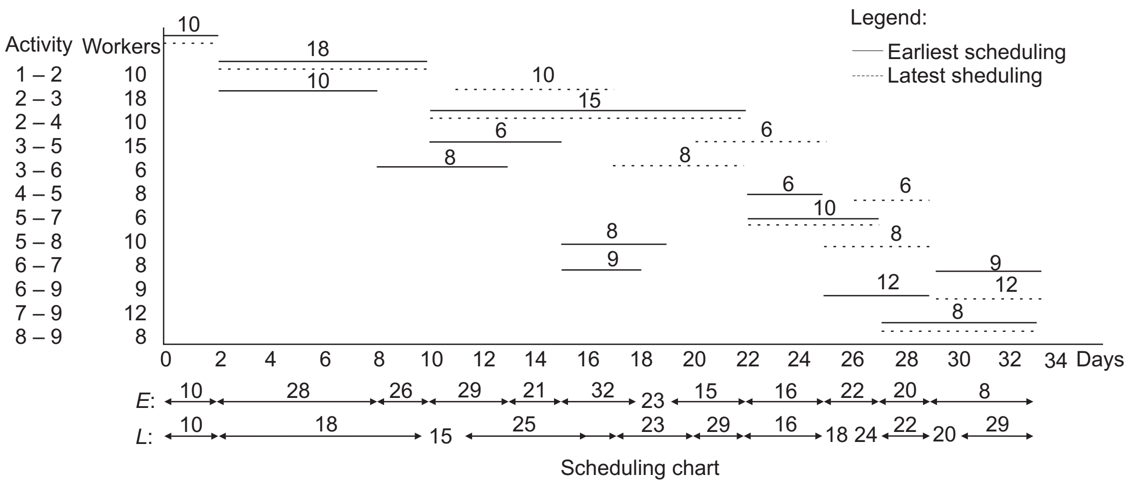
(c) Activity *G* is non-critical. Therefore, the proposal is unacceptable as no time saving would result.

(d) Reduction of 3 weeks in the time of activity *L* would reduce the duration of the project by 2 weeks while increase in *K*'s time would not affect the project duration. Thus, a saving of Rs 1,000 will result from the proposal.

24.

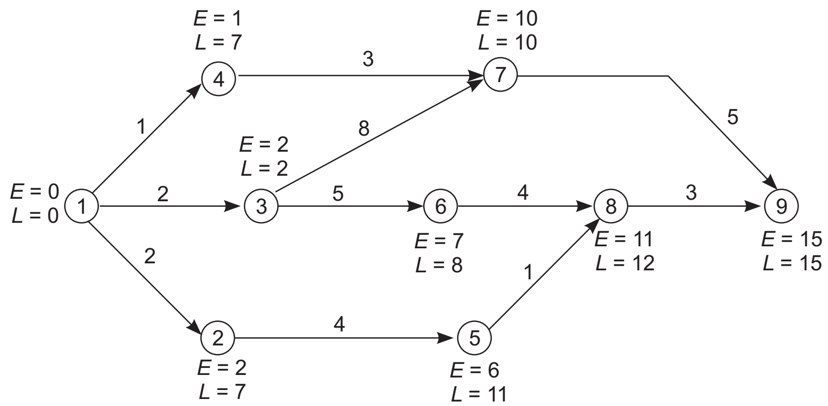


On the basis of the earliest and latest scheduling times shown in the network, the scheduling chart is drawn here. Manpower requirement on a day-to-day basis is indicated on the lower part of the chart.



Scheduling chart

25. (i) The project network is shown below.

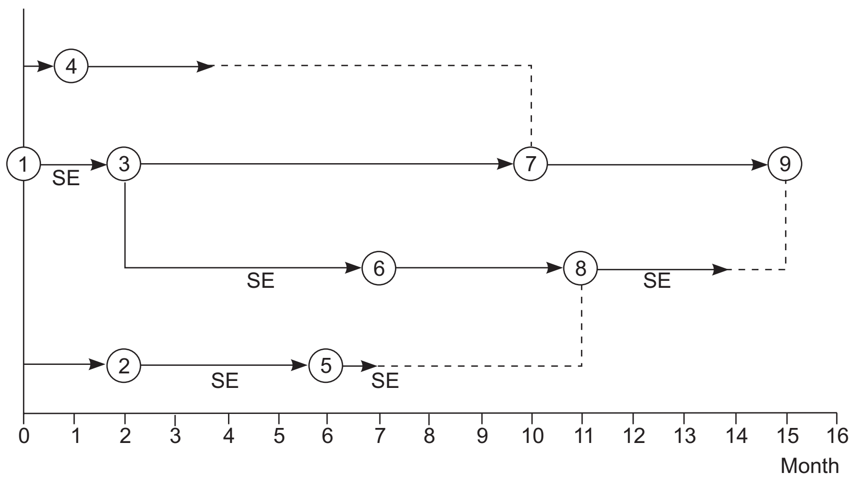


Network

(ii) From the network, we observe that the critical path is 1-3-7-9 and the project duration is 15 months.

(iii) The network given is redrawn on a time scale as follows. For the network, various paths and their lengths are:

Path	*Length*
1-3-7-9	15 months
1-3-6-8-9	14 months
1-2-5-8-9	10 months
1-4-7-9	9 months



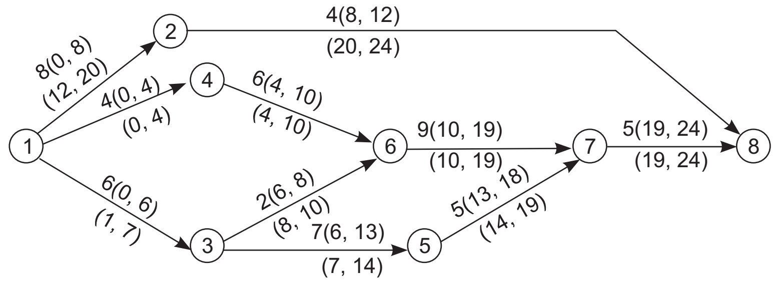
Network (Time-scaled)

It is given that the special equipment (SE) is required on activities 1-3, 3-6, 2-5, 5-8, and 8-9. Activities on paths 1-4-7-9 and 1-3-7-9 will continue as per schedule. The SE is required from the third month onwards on activities 3-6 and 2-5. Since a float of four months is available on path 1-2-5-8, we shift 2-5 by five months so that activity 3-6 may be completed within that time.

As a result of shifting 2-5 by five months, it would start in eighth month and completed by end of eleventh month. The following activity, 5-8, would be scheduled in the twelfth month. Finally, the SE would be employed on activity 8-9 that would begin in thirteenth month.

It is evident from above that the project would not be delayed by this rescheduling of activities.

26.



From the scheduling times, we determine:

Apply crane in this order (i) 4-6; days 4-10 No delay

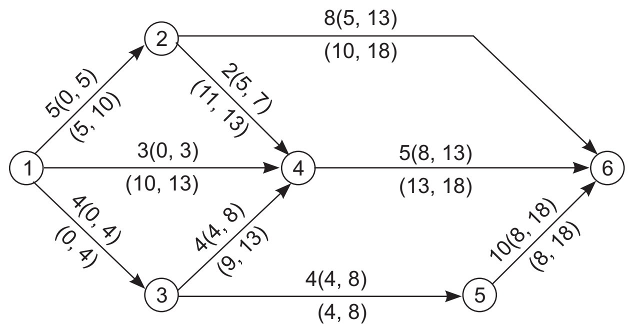
(ii) 3-6; days 10-12 Delay 2 days beyond LS

(iii) 5-17; days 13-18 or 14-19 No delay

(iv) 2-8; days to start: 18, 19, 20 No delay

Result: Delay in project competition = 2 days

27. The network for the given project is shown in the following figure.



Network

With unlimited resources, the project can be done in 18 days. However, when the resources are limited, as is the case here, the project may take longer than this. Thus, we will allocate the given resources to see how long the project will take to complete. This is done below.

When 8 workers are employed:

Halt 1 T = 0

EAS	:	1-2*	1-3*	1-4
ES	:	0	0	0
LS	:	5	0	0
OAS	:	1-3	1-2	1-4

Schedule : 1-3 for 4 days. No. of workers: 3

1-2 for 5 days. No. of workers: 4

Halt 2 T = 4

EAS	:	1-4*	2-4	2-6	3-4	3-5
ES	:	4	5	5	5	5
LS	:	10				
OAS	:	1-4				

Schedule : 1-4 for 3 days. No. of workers: 2

Halt 3 T = 5

EAS	:	2-4	2-6	3-4	3-5*
ES	:	5	5	5	5
LS	:	11	10	9	4
OAS	:	3-5	3-4	2-6	2-4

Schedule : 3-5 for 4 days. No. of workers: 5

Halt 4 T = 7

EAS	:	2-4	2-6*	3-4	5-6
ES	:	9	7	9	9
LS	:		10		
OAS	:	2-6			

Schedule : 2-6 for 8 days. No. of workers: 3

Halt 5 T = 9

EAS	:	2-4	3-4	5-6*
ES	:	15	9	9
LS	:		9	8
OAS	:	5-6	3-4	

Schedule : 5-6 for 10 days. No. of workers: 5

Halt 6 T = 19

EAS	:	2-4	3-4*
ES	:	19	19
LS	:	11	9
OAS	:	3-4	2-4

Schedule : 3-4 for 4 days. No. of workers: 5

Halt 7 T = 23

EAS : 2-4*

Schedule : 2-4 for 2 days. No. of workers: 6

Halt 8 T = 25

EAS : 4-6*

Schedule : 4-6 for 5 days. No. of workers: 3

Thus, project will complete in 30 days, as shown in part (a) of the following figure.

When 9 workers are employed:

Halt 1 T = 0

EAS	:	1-2*	1-3*	1-4*
ES	:	0	0	0
LS	:	5	0	10
OAS	:	1-3	1-2	1-4

Schedule : 1-3 for 4 days. No. of workers: 3
1-2 for 4 days. No. of workers: 4
1-4 for 4 days. No. of workers: 2

Halt 2 T = 4

EAS	:	2-4	2-6	3-4	3-5*
ES	:	5	4	4	4
LS	:		10	9	4
OAS	:	3-5	2-4	2-6	

Schedule : 3-5 for 4 days. No. of workers: 5

Halt 3 T = 5

EAS :	2-4	2-6*	3-4	5-6
ES :	8	5	8	8
LS :		10		
OAS :	2-6			

Schedule : 2-6 for 8 days. No. of workers: 3

Halt 4 T = 8

EAS :	2-4	3-4	5-6*
ES :	8	8	8
LS :	11	9	8
OAS :	5-6	3-4	2-4

Schedule : 5-6 for 10 days. No. of workers: 5

Halt 5 T = 18

EAS :	2-4	3-4*
ES :	18	18
LS :	11	9
OAS :	3-4	2-4

Schedule : 3-4 for 4 days. No. of workers: 5

Halt *6 T* = 22

EAS : 2-4*

Schedule : 2-4 for 2 days. No. of workers: 6

Halt 7 T = 24

EAS : 4-6*

Schedule : 4-6 for 5 days. No. of workers: 3

As shown in part (b) of the figure, the project would complete in 29 days, when 9 workers are employed.

When 10 workers are employed:

Halt 1 T = 0

EAS :	1-2*	1-3*	1-4*
ES :	0	0	0
LS :	5	0	0
OAS :	1-3	1-2	1-4

Schedule : 1-3 for 4 days. No. of workers: 3
1-2 for 5 days. No. of workers: 4
1-4 for 3 days. No. of workers: 2

Halt 2 T = 4

EAS :	2-4	2-6	3-4	3-5*
ES :	4	4	4	4
LS :	11	10	9	4
OAS :	3-5	3-4	2-6	2-4

Schedule : 3-5 for 4 days. No. of workers: 5

Halt *3 T = 5*

EAS :	2-4	2-6	3-4*	5-6
ES :	9	5	5	8
LS :		10	9	
OAS :	3-4	2-6		

Schedule : 3-4 for 4 days. No. of workers: 5

Halt 4 T = 8

EAS :	2-4	2-6	5-6*
ES :	9	8	8
LS :		10	8
OAS :	5-6	2-6	

Schedule : 5-6 for 10 days. No. of workers: 5

Halt 5 T = 9

EAS :	2-4	2-6*
ES :	18	9
LS :		10
OAS :	2-6	

Schedule : 2-6 for 8 days. No. of workers: 3

Halt 6 T = 18

EAS : 2-4

Schedule : 2-4 for 2 days. No. of workers: 6

Halt 7 T = 20

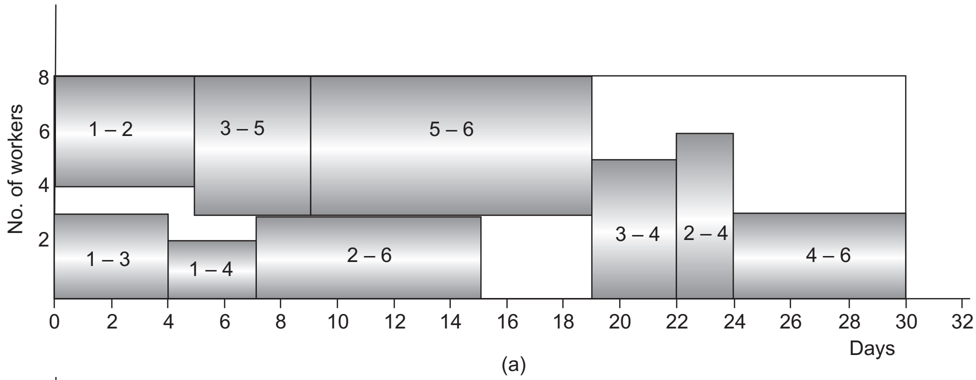
EAS : 4-6

Schedule : 4-6 for 5 days. No. of workers: 3

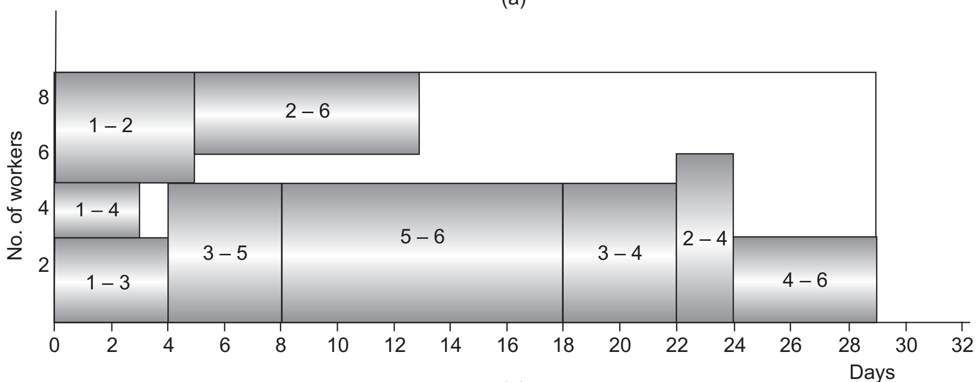
The loading chart, part (c) in the figure, shows the scheduling. From this, it is clear that the project would be completed in 25 days when 10 workers are employed. We can now work out the total cost of completing the project as follows:

No. of workers	*Duration*	*Labour cost*	*Overhead*	*Total cost*
8	30	1,920	1,500	3,420
9	29	2,088	1,450	3,538
10	25	2,000	1,250	3,250

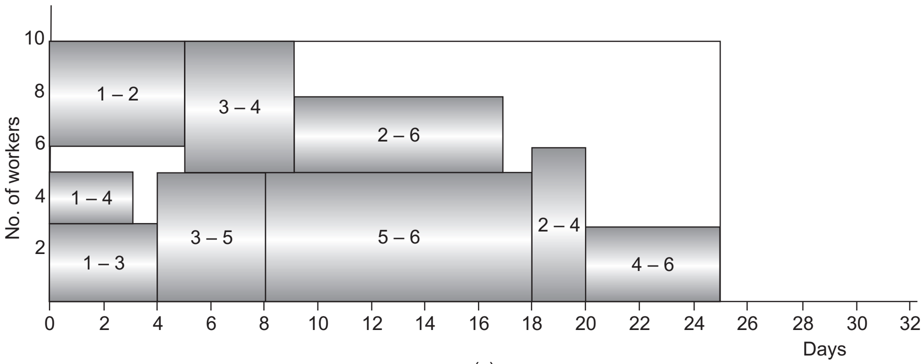
Thus, (a) the optimal number of workers to be employed is 10, and (b) the project duration is 25 days.



(a)



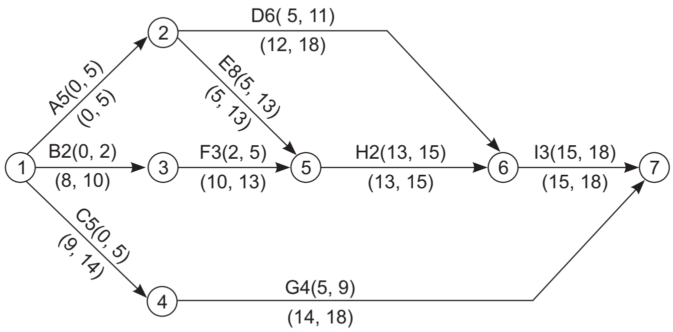
(b)



(c)

Loading Chart

28. The project network is shown in the figure. Also given are the earliest and the latest scheduling times of the activities.



Project Network

In accordance with principles stated earlier, the scheduling of activities follows:

Halt 1 T = 0

EAS	:	1-2*	1-3*	1-4	2-5	2-6	3-5
ES	:	0	0	0			
LS	:	0	8	9			
OAS	:	1-3	1-3	1-4			

Schedule : 1-2 for 5 days, and 1-3 for 2 days.

Resources : 30 workers and M_1 for 1-2, 20 workers and M_2 for 1-3.

Halt 2 T = 2

EAS	:	1-4	2-5	2-6	3-5*
ES	:	5	5	5	2
LS	:	9	5	12	10
OAS	:	3-5			

Schedule : 3-5 for 3 days. Resources: 20 workers and M_3.

Halt 3 T = 5

EAS	:	1-4	2-5*	2-6*	5-6
ES	:	5	5	5	
LS	:	9	5	12	
OAS	:	2-5	1-4	2-6	

Schedule : 2-5 for 8 days. Resources: 20 workers and M_1.
2-6 for 6 days. Resources: 20 workers and M_2.

Halt 4 T = 11

EAS	:	1-4	5-6*	6-7
ES	:	13	11	
LS	:	9	13	
OAS	:	5-6		

Schedule : 5-6 for 2 days. Resources: 20 workers and M_2.

Halt 5 T = 13

EAS	:	1-4*	6-7*	4-7
ES	:	13	13	
LS	:	9	15	
OAS	:	1-4	6-7	

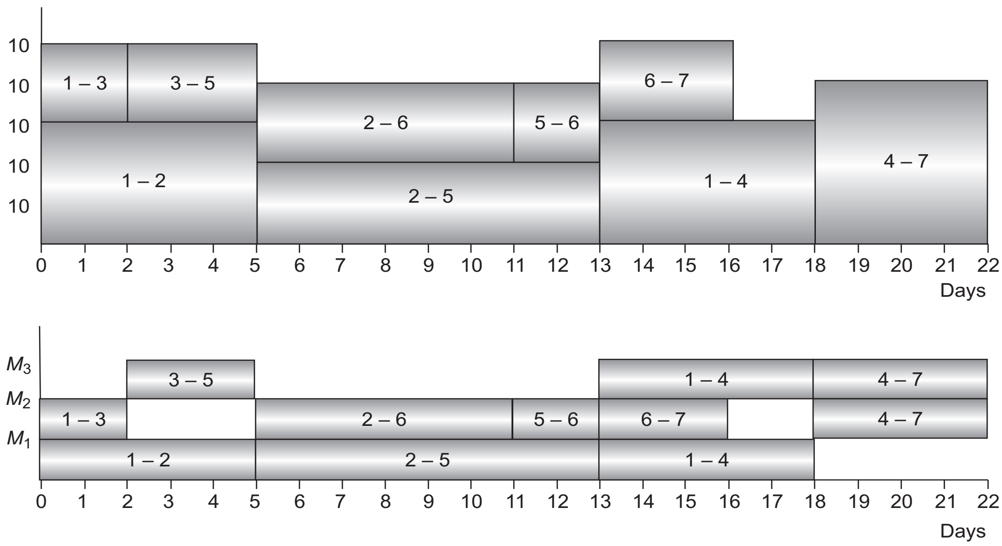
Schedule : 1-4 for 5 days. Resources: 30 workers, M_1 and M_3.
6-7 for 3 days. Resources: 20 workers and M_2.

Halt 6 T = 18

EAS : 4-7

Schedule : 4-7 for 4 days, Resources: 40 workers, M_2 and M_3.

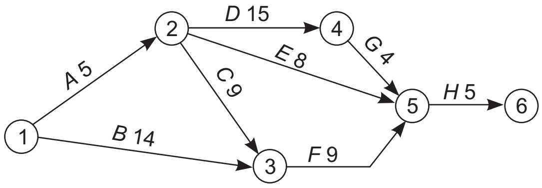
The loading chart is given below. It is clear that the project duration is 22 days.



Loading Chart

29. (i) The expected duration and variance for each of the activities are:

Activity	:	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
t_e	:	5	14	9	15	8	9	4	5
σ^2	:	$\frac{25}{9}$	$\frac{64}{9}$	$\frac{1}{9}$	$\frac{25}{9}$	$\frac{4}{9}$	0	$\frac{4}{9}$	0



Path	*Length*
1-2-4-5-6	29
1-2-5-6	18
1-2-3-5-6	28
1-3-5-6	28

(ii) Critical Path: 1-2-4-5-6 Activities: *A*, *D*, *G*, *H*
Expected duration: 29 weeks

Project variance: $\frac{25}{9} + \frac{25}{9} + \frac{4}{9} + 0 = 6$

Standard Deviation = $\sqrt{6}$ = 2.45 weeks

(iii) The required probability is given by area under the normal curve to the left of $X = 30$, when $\mu = 29$ and $\sigma = 2.45$.

$$Z = \frac{30 - 29}{2.45} = 0.41 \quad \text{Area} = 0.1591$$

∴ Required area = 0.5 + 0.1591 = 0.6591

30. (a) We have,

$$\text{Expected time} = \frac{a + 4m + b}{6}$$

Thus, $$15 = \frac{9.5 + 4m + b}{6}$$

or $$4m + b = 80.5 \qquad \text{(i)}$$

Also,

$$\left(\frac{b - a}{6}\right)^2 = 6.25$$

or $$\left(\frac{b - 9.5}{6}\right)^2 = 6.25$$

or $$b = 6 \times 2.25 + 9.5 = 24.5 \qquad \text{(ii)}$$

From equations (i) and (ii),

$$m = 14 \text{ and } b = 24.5$$

(b) Using the given data,

$$\mu = 12 + 3 + 8 + 7 + 5 + 6 = 41 \text{ weeks}$$

$$\sigma^2 = (2/3)^2 + (1/3)^2 + 2^2 + (5/3)^2 + (4/3)^2 = 82/9 \text{ or } 9.111$$

∴ $$\sigma = \sqrt{9.111} = 3.018 \text{ weeks}$$

(i) $Z = \frac{45 - 41}{3.018} = 1.33$

Area to the left of $Z = 1.33$ is 0.5 + 0.4075 = 0.9075. This is the required probability.

(ii) For Area (0.5 – 0.1) = 0.40, the Z-value is 1.28. Thus,

$$1.28 = \frac{X - 41}{3.018}$$

$\therefore \quad X = 1.28 \times 3.018 + 41 = 44.86$ weeks or 44 weeks and 6 days

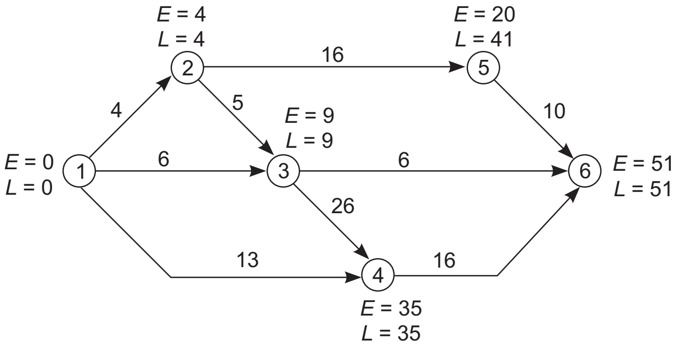
31. The expected time and variance for each activity is first calculated as:

$$t_e = \frac{o + 4m + p}{6} \quad \text{and} \quad \sigma^2 = \left(\frac{p - o}{6}\right)^2$$

These are given here:

Activity	:	1-2	1-3	1-4	2-3	2-5	3-4	3-6	4-6	5-6
t_e	:	4	6	13	5	16	26	6	16	10
σ^2	:	4/9	0	9	1	64/9	25	1	9	4

(i) The network is shown below and critical path is obtained by using expected times. Also given are the earliest and latest event times for nodes.

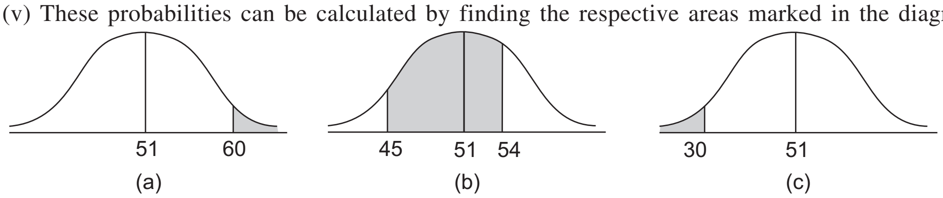


Network

The critical path is 1-2-3-4-6.

(ii) The expected duration of the project is 51 days. The critical activities being 1-2, 2-3, 3-4, and 4-6, the variance = 4/9 + 1 + 25 + 9 = 35.44. Thus, standard deviation = $\sqrt{35.44}$ = 5.95 days.

(iii) (iv) (v) These probabilities can be calculated by finding the respective areas marked in the diagram.



Calculation of Areas

In part (a), $Z = \frac{60 - 51}{5.95} = 1.51$ Area for $Z = 1.51$ is 0.4345

$\therefore$ Required area = 0.5 – 0.4345 = 0.0655

For part (b), $Z_1 = \frac{45 - 51}{5.95} = -1.01$ Area for Z = 1.01 is 0.3438

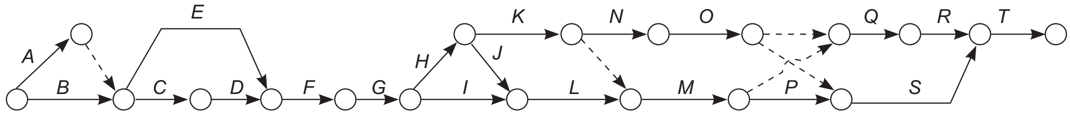
$Z_2 = \frac{54 - 51}{5.95} = 0.50$ Area for $Z = 0.50$ is 0.1915

$\therefore$ Required area = 0.5353

For part (c), $Z = \frac{30 - 51}{5.95} = -3.53$ Area for $Z = 3.53$ is 0.4998

$\therefore$ Required area = 0.5 – 0.4998
= 0.0002

32. (a)



Network Diagram

(b) Calculation of Mean and Variance for Activities

Activity	*Expected Time*	*Variance*	*Activity*	*Expected Time*	*Variance*
A	13/3	1	*K*	28/3	16/9
B	2	1/9	*L*	13/3	4/9
C	31/6	25/36	*M*	25/6	1/4
D	13/6	1/36	*N*	19/6	1/4
E	13/3	4/9	*O*	37/6	25/36
F	37/6	25/36	*P*	13/6	1/36
G	38/3	4	*Q*	11/2	49/36
H	19/6	1/4	*R*	4	4/9
I	13/6	1/36	*S*	22/3	16/9
J	49/6	49/36	*T*	3	1/9

Critical activities are: *A*, *C*, *D*, *F*, *G*, *H*, *K*, *N*, *O*, *Q*, *R*, *T*

(c) $\therefore$ Expected duration of the project = $\frac{13}{3} + \frac{31}{6} + \frac{13}{6} + \frac{37}{6} + \frac{38}{3} + \frac{19}{6} + \frac{28}{3} + \frac{19}{6} + \frac{37}{6} + \frac{11}{2} + 4 + 3$

$= 64\frac{5}{6}$ or 64.8333

(d) Variance, $\sigma^2 = 1 + \frac{25}{36} + \frac{1}{36} + \frac{25}{36} + 4 + \frac{1}{4} + \frac{16}{9} + \frac{1}{4} + \frac{25}{36} + \frac{49}{36} + \frac{4}{9} + \frac{1}{9} = 11\frac{11}{36}$ or 11.3056

$\therefore \sigma = \sqrt{11.3056} = 3.3624$

Thus, expected duration of the project is 64.8333 weeks with a standard deviation of 3.3624 weeks.

(e) For $X = 52$,

$$Z = \frac{52 - 64.8333}{3.3624} = -3.82$$

Area of the left of $Z = -3.82$ is 0.5 – 0.4999 = 0.0001. This is the probability that the project will be completed in 52 weeks.

(f) Probability that the project will be completed in 65 weeks is given by area to the left of $X = 65$.

$$Z = \frac{65 - 64.8333}{3.3624} = 0.05 \quad \text{Area} = 0.0199$$

$\therefore$ $P(X < 65) = 0.5 + 0.0199 = 0.5199$

(g) Probability of not completing the project within 70 weeks is given by the area to the right of $X = 70$.

$$Z = \frac{70 - 64.8333}{3.3624} = 1.54 \quad \text{Area} = 0.4382$$

$\therefore$ $P(X > 70) = 0.5 - 0.4382 = 0.0618$

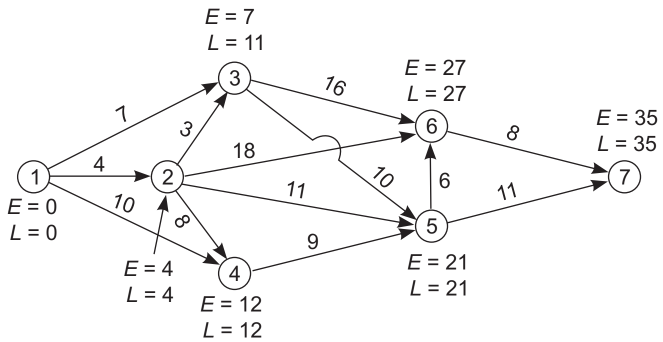
33. (a) Here a = 10 minutes, b = 60 minutes and m = 20 minutes. Accordingly,

(i) Expected duration = $\frac{10 + 4 \times 20 + 60}{6}$ = 25 minutes

(ii) Variance = $\left[\frac{60 - 10}{6}\right]^2$ = 69.44 minutes^2

(iii) Scheduling the project would need 25 minutes for this activity.

(b)



(i)

Activity	*Duration*	*ES*	*EF*	*LS*	*LF*	*Total Float*	*Free Float*
1-2	4	0	4	0	4	0	0
1-3	7	0	7	4	11	4	0
1-4	10	0	10	2	12	2	2
2-3	3	4	7	8	11	4	0
2-4	8	4	12	4	12	0	0
2-5	11	4	15	10	21	6	6
2-6	18	4	22	9	27	5	5
3-5	10	7	17	11	21	4	4
3-6	16	7	23	11	27	4	4
4-5	9	12	21	12	21	0	0
5-6	6	21	27	21	27	0	0
5-7	11	21	32	24	35	3	3
6-7	8	27	35	27	35	0	0

(ii) The critical path is 1-2-4-5-6-7 with the project duration of 35 days.

(iii) Activity 2-6 is not a critical activity. Hence, speeding it up would have no bearing on the project duration. On the other hand, the activity 4-5 lies on the critical path. This being a critical activity, speeding it up by 2 days would reduce the critical path length, and hence the project duration, by an equal amount.

(c) Expected duration of the project, μ = 35 days
Variance along the critical path, σ^2 = 81 days^2 (given)

$\therefore \sigma = \sqrt{81}$ = 9 days.

The probability of completing the project within 33 days is given by the area under the normal curve (with $\mu = 35$ and $\sigma = 9$) to the left of $X = 33$. Thus,

$$Z = \frac{X - \mu}{\sigma} = \frac{33 - 35}{9}$$
$$= -0.22$$

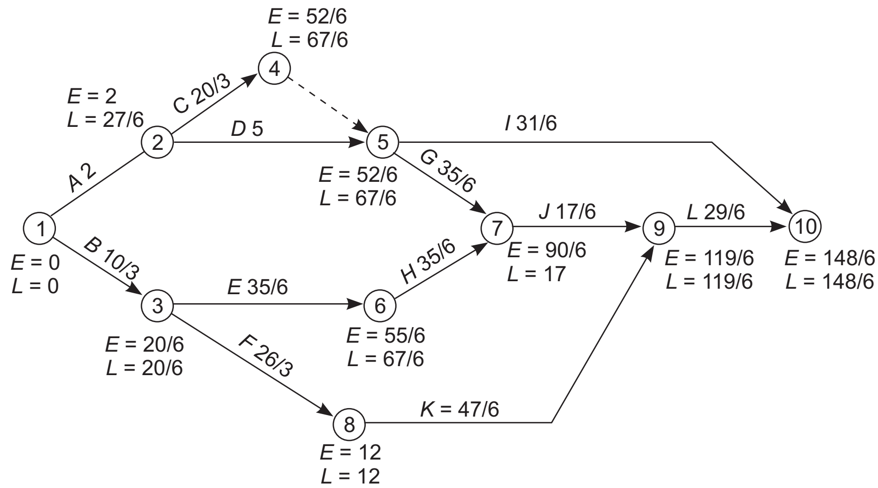
From the normal area table, area corresponding to $Z = 0.22$ is 0.0871. Accordingly, the required probability is 0.5 – 0.0871 = 0.4129.

For probability of completing the project in 44 days,

$$Z = \frac{44 - 35}{9} = 1.00 \qquad \text{Area} = 0.3413$$

$$P(X \leq 44) = 0.5 + 0.3413 = 0.8413$$

34.



Activity		*a*	*m*	*b*	*te*	σ^2	*ES*	*EF*	*LS*	*LF*	*Total stock*
A	1-2	2	2	2	2	0	0	2	15/6	27/6	15/6
B	1-3	1	3	7	20/6	1	0	20/6	0	20/6	0
C	2-4	4	7	8	40/6	4/9	2	52/6	27/6	67/6	15/6
D	2-5	3	5	7	5	4/9	2	7	37/6	67/6	25/6
E	3-6	2	6	9	35/6	49/36	20/6	55/6	32/6	67/6	2
F	3-8	5	9	11	52/6	1	20/6	72/6	20/6	12	0
G	5-7	3	6	8	35/6	25/36	52/6	87/6	67/6	17	15/6
H	6-7	2	6	9	35/6	49/36	55/6	90/6	67/6	17	2
I	5-10	3	5	8	31/6	25/36	52/6	83/6	117/6	148/6	65/6
J	7-9	1	3	4	17/6	9/36	15	107/6	17	119/6	2
K	8-9	4	8	11	47/6	49/36	12	119/6	12	119/6	0
L	9-10	2	5	7	29/6	25/36	119/6	148/6	119/6	148/6	0

Critical path : 1-3-8-9-10 (*B-F-K-L*)

Expected completion time = 148/6 = 24.67 days

Variance = 146/36 $\quad\therefore\ \sigma = \sqrt{146/36} = 2.014$ days

If X number of days give a 99% probability of completion, we have

$$2.33 = \frac{X - 24.67}{2.014}$$

Thus,

$$X = 2.33 \times 2.014 + 24.67$$
$$= 29.36 \simeq 30 \text{ days}$$

35. (a) Here $\mu = 21$ months, $\sigma = 2$ months.

(i) $P(X > 22)$ is given by area under the normal curve to the right of $X = 22$.

$$Z = \frac{22 - 21}{2} = 0.5$$

For $Z = 0.5$, the area is 0.1915. Accordingly, area to the right = 0.5 – 0.1915 = 0.3085.

(ii) The probability of the project being completed in the 24th month is given by the area included between $X = 23$ and $X = 24$.

For $X = 23$, Area

$$Z = \frac{23 - 21}{2} = 1.00 \qquad 0.3413$$

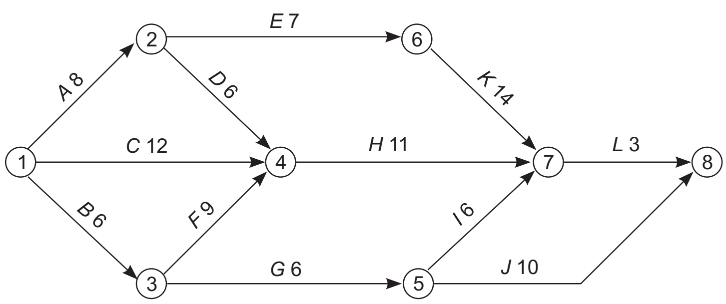
For $X = 24$,

$$Z = \frac{24 - 21}{2} = 1.50 \qquad 0.4332$$

Thus, required area = 0.4332 – 0.3413 = 0.0919

(b) (i) The expected time and standard deviation of each of the activities of the project are given here:

Activity	*te*	σ	*Activity*	*te*	σ
A	8	2	*G*	6	4/3
B	6	2/3	*H*	11	1/3
C	12	7/3	*I*	6	2/3
D	6	0	*J*	10	0
E	7	2/3	*K*	14	4/3
F	9	3/2	*L*	3	2/3



(ii) Critical path 1-2-6-7-8 (*A E K L*)

Expected duration : 8 + 7 + 14 + 3 = 32 weeks

Variance, $\sigma^2 = 2^2 + (2/3)^2 + (4/3)^2 + (2/3)^2 = 20/3$ weeks^2

$\therefore$ Standard deviation, $\sigma = \sqrt{20/3} = 2.582$ weeks

(iii) The area under the normal curve, with $\mu = 32$ and $\sigma = 2.582$, to the left of $X = 38$ gives the desired probability. Here,

$$Z = \frac{38 - 32}{2.582} = 2.32 \qquad \frac{\text{Area}}{0.4898} \text{ (From Table)}$$

$\therefore$ Total area to the left of $X = 0.5 + 0.4898 = 0.9898$

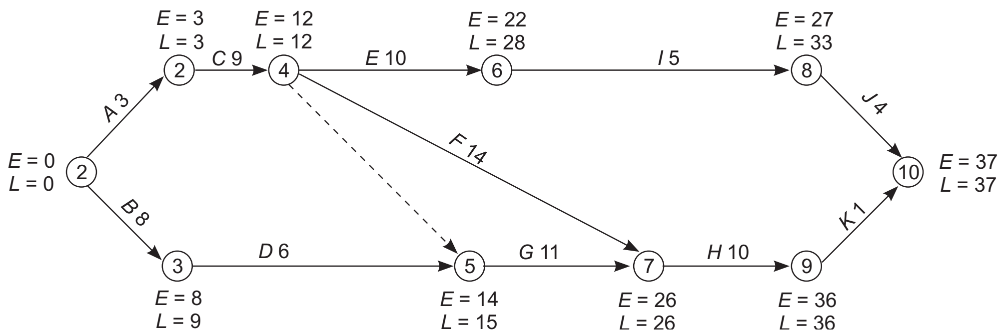
(iv) For 90 per cent probability, area between mean and X is 40% or 0.40. Corresponding to this, the Z value is 1.28. Thus,

$$1.28 = \frac{X - 32}{2.582}$$

or $\quad X = 1.28 \times 2.582 + 32$

$\quad\quad = 35.3$ weeks or 35 weeks 2 days.

36. (i) The PERT network is drawn in the following figure.



PERT Network

The critical path is 1-2-4-7-9-10, involving activities *A*, *C*, *F*, *H*, and *K*. It is obtained using expected times as shown calculated in table. The table also gives variances for the activities, the earliest and latest scheduling times based on expected durations, and the expected float associated.

Expected Durations, Variances, and Floats

Activity	*a*	*b*	*m*	*Expected Time*	σ^2	*Earliest*		*Latest*		*Total Float*
						Start	*Finish*	*Start*	*Finish*	
A	2	4	3	3	1/9	0	3	0	3	0
B	8	8	8	8	0	0	8	1	9	1
C	7	11	9	9	4/9	3	12	3	12	0
D	6	6	6	6	0	8	14	9	15	1
E	9	11	10	10	1/9	12	22	18	28	6
F	10	18	14	14	16/9	12	26	12	26	0
G	11	11	11	11	0	14	25	15	26	1
H	6	14	10	10	16/9	26	36	26	36	0
I	4	6	5	5	1/9	22	27	28	33	6
J	3	5	4	4	1/9	27	31	33	37	6
K	1	1	1	1	0	36	37	36	37	0

(ii) This project has expected completion time equal to 37 weeks with a standard deviation $= \sqrt{(1/9 + 4/9 + 16/9 + 16/9 + 0)}$ = 2.028 weeks. In order to calculate the probability that a maximum penalty of Rs 15,000 would be payable, we need to compute the chances that the project would be completed within 40 weeks (Since (40 – 37) × 5,000 = 15,000). For this, we have

$$Z = \frac{40 - 37}{2.028} = 1.48$$

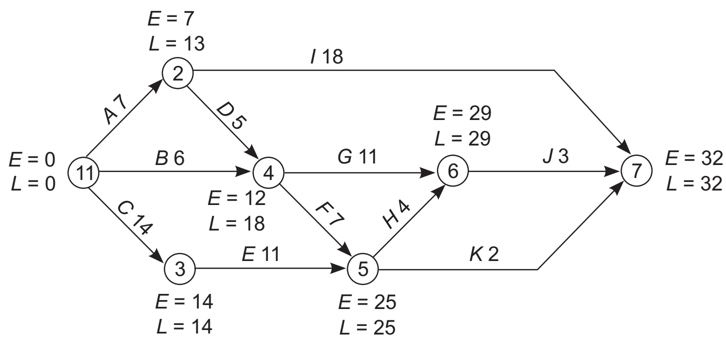
From the Normal Area Table, the area for Z = 1.48 is obtained as 0.4306. Thus, the required probability = 0.5 + 0.4306 = 0.9306.

37. **Calculation of Expected Times and Variances**

Activity	*a*	*m*	*b*	*Expected Time*	*Variance*	
A	3	6	15	7	4	
B	2	5	14	6	4	
C	6	12	30	14	16	*C*
D	2	5	8	5	1	
E	5	11	17	11	4	*C*
F	3	6	15	7	4	
G	3	9	27	11	16	
H	1	4	7	4	1	*C*
I	4	19	28	18	16	
J	1	2	9	3	16/9	*C*
K	2	4	12	5	25/9	

The network diagram is shown on next page. The critical path is obtained as 1-3-5-6-7, comprising activities *C*, *E*, *H*, *J*. Also given are the earliest and the latest event times, *E* and *L*, in the diagram. They are all calculated using expected times.

The project has an expected completion time of 32 days and variance = 16 + 4 + 1 + 16/9 = 22.778. The standard deviation = $\sqrt{22.778}$ = 4.77 days.



Network Diagram

To calculate the probability that the project will be completed within two days later than expected, we find area under normal curve to the left of $X = 34$. Thus,

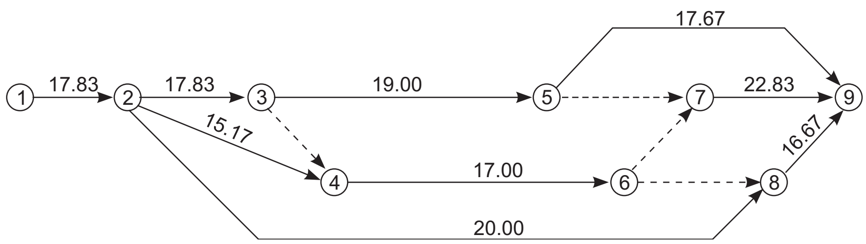
$$Z = \frac{34 - 32}{4.77} = 0.42$$

For $Z = 0.42$, the area is given as 0.1628. Thus, the total area to left of $X = 34$ is $0.5 + 0.1628 = 0.6628$, which is the desired probability.

38. **Calculation of Expected Duration and Variances**

Activity	*Duration*			*Expected Duration*	*Variance*	
	a	*m*	*b*			
1-2	14	17	25	17.83	3.36	*C*
2-3	14	18	21	17.83	1.36	*C*
2-4	13	15	18	15.17	0.69	
2-8	16	19	28	20.00	4.00	
3-4	—	—	—	—	—	
3-5	15	18	27	19.00	4.00	*C*
4-6	13	17	21	17.00	1.78	
5-7	—	—	—	—	—	*C*
5-9	14	18	20	17.67	1.00	
6-7	—	—	—	—	—	
6-8	—	—	—	—	—	
7-9	16	20	41	22.83	17.36	*C*
8-9	14	16	22	16.67	1.78	

The PERT network is shown below and critical path is found there from using expected times for various activities.



Network Diagram

The various paths and their lengths are:

Path	*Length*	*Path*	*Length*
1-2-3-5-7-9	77.49	1-2-8-9	54.50
1-2-3-5-9	72.33	1-2-4-6-8-9	66.67
1-2-3-4-6-7-9	75.49	1-2-4-6-7-9	72.83
1-2-3-4-6-8-9	69.33		

Thus, critical path is 1-2-3-5-7-9 with project duration expected to be 77.49 days. The summation variances of critical activities gives 26.08. Thus,

Expected project duration, μ = 77.49, and standard deviation, $\sigma = \sqrt{26.08}$ = 5.12 days.

We now determine within how many days should the project be completed so as to break-even was a 95% probability. We have, $Z(0.95)$ = 1.65. Thus,

$$1.65 = \frac{X - \mu}{\sigma} = \frac{X - 77.49}{5.12}$$

or $\quad X = 1.65 \times 5.12 + 77.49 = 85.9$ or 86 days

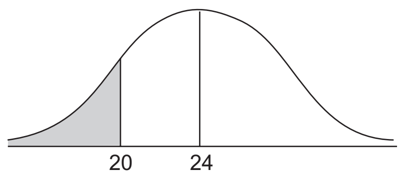
The fixed cost of the project being Rs 8,00,000 and the variable cost being Rs 9,000 per day, the amount to bid is calculated below:

Bid amount = Rs 8,00,000 + Rs 9,000 × 86
= Rs 15,74,000

39. (a) With $\mu = 24$ and $\sigma = \sqrt{9}$ = 3, the probability of completing the project in 20 months is given by the area under normal curve as shown in figure.

Now, $$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{20 - 24}{3} = 1.33$$



Normal Curve

The area corresponding to Z = 1.33 is 0.4082. Thus, the required probability = 0.5 – 0.4082 = 0.0918.

Further, let the required time in which the work be completed with 0.90 probability be X. Since the area between μ and X is 0.40, the Z-value corresponding to this area is 1.28. Accordingly,

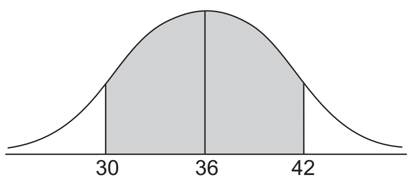
$$1.28 = \frac{X - \mu}{\sigma}$$

or $\quad X = 1.28 \times 3 + 24$
$\quad = 27.84$ months or 2 years 3 months and 25 days

(b) The require probability is given by the area under the normal curve (with $\mu = 36$ and $\sigma = 6$) between $X = 30$ and $X = 42$. This is as shown in figure. We have, now

$$Z_1 = \frac{30 - 36}{6} = -1.00$$

$$Z_2 = \frac{42 - 36}{6} = 1.00$$



Normal Curve

Area corresponding to Z = 1.00 is 0.3413. Thus, the required area = 0.3413 + 0.3413 = 0.6826.

(c) With μ = 42 and $\sigma = \sqrt{36}$ = 6, we have

$$Z_1 = \frac{36-42}{6} = -1.00 \text{ and } Z_2 = \frac{48-42}{6} = 1.00.$$

Since area included in the range $\mu \pm 1\ \sigma$ is about 68%, option (ii) is the correct answer.

(d) Statement (ii) is correct.

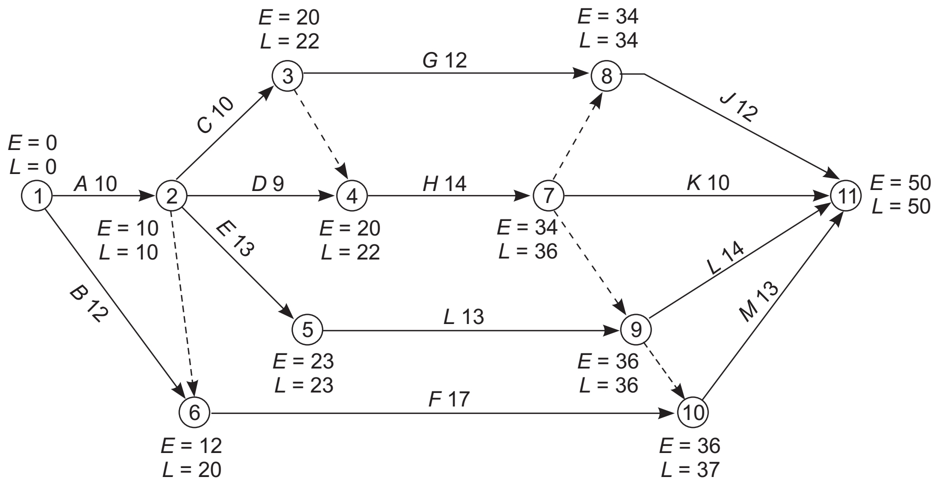
(e) If a and b be the optimistic and pessimistic times respectively, we have

$$\frac{19}{3} = \frac{a+b+4\times 6}{6} \text{ and } \frac{b-a}{6} = 1.$$

Solving these two equations, we get a= 4 and b = 10.

(f) Substituting the known values in the expressions to calculate expected time and standard deviation, and then solving for a and b, we get a = 7 and b = 31. Hence, option (ii) is correct.

40. (a) The network is shown below. The expected project duration is 50 days and the critical activities are



A, *E*, *I* and *L*.

Network

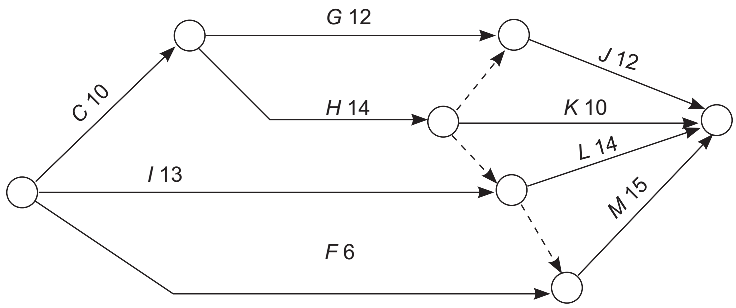
(b) The revised network is shown in the following figure. It may be mentioned that after fire (with 23 days gone), 37 days remain and the following tasks are to be done.

Tasks	*Immediate predecessors*	*Duration*
C	—	10
F	—	6 (remaining)
G	*C*	12
H	*C*	14
I	—	13
J	*G*, *H*	12
K	*H*	10
L	*H*, *I*	14
M	*H*, *I*, *F*	15 (revised)

(c) For the revised project, we have

Critical activity	*Duration*	*Standard deviation*	*Variance*
C	10	0.33	0.1089
H	14	1.33	1.7689
M	15	0.33	0.1089
	39		1.9867

Standard deviation = $\sqrt{1.9867}$ = 1.4095



Revised Network

Now, we have to find the probability of completing the total project in 60 days, which implies completing the revised project in 37 days. We have,

$$Z = \frac{37 - 39}{1.4095} = -1.42.$$ Area for $Z = 1.42$ is 0.4222.

$\therefore$ Required area = 0.5 – 0.4222
= 0.0778

CHAPTER 13

1. **Conditional Pay-off Matrix**

Estimated Sales (Units)	*Type of Shampoo*		
	Egg	*Clinic*	*Deluxe*
15,000	30	40	55
10,000	10	15	20
5,000	10	5	3
Minimum	10*	5	3
Maximum	30	40	55*
Average	50/3	20	78/3*

Conditional Regret Matrix

Estimated Sales (Units)	*Type of Shampoo*		
	Egg	*Clinic*	*Deluxe*
15,000	25	15	0
10,000	10	5	0
5,000	0	5	7
Maximum	25	15	7*

From the maxima, minima, and average values derived, we have

Criterion	*Decision*	*Pay-off*
Maximin	Egg shampoo	Rs 10 m
Maximax	Deluxe shampoo	Rs 55 m
Laplace	Deluxe shampoo	Rs 78/3 = 26 m
Regret	Deluxe ahampoo	Rs 7 m (Regret)

2. **Conditional Profit (in ’000 Rs)**

Product Acceptance	*Product Line*		
	Full	*Partial*	*Minimal*
Good	80	70	50
Fair	50	45	40
Poor	–25	–10	0
Maximum	80*	70	50
Minimum	–25	–10	0*
Average	35*	35*	30

Conditional Regret (in ’000 Rs)

Product Acceptance	*Product Line*		
	Full	*Partial*	*Minimal*
Good	0	10	30
Fair	0	5	10
Poor	25	10	0
Maximum	25	10*	30

Decision Rule	*Decision*	*Pay-off/Regret*
Maximax	Full	80
Maximin	Minimal	0
Laplace	Full or Partial	35
Minimax Regret	Partial	10

3. The conditional pay-off matrix is reproduced in the table. Given alongwith in parantheses are the regret values.

Event	*Prob.*	*Courses of Action* By Land	Obtain option	No action
Large reserves	0.2	40(0)	28(12)	0(40)
Minor reserves	0.5	10(0)	1(9)	0(10)
No oil	0.3	–25(25)	2(2)	0(0)
Average		8.33	9.00*	0
Minimum		–25	–2	0*
Maximum		40*	28	0
Max. Regret		25	12*	40
Expected Payoff		5.5*	5.5*	0

Decision Rule	*Decision*	*Payoff/Regret*
Laplace	Obtain option	9
Maximin	No action	0
Maximax	Buy land	40
Minimax Regret	Obtain option	12
Expected Payoff	Buy land/Obtain option	5.5

4. The pay-off matrix and the regret matrix based thereon are given below:

Pay-off and Regret Matrices

Event	*Pay-off for Action* A_1	A_2	A_3	*Regret for Action* A_1	A_2	A_3
E_1	250	100	200	0	150	50
E_2	250	125	300	50	175	0
E_3	250	625	450	375	0	175
Average	250	283.3	316.7	Max. 375	175	175
Minimum	250	100	200			

From this information, we have

Criterion	*Decision*
Laplace	A_3: as average pay-off is highest
Maximin	A_1: as highest of the minimum values is 250
Hurwicz	A_2: as it is the highest-value alternative with $\alpha = 0.5$
	A_1: $250 \times 0.5 + 250 \times 0.5 = 250$
	A_2: $625 \times 0.5 + 100 \times 0.5 = 362.5$
	A_3: $450 \times 0.5 + 200 \times 0.5 = 325$
Minimum regret (Minimax)	A_2 or A_3: as they both have the least maximum regret values.

5. **Pay-off Matrix**

State of Nature	*Course of Action*				
	a_1	a_2	a_3	a_4	a_5
S_1	26	22	13	22	18
S_2	26	22	34	30	20
S_3	18	22	18	18	20
S_4	22	22	18	18	18
(R_1) Max.	26	22	34	30	20
(R_2) Min.	18	22	13	18	18
(R_3) Average	23	22	20.75	22	19

(a) When payoffs are in terms of profit:

	Criterion	*Decision*	*Pay-off*
(i)	Maximax (R_1)	a_3	34
(ii)	Maximin (R_2)	a_2	22
(iii)	Laplace (R_3)	a_1	23

(b) When the payoffs represent costs:

(i)	Minimin (R_2)	a_3	13
(ii)	Minimax (R_1)	a_5	20
(iii)	Laplace (R_3)	a_5	19

6. Expected payoffs:

a_1 : $0.6 \times 0 + 0.1 \times 2 + 0.2 \times 5 \times 0.1(-4) = 0.8$

a_2 : $0.6 \times 3 + 0.1 \times 2 + 0.2(-1) + 0.1(-3) = 1.5$

a_3 : $0.6(-3) + 0.1 \times 2 + 0.2 \times 3 + 0.1 \times 1 = -0.9$

Expected Regrets:

a_1 : $0.6 \times 3 + 0.1 \times 0 + 0.2 \times 0 + 0.1 \times 5 = 2.3$

a_2 : $0.6 \times 0 + 0.1 \times 0 + 0.2 \times 6 + 0.1 \times 4 = 1.6$

a_3 : $0.6 \times 6 + 0.1 \times 0 + 0.2 \times 2 + 0.1 \times 0 = 4.0$

Best alternative under both the criteria is a_2.

7. It is assumed for solving this problem that all the demand levels are equally likely.

(a)

Demand	*Buy*	*Do not buy*
0	(5000)	0
1	(2800)	0
2	(600)	0
3	1600	0
4	3800	0
5	6000	0
Expected value	500	0

Decision: Buy

(b) **Conditional Payoff (Rs)**

Demand	*No. of machines to buy*					
	0	*1*	*2*	*3*	*4*	*5*
0	0	(1,000)	(2,000)	(3,000)	(4,000)	(5,000)
1	0	1,200	200	(800)	(1,800)	(2,800)
2	0	1,200	2,400	1,400	400	(600)
3	0	1,200	2,400	3,600	2,600	1,600
4	0	1,200	2,400	3,600	4,800	3,800
5	0	1,200	2,400	3,600	4,800	6,000
Expected Value	0	833.3	1,300	1,400	1,133.3	500

Decision: Buy 3 machines

8. The conditional opportunity loss values are shown in table. For each row of the pay-off matrix, the various values are subtracted from the largest value to obtain corresponding values in the opportunity loss table. The expected opportunity loss values for various strategies are also shown calculated. It is evident that Type I souvenir should be bought.

Conditional Opportunity Loss Table

Event	*Probability*	*Course of Actiion*		
		Type I	*Type II*	*Type III*
Team *A* wins	0.6	0	400	900
Team *B* wins	0.4	850	400	0
Expected loss		340	400	540

9. **Conditional Pay-off Matrix**

Demand	*Prob.*	*Production (Units)*		
		2000	*3000*	*6000*
2000	8/36	90,000	55,000	(50,000)
3000	16/36	90,000	135,000	30,000
6000	12/36	90,000	135,000	2,70,000
Expected Value		90,000	117,222	92,222

Optimal Policy: Produce 3,000 Units.

10. (a) The given frequencies are converted into probabilities by dividing each of them by 200—the total frequency. The expected demand, given by ΣpX, is equal to 205 loaves, is shown calculated in table here.

Calculation of Expected Demand

Daily Demand (X)	*Probability (p)*	*pX*
0	0.05	0
100	0.30	30
200	0.30	60
300	0.25	75
400	0.10	40
	Total	205

(b) From the given data, it is evident that
Profit on the sale of a loaf = Rs 10 – 5 = Rs 5
Loss on an unsold loaf = Rs 5 – 2 = Rs 3
With these values, the pay-off matrix is shown in table below.

Conditional Pay-off Matrix

Demand	*Prob.*	*Course of Action (Loaves)*				
		0	*100*	*200*	*300*	*400*
0	0.05	0	(300)	(600)	(900)	(1,200)
100	0.30	0	500	200	(100)	(400)
200	0.30	0	500	1,000	700	400
300	0.25	0	500	1,000	1,500	1,200
400	0.10	0	500	1,000	1,500	2,000
	Expected pay-off	0	475	8,600	660	440

(c) The expected profit arising from each level of production is given in last row of the table. The optimal policy is to produce 200 loaves.

11. **Conditional Pay-off Matrix**

Doses per week	*Prob.*	*No. of Doses*			
		20	*25*	*40*	*60*
20	0.10	800	700	400	0
25	0.30	800	1,000	700	300
40	0.50	800	1,000	1,600	1,200
60	0.10	800	1,000	1,600	2,400
	Expected value	800	970	1,210	930

Conditional Regret Matrix

Doses Per week	*Prob.*	*No. of Doses*			
		20	*25*	*40*	*60*
20	0.10	0	100	400	800
25	0.30	200	0	300	700
40	0.50	800	600	0	400
60	0.10	1,600	1,400	800	0
	Expected value	620	450	210	490

Optimum number of doses to buy = 40
Expected value of perfect information = Rs 210.

12. From the given information,
Profit on sale of a case = Rs 50 – Rs 20 = Rs 30
Loss on an unsold case = Rs 20 – 0 = Rs 20
On the basis of this information, the pay-off matrix is drawn for various strategies.

Conditional Pay-off Matrix

Sales (cases)	*Frequency*	*Prob.*	*Stock (cases)*			
			10	*11*	*12*	*13*
10	15	0.15	300	280	260	240
11	20	0.20	300	330	310	290
12	40	0.40	300	330	360	340
13	25	0.25	300	330	360	390
	Expected Value		300	322.5	335	327.5

The probability for each level of sales is calculated by dividing the given frequency by total frequency (= 100). The expected pay-offs for all strategies are shown calculated in the last row. Since the expected pay-off is maximum for 12, the optimal policy is to stock 12 cases.

13. **Calculation of Expected Pay-off**

Demand	*Prob.*	*Small*	*Large*
10,000	0.25	–1,00,000	–3,00,000
20,000	0.25	1,00,000	0
50,000	0.25	3,00,000	1,00,000
1,00,000	0.25	3,00,000	6,00,000
	Expected Value =	1,50,000	1,00,000

Since the EMV for a small-sized factory is higher, the manufacturer should build a small factory.

Here, EPPI = 0.25 (–1,00,000) + 0.25 × 1,00,000 + 0.25 × 3,00,000 + 0.25 × 6,00,000
= Rs 2,25,000

∴ EVPI = 2,25,000 – 1,50,000 = Rs 75,000

EVPI is the maximum price a decision-maker is willing to pay for a perfect forecast of the events (demand in this example).

14. Expected return from bank = 0.06

Expected return from business = $0.12p + (-0.02)(1 - p)$
$= 0.14p - 0.02$

The investments are equally attractive when

$$0.14\, p - 0.02 = 0.06$$

or $p = 0.08/0.14 = 4/7$

which is the required probability.

15. Expected return from Reliable Company's bonds = 0.08

$$\text{Expected return from business} = \frac{12{,}000}{1{,}00{,}000}\, p + \left(\frac{-2{,}000}{1{,}00{,}000}\right)(1 - p)$$

$= 0.14p - 0.02$

To be neutral between the two, we have

$$0.14p - 0.02 = 0.08$$

or $p = 0.10/0.14$ or $5/7$.

Thus, $p = 5/7$ would make the two alternatives equally attractive. This probability value can serve as the benchmark for investment decision. If it is felt that the chances of favourable condition of economy are less than 5/7, then bonds be purchased while if the chances are reckoned to be more than this value, the proposal of investing in equipment should be accepted.

16. **Calculation of Expected Pay-off**

Demand (Units)	*Prob.*	*Production Level*	
		Limited	*Full*
5000	0.30	42,00,000	50,00,000
4000	0.40	32,00,000	30,00,000
3000	0.30	22,00,000	10,00,000
	Expected Value	32,00,000	30,00,000

Optimal decision: Go for limited production.

17. (a) $25\alpha - 50 = 35\alpha - 90$ implies $\alpha = 40/10 = 4$.
    Thus, for $\alpha = 4$, $P_{A1} = P_{A2}$.
    (b) For $\alpha = 10$, $P_{A1} = 25 \times 10 - 50 = 200$, and $P_{A2} = 35 \times 10 - 90 = 260$.
    Thus, for $\alpha = 10$, P_{42} is better
    (c) For $\alpha = 15$, $P_{A1} = 25 \times 15 - 50 = 325$ and $P_{A2} = 35 \times 15 - 90 = 435$.
    Regret for $P_{A1} = 435 - 325 = 110$
    For $\alpha = 4$, P_{A1} and P_{A2} are equally attractive and neither has regret.

18. EMV for hot snack stall = 0.6 × 5,000 + 0.4 × 1,000 = Rs 3,400
    EMV for ice cream stall = 0.6 × 1,000 + 0.4 × 6,500 = Rs 3,200
    Thus, hot snack stall is preferable.

19. **Calculation of Expected Cost** (*Cost in Rs lakhs*)

No. of Breakdowns	*Prob.*	*No. of Spare Components*				
		0	*1*	*2*	*3*	*4*
0	0.80	0	5	10	15	20
1	0.08	40	0	5	10	15
2	0.06	80	40	0	5	10
3	0.04	120	80	40	0	5
4	0.02	160	120	80	40	0
	Expected Value	16.0	8.4	11.6	13.9	18.0

Optimal number of spares to order = 1.

20. The conditional total-cost matrix is given in the following table. Also, the expected cost for each of the alternatives is given. From the expected cost values, it is clear that three units should be stocked every week.

Conditional-cost Matrix

Sales (Units)	*Prob.*	*Course of Action: Stock (Units)*						
		0	*1*	*2*	*3*	*4*	*5*	*6*
0	0.10	0	30	60	90	120	150	180
1	0.10	70	30	60	90	120	150	180
2	0.20	140	100	60	90	120	150	180
3	0.25	210	170	130	90	120	150	180
4	0.15	280	240	200	160	120	150	180
5	0.15	350	310	270	230	190	150	180
6	0.05	420	380	340	300	260	220	180
	Expected cost:	203	170	144	132	137.5	153.5	180

Calculation of EVPI: Here expected cost of perfect information,

ECPI = $0.10 \times 0 + 0.10 \times 30 + 0.20 \times 60 + 0.25 \times 90 + 0.15 \times 120 + 0.15 \times 150 + 0.05 \times 180 = 87$

EVPI = Expected cost for optimal decision – ECPI

= 132 – 87 = Rs 45

21. Let x be the level of demand that would make the two alternatives equally attractive. We have,

$$24,00,000 + 8x = 24x$$

or $$x = 1,50,000$$

For demand > 1,50,000 units, set up own facilities.

22. The expected profit for each of the alternatives is shown calculated below:

Alternative 1: Invest in project 1 of this month only

Expected profit = 0.5 × 20,000 + 0.5 (–10,000) = Rs 5,000

Alternative 2: Invest in project 2 only

Expected profit = 0.5 × 15,000 + 0.5(–5,000) = Rs 5,000

Alternative 3: Invest in projects 1 and 2 together.

Outcome	*Prob.*	*Pay-off*	*Expected profit (Rs)*
(a) Project 1 succeeds			
Project 2 succeeds	0.5 × 0.5 = 0.25	35,000	8,750
(b) Project 1 succeeds			
Project 2 fails	0.5 × 0.5 = 0.25	15,000	3,750
(c) Project 1 fails*	0.50	(10,000)	(5,000)
		Total	7,500

*If project 1 fails, enough cash would not be available to launch project 2.

Conclusion: To maximise its profits, the company should adopt alternative 3 and, thus, invest in projects 1 and 2 together.

23. **Conditional Pay-off Matrix**

(*Cost in '000 Rs*)

No. of spares Required	*Prob.*	*No. of Spares to order*				
		0	*1*	*2*	*3*	*4*
0	0.93	0	10	20	30	40
1	0.04	200	10	20	30	40
2	0.01	400	210	20	30	40
3	0.01	600	410	220	30	40
4	0.01	800	610	420	230	40
	Expected Value	26	22	26	32	40

∴ Optimal number of spares to order = 1

24. (a) From the given information, we have

Unit contribution = Rs 130 – (80 + 5) = Rs 45

Unit loss when surplus is sold = Rs 85 – 50 = Rs 35

Unit penalty for unsatisfied demand = Rs 20/outfit

Contribution calculations may be done as follows:

When 1,100 units are purchased:

Demand	*Contribution*	
1,100	1,100 × 45	= Rs 49,500
1,200	1,100 × 45 – 100 × 20	= Rs 47,500
1,300	1,100 × 45 – 200 × 20	= Rs 45,500
1,400	1,100 × 45 – 300 × 20	= Rs 43,500

When 1,200 units are purchased:

Demand	*Contribution*	
1,100	1,100 × 45 – 100 × 35	= Rs 46,000
1,200	1,200 × 45	= Rs 54,000
1,300	1,200 × 45 – 100 × 20	= Rs 52,000
1,400	1,200 × 45 – 200 × 20	= Rs 50,000

Similarly, other calculations may be done. The pay-offs (in '000 Rs) are shown in table. It may be mentioned that the ordering and receiving cost of Rs 800 is constant throughout. As such, it has not been considered in making calculations.

Determination of Optimal Order Quantity

Demand	*Prob.*	*Order Quantity*			
		1,100	*1,200*	*1,300*	*1,400*
1,100	0.3	49.5	46.0	42.5	39.0
1,200	0.4	47.5	54.0	50.5	47.0
1,300	0.2	45.5	52.0	58.5	55.0
1,400	0.1	43.5	50.0	56.5	63.0
Expected contribution		47.3	50.8	50.3	47.8

Using the given probabilities, expected contribution for each of the order quantities is also shown calculated in the table. On the basis of the values obtained, the optimal order quantity is 1,200 units.

(b) The model used here differs from the classical economic order quantity (*EOQ*) model on a fundamental level in that whereas it deals with uncertain demand, the *EOQ* model deals with demand that is known and certain. Thus, while this model does not have much mathematical sophistication, it does have the capability of handling uncertainty. Further, the model used here considers and analyses stock-outs, the classical *EOQ* model in its original format does not permit the out-of-stock situations. The *EOQ* model is basically used in the manufacturing environment where an item is constantly used and replenished periodically.

25. From the given information,

Profit per pack sold = Rs 20, ∴ Profit per case = 20 × 50 = Rs 1,000

Loss per pack unsold = Rs 10, ∴ Loss per case = 10 × 50 = Rs 500

The profit function is:

$$P = 1,000\, s \qquad \text{for } s \le d$$
$$= 1,000\, d - 500(s - d) \qquad \text{for } s > d$$

Accordingly, the conditional pay-off matrix is as shown in table below.

Conditional Pay-off Matrix

Demand (Cases)	*Prob.*	*No. of Cases Ordered*				
		5	*10*	*15*	*20*	*25*
5	0.20	5,000	2,500	0	–2,500	–5,000
10	0.20	5,000	10,000	7,500	5,000	2,500
15	0.30	5,000	10,000	15,000	12,500	10,000
20	0.20	5,000	10,000	15,000	20,000	17,500
25	0.10	5,000	10,000	15,000	20,000	25,000
Expected Pay-off		5,000	8,500	10,500	10,250	8,500
Simple Average		5,000	8,500	10,500	11,000	10,000

(a) Maximum expected pay-off corresponds to 15 cases. It is equal to Rs 10,500. Thus, optimal policy is to order 15 cases.

(b) When the manager is completely uncertain, we obtain simple average pay-off (ignoring probabilities, that is), Since it is the highest for 20, she should buy 20 cases.

26. **Conditional Pay-off Matrix**

Act, a_i	*Demand, d_j*						*Expected Value*
	3	*4*	*5*	*6*	*7*	*8*	
	0.05	0.10	0.30	0.40	0.10	0.05	
3	1,200	1,200	1,200	1,200	1,200	1,200	1,200
4	900	1,600	1,600	1,600	1,600	1,600	1,565
5	600	1,300	2,000	2,000	2,000	2,000	1,860
6	300	1,000	1,700	2,400	2,400	2,400	1,945
7	0	700	1,400	2,100	2,800	2,800	1,750
8	(300)	400	1,100	1,800	2,500	3,200	1,485

Optimal policy: $a_i = 6$, Expected value = 1945.

27. **Conditional Pay-off Matrix**

Demand (Units)	*Prob.*	*Units Manufactured*			
		20,000	*30,000*	*40,000*	*50,000*
20,000	0.10	80,000	(10,000)	(1,20,000)	(2,10,000)
30,000	0.40	80,000	1,90,000	80,000	(10,000)
40,000	0.30	80,000	1,90,000	2,80,000	1,90,000
50,000	0.20	80,000	1,90,000	2,80,000	3,90,000
Expected Value		80,000	1,70,000	1,60,000	1,10,000

Optimal size of production run = 30,000 units.

28. Let us call it situation *A* when selling price is Rs 15 and situation *B* when selling price is Rs 20. From the information given, we have

	Situation A	*Situation B*
Contribution margin per unit	Rs 15 – 3 = Rs 12	Rs 20 – 3 = Rs 17
Total fixed cost ('000 Rs)	Rs 25 + 40 = Rs 65	Rs 96 + 40 = Rs 136

We may first calculate the materials cost under each of the three purchase options. This is done below:

		Situation A			*Situation B*	
Sale ('000 kg)	36	28	18	28	23	13
Purchase option 1: Any Quantity @ Rs 3 per kg (No sales needed):						
Materials buy ('000 kg)	36	28	18	28	23	13
Materials cost ('000 Rs) (3 kg @ Rs 3/kg)	324	252	165	252	207	111
Purchase option 2: Price @ Rs 2.75 per kg, minimum quantity 5,000 kg:						
Materials required (in '000 kg)	108	84	54	84	69	39
Materials buy (in '000 kg)	108	84	54	84	69	50
Materials sale	—	—	—	—	—	11
Materials cost ('000 Rs)	297	231	148.5	231	189.75	126.5
Purchase option 3: Price @ Rs 2.50 per kg, minimum quantity 70,000 kg:						
Materials required ('000 kg)	108	84	54	84	69	39
Materials buy ('000 kg)	108	84	70	84	70	70
Materials sale ('000 kg)	—	—	16	—	1	31
Materials cost ('000 Rs)	270	210	156	210	174	144

Now we can calculate the conditional and expected profit/loss for each of the situations and purchase options.

All values in '000s

		Situation A			*Situation B*	
Sale (in '000 units)	36	28	18	28	23	13
Purchase option 1:						
Gross contribution	432	336	216	476	391	221
Less Materials cost	324	252	162	252	207	117
Fixed cost	65	65	65	65	65	65
Conditional profit/loss	43	19	(11)	88	48	(32)
Probability	0.3	0.5	0.2	0.2	0.5	0.2
Expected P/L	12.9	9.5	(2.2)	26.4	24.0	(6.4)
			20.2			44.0
Purchase option 2:						
Gross contribution	432	336	216	476	391	221
Less Materials cost	297	321	148.5	231	189.75	126.5
Fixed cost	65	65	65	65	65	65
Conditional profit/loss	70	40	2.5	109	65.25	(41.5)
Probability	0.3	0.5	0.2	0.3	0.5	0.2
Expected P/L	21.0	20.0	0.5	32.7	32.625	(8.3)
			41.5			57.025
Purchase option 3:						
Gross Contribution	432	336	216	476	391	221
Less Materials cost	270	210	159	210	174	144
Fixed cost	65	65	65	65	65	65
Conditional profit/loss	97	61	(8)	130	81	(59)
Probability	0.3	0.5	0.2	0.3	0.5	0.2
Expected P/L	29.1	30.5	(1.6)	39.0	40.5	(11.8)
			58.0			67.7

From the expected profit values calculated, it may be observed that the optimal policy is to have selling price of Rs 20 and exercise purchase option 3, involving a cost of Rs 2.75 per kg for a minimum quantity of 70,000 units.

(b) The conditional pay-offs in each of the situations under optimistic, most likely, and pessimistic conditions for each of the purchase options are tabulated below:

Purchase option	*Conditional Profit/loss ('000 Rs)*					
	Optimistic		*Most likely*		*Pessimistic*	
1	43	48	19	48	(11)	(32)
2	70	109	40	65.25	2.5	(41.5)
3	97	130	61	81	(8)	(59)

Selection of best result under each condition gives:

Conditional profit	130	81	2.5
Probability	0.3	0.5	0.2
Expected profit	39.0	40.5	0.5

Thus, expected profit under perfect information,

EPPI = 39.0 + 40.5 + 0.5 = 80 (thousand)

Expected profit under optimal policy (obtained in *a* above) = 67.7 (thousand)

$\therefore$ Maximum price for perfect information,

EVPI = 80.0 – 67.7 = 12.3 (thousand)

Thus, EVPI = Rs 12,300.

29. From the given data,

Expected cost per machine if policy is not taken

$= 0.20 \times 30 + 0.44 \times 70 + 0.36 \times 120 =$ Rs 80

Cost of buying policy = 8×45 = Rs 360

The calculation of expected cost under each of the two strategies is shown in table here.

Calculation of Expected Cost

No. of Machines Failing	*Prob.*	*Course of Action*	
		Buy Policy	*Don't Buy Policy*
3	0.15	360	240
4	0.30	360	320
5	0.50	360	400
6	0.05	360	480
	Expected cost	360	356

Conclution: Don't buy maintenance policy.

30. Without research:

Calculation of Expected Pay-off (*Value in Rs lakhs*)

Demand	*Prob.*	*Size of Plant*	
		2,500 Tonnes	*5,000 Tonnes*
Low	0.30	30	–20
High	0.70	40	55
	Expected value	37.0	32.5

EPPI = 0.30 × 30 + 0.70 × 55 = Rs 47.5 lakhs

EVPI = Rs 47.5 lacs – Rs 37.0 lacs = Rs 10.5 lakhs

With research:

Let I_1 : A low demand is indicated by research,

I_2 : A high demand is indicated by research,

E_1 : The event of low demand, and

E_2 : The event of high demand.

$$P(I_1) = P(E_1) \times P(I_1/E_1) + P(E_2) \times P(I_1/E_2)$$
$$= 0.3 \times 0.8 + 0.7 \times 0.1 = 0.31$$
$$P(I_2) = P(E_1) \times P(I_2/E_1) + P(E_2) \times P(I_2/E_2)$$
$$= 0.3 \times 0.2 + 0.7 \times 0.9 = 0.69$$

Calculation of posterior probabilities:

(a) For I_1 :

$$P(E_1/I_1) = \frac{0.24}{0.31} = 0.77$$

$$P(E_2/I_1) = \frac{0.07}{0.69} = 0.23$$

Expected Pay-off:

For 2,500-Tonnes Plant: 0.77 × 30 + 0.23 × 40 = 32.3 (Rs lakhs)

For 5,000-Tonnes Plant: 0.77 (–20) + 0.23 × 55 = –2.75 (Rs lakhs)

Decision: 2,500-Tonnes Plant

(b) For I_2:

$$P(E_1/I_2) = \frac{0.06}{0.69} = 0.09$$

$$P(E_2/I_2) = \frac{0.63}{0.69} = 0.91$$

Expected pay-off:

For 2,500-Tonnes Plant: 0.09 × 30 + 0.91 × 40 = 39.1 (Rs lakhs)

For 5,000-Tonnes Plant: 0.09 (–20) + 0.91 × 55 = 48.25 (Rs lakhs)

Decision: 5,000-Tonnes Plant.

Now, Overall expected pay-off = 0.31 × 32.3 + 0.69 × 48.25

= Rs 43.306 lakhs

Expected value of sample in formation,

EVSI = 43.306 – 37 = Rs 6.306 lakhs.

Since EVSI is greater than Rs 2 lakhs, the cost of research, it is advisable to spend money on research. If 'low' is indicated by research, build a 2,500-Tonnes plant, and if 'high' is indicated then build a 5,000-Tonnes plant.

31. First we calculate posterior probabilities in light of the sample information.

Calculation of Posterior Probabilities

Lot Type H_i	*Prior Prob.* $P(H_i)$	*Conditional Prob.* $P(E/H_i)$	*Joint Prob.* $P(H_i \cap E)$	*Posterior Prob.* $P(H_i/E)$
1% def.	0.5	0.083	0.0415	0.2767
2% def.	0.3	0.185	0.0555	0.3700
5% def.	0.2	0.265	0.0530	0.3533
			$P(E)$ = 0.1500	

Next, compute expected cost of the alternatives.

Calculation of Expected Cost

Outcome	*Prob.*	*Accept*	*Reject*
1% *def. lot*	0.2767	0	600
2% *def. lot*	0.3700	400	0
5% *def. lot*	0.3533	600	0
	Expected value	360	166.2

Conclusion: Reject the lot.

32. (a) **Calculation of Expected Cost** (*'000 Rs*)

Lot Type	*Prob.*	*Accept*	*Reject*
D_1	0.7	0	3
D_2	0.3	5	0
	Expected cost	1.5	2.1

Conclusion: Accept the lot.

(b) Calculate posterior probabilities in light of the sample information and recalculate the expected cost.

Calculation of Posterior Probabilities

Lot Type H_i	*Prior Prob.* $P(H_i)$	*Condition Prob.* $P(E/H_i)$	*Joint Prob.* $P(H_i \cap E)$	*Posterior Prob.* $P(H_i/E)$
D_1	0.7	$(0.05)^2 = 0.0025$	0.00175	0.368
D_2	0.3	$(0.10)^2 = 0.0100$	0.00300	0.632
			$P(E) = 0.00475$	

Calculation of Expected Cost

Lot Type	*Prob.*	*Accept*	*Reject*
D_1	0.368	0	3
D_2	0.632	5	0
	Expected cost	3.160	1.104

Conclusion: Reject the lot.

(c) Expected value of the sample information,

EVS1 = Expected cost without Information – Expected cost with Information

= 1.5 – 1.104 = 0.396 (thousand Rs) = Rs 396

(d) At testing cost of Rs 40 per unit, total cost testing = Rs 80. Since it is less than EVS1, testing should be done.

33. (a) The best option to the company, before the test, is given by expected profit. From the expected profit values shown calculated below, it is evident that optimal act is A_1.

Calculation of Expected Profit

Event	*Probability*	*Course of Action*	
		A_1	A_2
d_1	0.7	10 m	0
d_2	0.1	1 m	0
d_3	0.2	(5 m)	0
	Expected profit	6.1 m	0

(b) Expected pay-off under perfect information,

$$EPPI = 0.7 \times 10 + 0.1 \times 1 + 0.2 \times 0 = \text{Rs } 7.1 \text{ m}$$

$$\therefore \quad EVPI = 7.1 - 6.1 = \text{Rs } 1 \text{ m}$$

(c) First of all, we calculate posterior probabilities, as given in table below:

Calculation of Posterior Probabilities

Event	*Prior Prob.* *P(d)*	*Conditional Prob.* *P(I/d)*	*Joint Prob.* $P(I \cap D)$	*Posterior Prob.* *P(d/I)*
For I_1:				
d_1	0.7	0.6	0.42	42/47
d_2	0.1	0.3	0.03	3/47
d_3	0.2	0.1	0.02	2/47
			0.47	
For I_2:				
d_1	0.7	0.3	0.21	21/27
d_2	0.1	0.6	0.06	6/27
d_3	0.2	0.1	0.02	2/27
			0.27	
For I_3:				
d_1	0.7	0.1	0.07	7/24
d_2	0.1	0.1	0.06	1/24
d_3	0.2	0.8	0.16	16/24
			0.24	

Next, we determine optimal course of action under each of the three outcomes of test marketing. This is shown in the next table.

Determination of Conditional Optimal Actions

Event	*Indication* I_1			*Indication* I_2			*Indication* I_3		
	Prob.	A_1	A_2	*Prob.*	A_1	A_2	*Prob.*	A_1	A_2
d_1	42/47	10	0	21/27	10	0	7/24	10	0
d_2	3/47	1	0	6/27	1	0	1/24	1	0
d_3	2/47	(5)	0	2/27	(5)	0	16/24	(5)	0
Exp. Value		413/47	0		206/29	0		(9/24)	0

Thus, optimal course of action when I_1 is indicated: A_1
I_2 is indicated: A_1
I_3 is indicated: A_2

Now, we can calculate the expected value with sample information, as follows:

Event	*Probability*	*Pay-off*	*Expected value*
I_1	47/100	413/47	4.13
I_2	29/100	206/29	2.06
I_3	24/100	0	0.00
		EPSI	= 6.19

(d) Expected value of test marketing
= 6.19 – 6.10 = 0.09 million or Rs 90,000.

34. *With no Survey:*

Calculation of Expected Profit

Event	*Prob.*	S_1*: produce*	S_2*: Do not produce*
N_1: Success	0.60	150	0
N_2: Failure	0.40	–100	0
	Expected value	50	0

EPPI = 0.60 × 150 + 0.40 × 0 = Rs 90 lakhs
EVPI = 90 – 50 = Rs 40 lakhs

Survey by Alpha:

Calculation of Posterior probabilities:

We have

$$P(Z_1) = P(N_1) \times P(Z_1/N_1) + P(N_2) \times P(Z_1/N_2)$$
$$= 0.60 \times 0.90 + 0.40 \times 0.10 = 0.58$$

$$\therefore \quad P(N_1/Z_1) = \frac{0.54}{0.58} = 0.93 \text{ and } P(N_2/Z_1) = \frac{0.04}{0.58} = 0.07$$

Also,

$$P(Z_2) = P(N_1) \times P(Z_2/N_1) + P(N_2) \times P(Z_2/N_2)$$
$$= 0.60 \times 0.10 + 0.40 \times 0.90 = 0.42$$

$$\therefore \quad P(N_1/Z_2) = \frac{0.06}{0.42} = 0.14 \text{ and } P(N_2/Z_2) = \frac{0.36}{0.42} = 0.86$$

Expected Pay-offs

In case of Z_1:

For S_1: 0.93 × 150 + 0.07 (–100) = 132.5 Decision: S_1

For S_2: 0.93 × 0 + 0.07 × 0 = 0

In case of Z_2:

For S_1 : 0.14 × 150 + 0.86(–100) = –65 Decision : S_2

For S_2 : 0.14 × 0 + 0.86 = 0

Overall expected pay-off = 0.58 × 132.5 + 0.42 × 0 = 76.85 (Rs lakhs)

EVS1 = 76.85 – 50 = Rs 26.85 lakhs

Net increase in expected profit = 26.85 – 0.5 = Rs 26.35 lakhs

Survey by Beta:
Calculation of Posterior Probabilities:
We have

$$P(Z_1) = 0.60 \times 0.70 + 0.40 \times 0.30 = 0.54$$
$$P(Z_2) = 0.60 \times 0.30 + 0.40 \times 0.70 = 0.46$$

$$\therefore \quad P(N_1/Z_1) = \frac{0.42}{0.54} = 0.78 \text{ and } P(N_2/Z_1) = \frac{0.12}{0.54} = 0.22$$

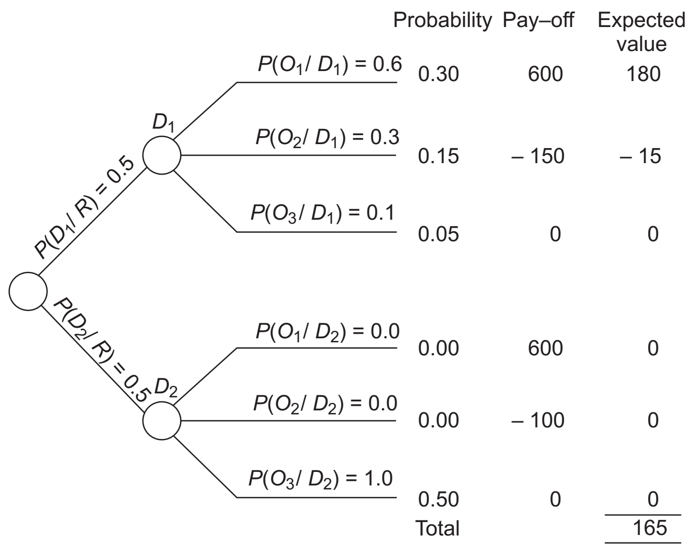
$$P(N_1/Z_2) = \frac{0.18}{0.46} = 0.39 \text{ and } P(N_2/Z_2) = \frac{0.28}{0.46} = 0.61$$

Expected Pay-offs: In case of Z_1
For S_1: $0.78 \times 150 + 0.22\ (-100) = 95$ Decision: S_1
For S_2: $0.78 \times 0 + 0.22 \times 0 = 0$
In case of Z_2
For S_1: $0.39 \times 150 + 0.61\ (-100) = -2.5$ Decision: S_2
For S_2: $0.39 \times 0 + 0.61 \times 0 = 0$
Overall expected pay-off = $0.54 \times 95 + 0.46 \times 0$ = Rs 51.3 lakhs

EVSI = 51.3 – 50 = Rs 1.3 lakhs

Net increase in expected profit = 1.3 – 0.3 = Rs 1 lakh
Thus, expected pay-offs are
(a) No survey: Rs 50 lakhs
(b) Survey by Alpha: Rs 76.85 – Rs 0.5 = Rs 76.35 lakhs
(c) Survey by Beta: Rs 51.3 – Rs 0.3 = Rs 51 lakhs
Best option: Survey by Alpha Company.
EVPI = Rs 40 lakhs

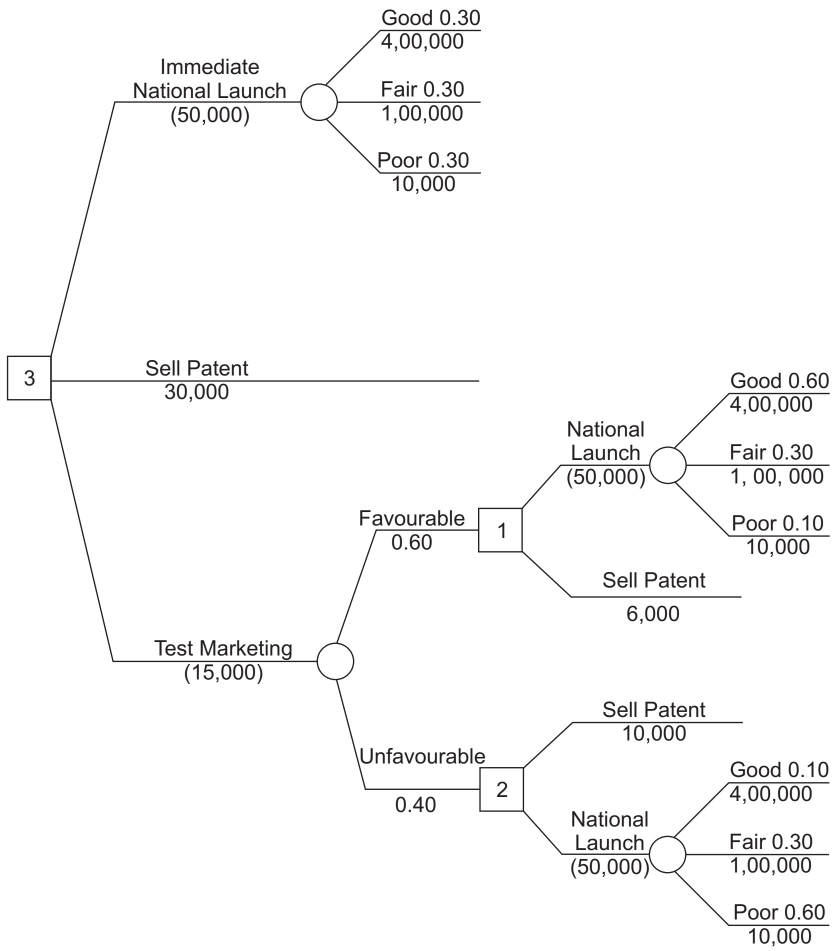
35.



Tree Diagram

Thus, expected pay-off is Rs 1,65,000.

36.



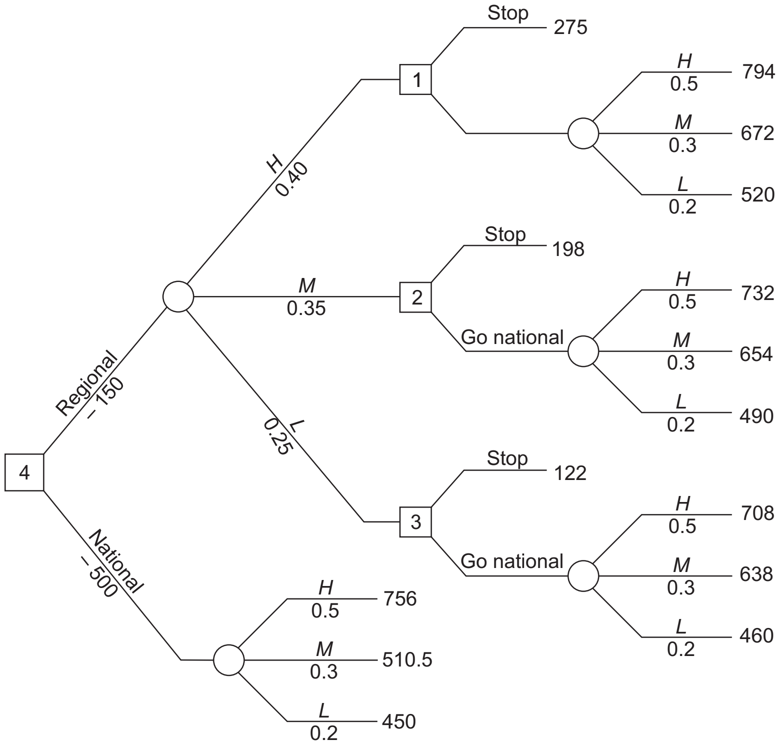
Decision Tree

Analysis Table

Decision Node	*Alternatives*	*EMV*	*Decision*
1	National Launch	220,000	National Launch
	Sell Patent	60,000	
2	National Launch	26,000	National Launch
	Sell Patent	10,000	
3	Immediate National Launch	113,000	Test Marketing
	Sell Patent	30,000	
	Test marketing	128,000	

Conclusion: Go for test marketing and then launch nationally, whether it is (test marketing) is favourable or unfavourable.

37. The decision-tree corresponding to the given information is shown in the figure.



(i) The probabilities given indicate the likelihood of different regional outcomes, that is, $P(h) = 0.40$, $P(m) = 0.35$ and $P(l) = 0.25$, and the chances of particular national outcomes, given high regional demand, that is, $P(H/h) = 0.5$, $P(M/h) = 0.3$ and $P(L/h) = 0.2$. Considering the top branch of the decision-tree relating to a high regional demand, the expected value of going national is $0.5 \times 794 + 0.3 \times 672 + 0.2 \times 520 - 450 = 252.6$. The return from not going national and staying with regional distribution only is 275, so that it would be more profitable to stay regional.

Now, if this is the situation following a high regional demand, it can presumably be inferred that it would be more profitable to go national if the regional demands were medium or low (nodes 2 and 3). In other words, if we start regional, we should probably not go beyond that.

Here we are not given any information about the probabilities of high, medium and low demand if we go national at the outset. If we assume that they would be similar to those regional, the expected value of going national at the outset is $0.4 \times 756 + 0.35 \times 510 + 0.25 \times 450 - 500 = 93.575$.

On the other hand, the expected value of regional distribution only is $0.40 \times 275 + 0.35 \times 198 + 0.25 \times 122 - 150 = 59.8$. Thus, it seems to suggest a national distribution from the outset. With national distribution, however, we would be gambling somewhat on the occurrence of high demand—regional distribution is less risky.

(ii) To compare the relative riskiness of the two alternatives, we shall compute their coefficients of variation.

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{X}} \times 100$$

For regional distribution:

$$\text{Variance} = \Sigma pX^2 - (\Sigma pX^2)$$

$= 0.4 \times 275^2 + 0.35 \times 1.98^2 + 0.25 \times 122^2 - 209.8^2$
$= 3676.36$

$\sigma = \sqrt{\text{Variance}}$

$= \sqrt{3676.36} = 60.63$

Expected value (net of cost) = 59.8

$\therefore$ Coefficient of variation = $\frac{60.63}{59.8} \times 100 = 101.4\%$

For national distribution:

Variance $= 0.4 \times 756^2 + 0.35 \times 510.5^2 + 0.25 \times 450^2 - 593.575^2$
$= 18{,}121.7$

$\sigma = \sqrt{18{,}121.7} = 134.62$

Expected value (net of cost) = 593.557 – 500 = 93.575

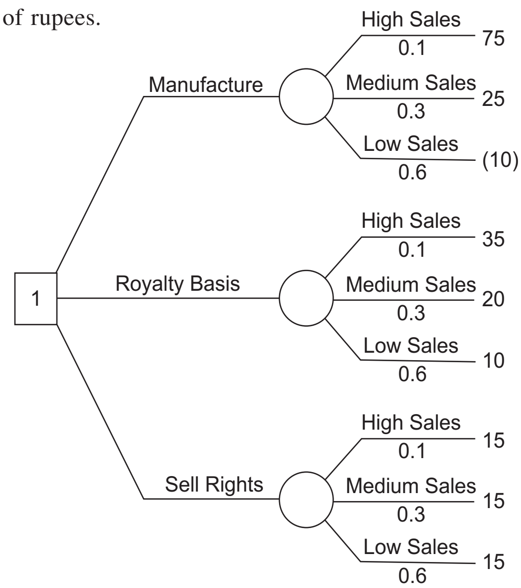
$\therefore$ Coefficient of variation = $\frac{134.62}{93.575} \times 100 = 143.9\%$

Thus, the second alternative is more risky.

38. **Analysis Table**

Decision Node	*Alternatives*	*EMV*	*Decision*
1	Manufacture	$75 \times 0.1 + 25 \times 0.3 + (-10) \times 0.6$ = Rs 9	Royalty Basis
	Royalty Basis	$35 \times 0.1 + 20 \times 0.3 + 10 \times 0.6$ = Rs 15.5	
	Sell Rights	Rs 15	

All amounts in thousands of rupees.



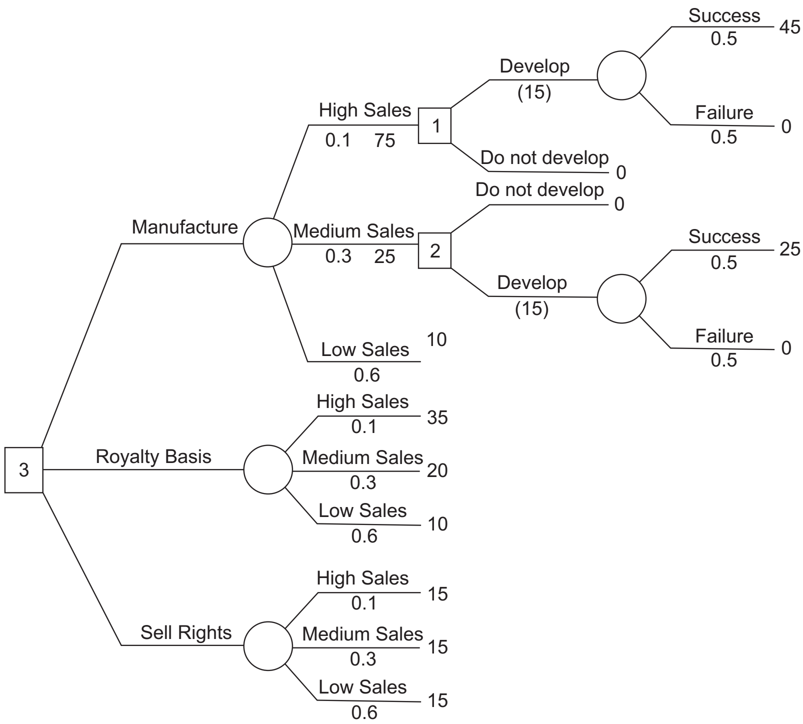
Decision Tree

Analysis Table

Decision Node	*Alternatives*	*EMV*	*Decision*
1	Develop Do not develop	7.5 0	Develop
2	Develop Do not develop	–2.5 0	Do not develop
3	Manufacture Royalty Basis Sell Rights	9.75* 15.5 15	Royalty Basis

* $82.5 \times 0.1 + 25 \times 0.3 - 10 \times 0.6 = 9.75$

Result: No change in decision

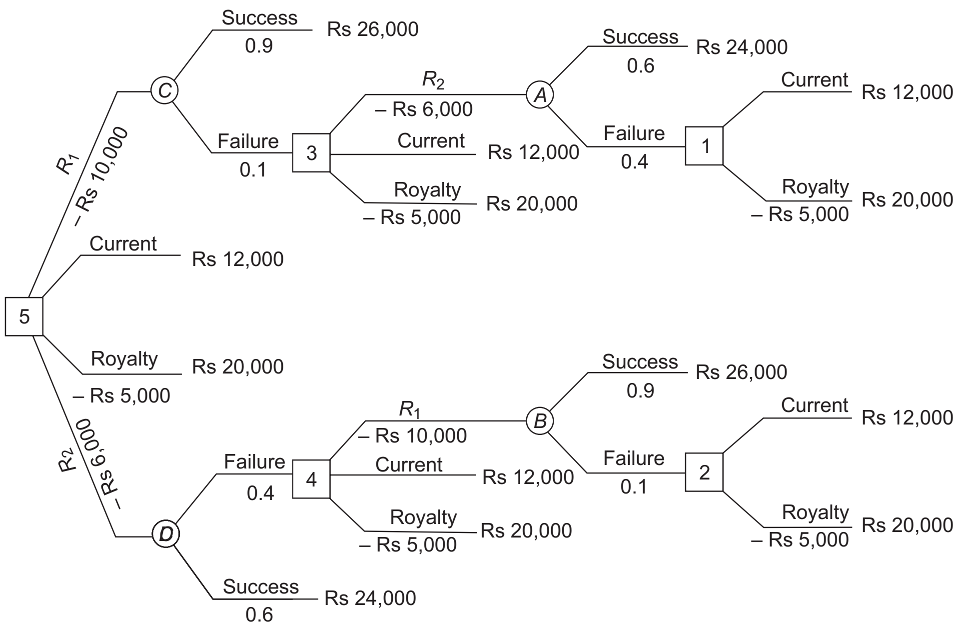


Revised Decision Tree

39. The decision tree is shown below.
    The analysis of tree follows.
    Expected monetary value (EMV) at nodes 1 and 2:
    Max {(Rs 20,000 – Rs 5,000), Rs 12,000} = Rs 15,000
    $\therefore$ Conditional decision at each of these nodes is to pay royalty of new process.
    EMV at chance node $A = 0.4 \times 15{,}000 + 0.6 \times 24{,}000 =$ Rs 20,400
    EMV at node 3: Max. {(Rs 20,400 – Rs 6,000), Rs 12,000, (Rs 20,000 – Rs 5,000)}
    = Max. {Rs 14,400, Rs 12,000, Rs 15,000}
    = Rs 15,000

∴ Conditional decision when R_1 fails is to pay royalty of new process.
EMV at chance node $B = 0.9 \times 26{,}000 + 0.1 \times 15{,}000 =$ Rs 24,900
EMV at node 4: Max. {(Rs 24,900 – Rs 10,000), Rs 12,000, (Rs 20,000 – Rs 5,000)}
= Max. {Rs 14,900, Rs 12,000, Rs 15,000}
= Rs 15,000
∴ Conditional decision when R_2 fails is to pay royalty of new process.
EMV at node 5: Max{((0.9 × 26,000 + 0.1 × 15,000) – 10,000), 12,000, (20,000 – 5,000),
((0.6 × 24,000 + 0.4 × 15,000) – 6,000)}
= Max. {Rs 14,900, Rs 12,000, Rs 15,000, Rs 14,400}
= Rs 15,000

Thus, optimal strategy for the company is that it should pay Rs 5,000 as royalty of a new process, which would result in maximum expected pay-off of Rs 15,000.



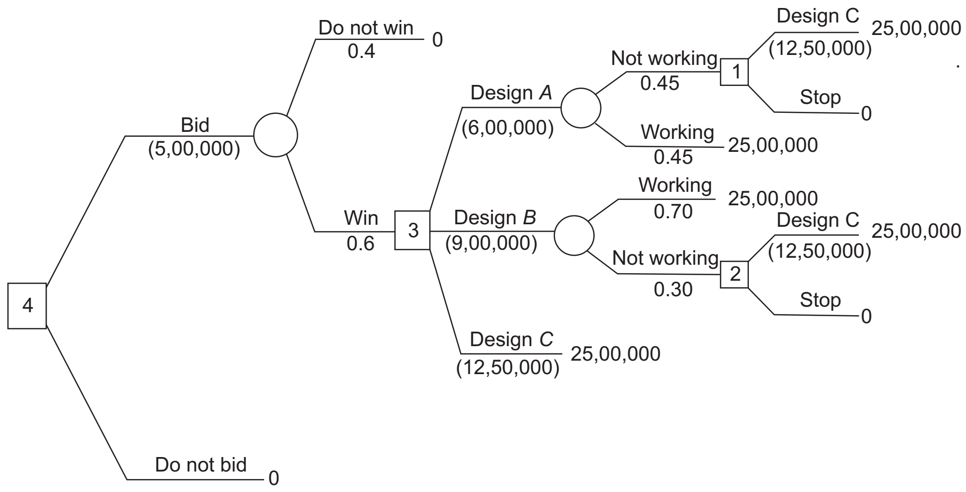
Decision Tree

40.

Analysis Table

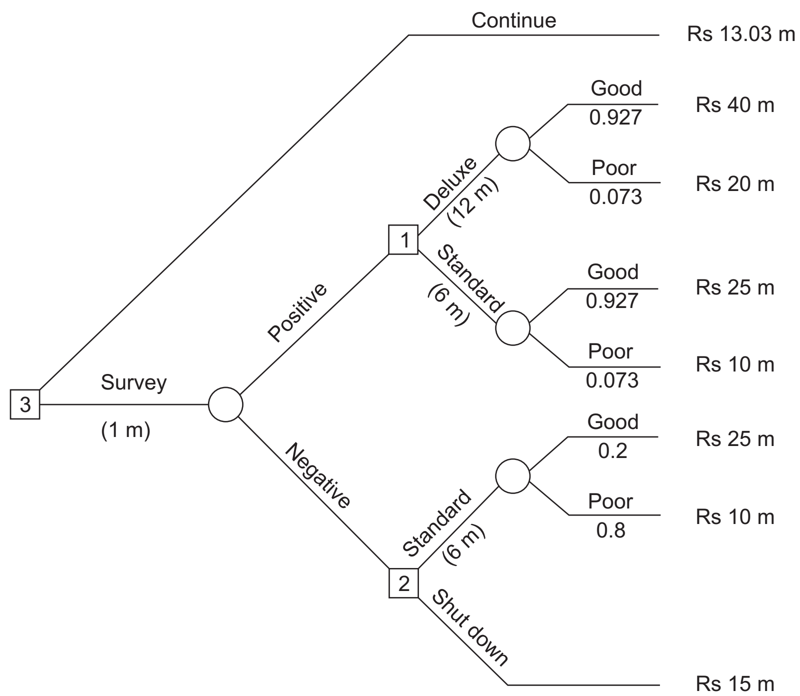
Decision Node	*Alternatives*	*EMV*	*Decision*
1	Design C	12,50,000	Design C
	Stop	0	
2	Design C	12,50,000	Design C
	Stop	0	
3	Design A	13,37,500	Design A
	Design B	12,25,000	
	Design C	12,50,000	
4	Bid	3,02,500	Bid
	Do not bid	0	

Conclusion: Bid and use design A. If it fails, use design C.



Decision Tree

41.



Decision – Tree Diagram

Notes and Working:

1. In case of continuation,
   Income = 2 + 2(1 – 10%) + 2(1 – 10%)^2 + … + 2(1 – 10%)^9
   = 2 + 2(0.9) + 2(0.9)^2 + … + 2(0.9)^9
   $$= \frac{2(1-0.9^{10})}{1-0.9} = \text{Rs } 13.03 \text{ m}$$
2. The probability required for calculating the expected value of each branch may be obtained as follows: Using Bayes' Theorem,
   $$P(\text{Good/Positive}) = \frac{P(\text{Good}) \times P(\text{Positive/Good})}{P(\text{Good}) \times P(\text{Positive/Good}) + P(\text{Poor}) \times P(\text{Positive/Poor})}$$
   $$= \frac{0.85 \times 0.60}{0.85 \times 0.60 + 0.10 \times 0.40} = 0.927$$
   P(Poor/Positive) = 1 – P(Good/Positive)
   = 1 – 0.927 = 0.073
   Similarly,
   $$P(\text{Good/Negative}) = \frac{P(\text{Good}) \times P(\text{Negative/Good})}{P(\text{Good}) \times P(\text{Negative/Good}) + P(\text{Poor}) \times P(\text{Negative/Poor})}$$
   $$= \frac{0.15 \times 0.60}{0.15 \times 0.60 + 0.90 \times 0.40} = 0.2$$
   P(Poor/Negative) = 1 – P(Good/Negative)
   = 1 – 0.2 = 0.8
   Decisions at various nodes are analysed and given below:

Decision node	*Options*	*EMV*	*Decision*
1	Deluxe	Rs 26.54 m	
	Standard	Rs 17.91 m	Deluxe
2	Standard	Rs 7 m	
	Shut down	Rs 15 m	Shut down
3	Continue	Rs 21.38 m	
	Survey	Rs 13.03 m	Survey

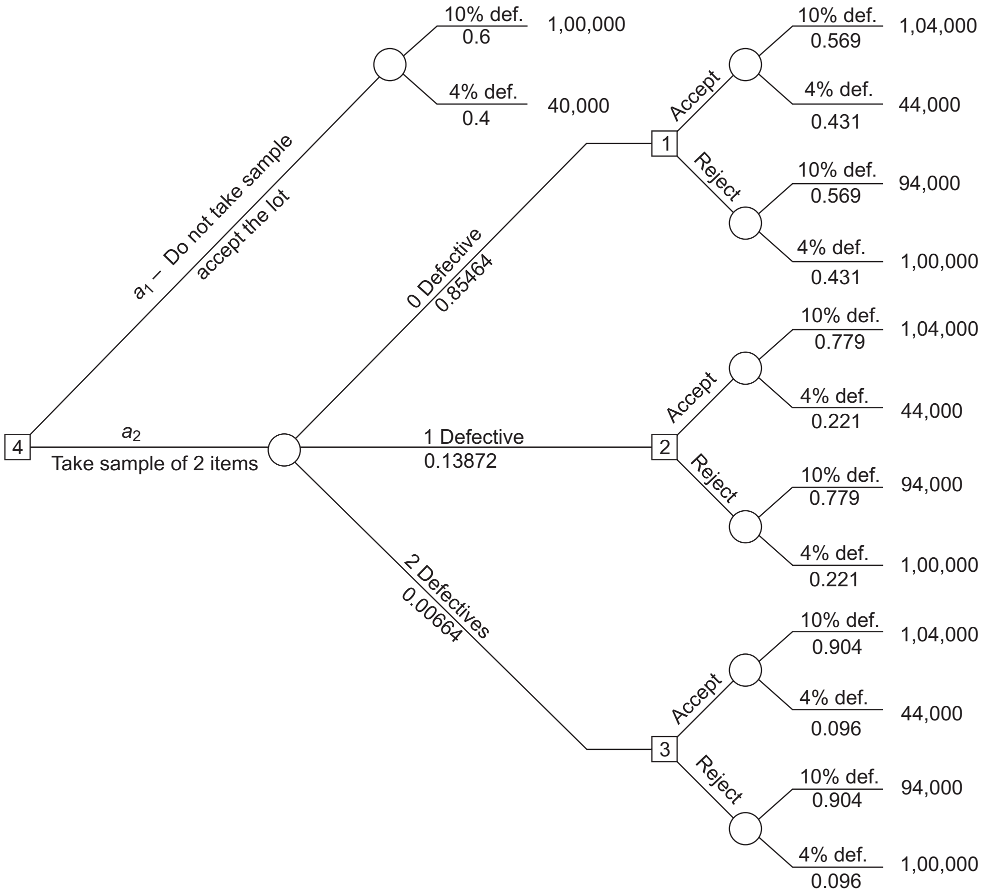
The optimal decision, therefore, is to choose survey. If the survey is positive, choose deluxe upgrade and if it is negative, close down.

42. From the given data:
    Prior Probability: P(10% defective lot) = 3/5 = 0.6
    P(4% defective lot) = 2/5 = 0.4
    The decision tree is shown in figure for which conditional pay-offs and the probabilities are shown calculated below.
    For branch a_1: accept a lot without sampling:
    (i) The expected cost of accepting a lot that is 10% defective = Rs 20,000 per defective unit × 0.10 × 50 units per lot = Rs 1,00,000.
    (ii) The expected cost of accepting a lot that is 4% defective = Rs 20,000 per defective unit × 0.04 × 50 units per lot =- Rs 40,000
    For branch a_2: take a sample of two items:
    (i) The expected cost of accepting a lot that is 10% defective = Rs 1,00,000 + 2 × 2,000 = Rs 1,04,000

(ii) The expected cost of accepting a lot that is 4% defective = Rs 40,000 + 2 × 2,000 = Rs 44,000

(iii) The expected cost of rejecting a lot that is 10% defective = Rs 2,000 (0.90 × 50) + 2 × 2,000 = Rs 94,000

(iv) The expected cost of rejecting a lot that is 4% defective = Rs 2,000 (0.96 × 50) + 2 × 2,000 = Rs 1,00,000



Decision Tree

Determination of Probabilities of Outcomes

Let H_1 and H_2 be the events that a lot contains, respectively, 10% and 4% defectives; and E_1, E_2, and E_3 be the events that, respectively, none, one, and both of the items sampled would be defective. Accordingly,

$$P(E_1) = P(H_1 \cap E_1) + P(H_2 \cap E_1)$$
$$= 0.6 \times 0.90^2 + 0.4 \times 0.96^2 = 0.85464$$
$$P(E_2) = P(H_1 \cap E_2) + P(H_2 \cap E_2)$$
$$= 0.6 \times 0.90 \times 0.10 + 0.4 \times 0.96 \times 0.04 \times 2 = 0.13872$$
$$P(E_3) = P(H_1 \cap E_3) + P(H_2 \cap E_3)$$
$$= 0.6 \times 0.10^2 + 0.4 \times 0.04^2 = 0.00664$$

From these, posterior probabilities can be obtained as follows:

$$P(H_1/E_1) = 0.486/0.85464 = 0.569$$
$$P(H_2/E_1) = 0.36864/0.85464 = 0.431$$

$P(H_1/E_2) = 0.108/0.13872 = 0.779$
$P(H_2/E_2) = 0.03072/0.13872 = 0.221$
$P(H_1/E_3) = 0.006/0.00664 = 0.904$
$P(H_2/E_3) = 0.00064/0.00664 = 0.096$

Using principle of minimising expected cost, we arrive at the best decisions at nodes 1, 2, and 3.

At node 1:

$E(\text{Cost of accept}/E_1) = 0.569 \times 1{,}04{,}000 + 0.431 \times 44{,}000 = 78{,}140$
$E(\text{Cost of reject}/E_1) = 0.569 \times 94{,}000 + 0.431 \times 1{,}00{,}000 = 96{,}586$

Decision : Accept

At node 2:

$E(\text{Cost of accept}/E_2) = 0.779 \times 1{,}04{,}000 + 0.221 \times 44{,}000 = 90{,}740$
$E(\text{Cost of reject}/E_2) = 0.779 \times 94{,}000 + 0.221 \times 1{,}00{,}000 = 95{,}326$

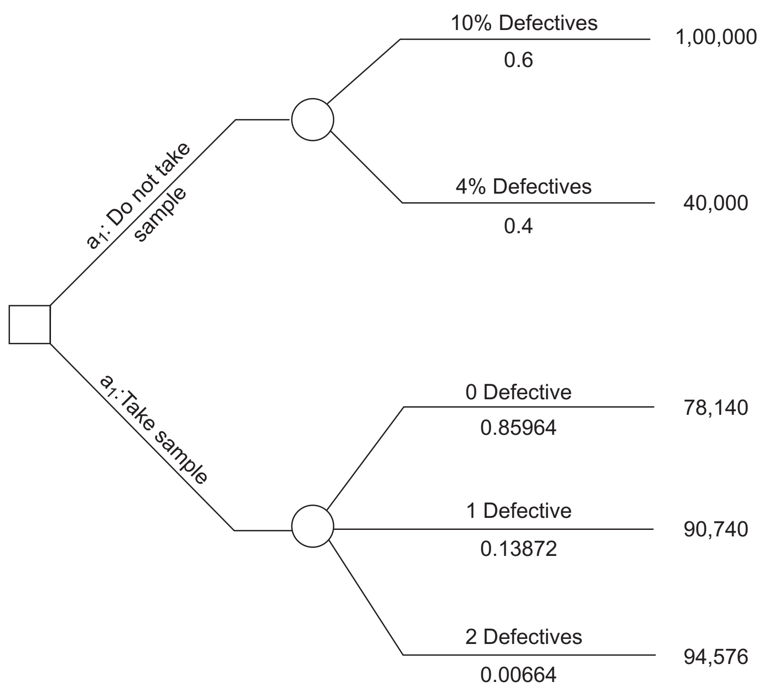
Decision : Accept

At nodes 3:

$E(\text{Cost of accept}/E_3) = 0.904 \times 1{,}04{,}000 + 0.096 \times 44{,}000 = 98{,}240$
$E(\text{Cost of reject}/E_3) = 0.904 \times 94{,}000 + 0.096 \times 1{,}00{,}000 = 94{,}576$

Decision : reject

Next, the decision nodes are replaced by their best values to yield the decision tree in reduced form as shown in the figure below.



Revised decision tree

Expected cost of 'do not take sample': a_1

= 0.6 × 1,00,000 + 0.4 × 40,000 = Rs 76,000

Expected cost of 'take sample': a_2

= 0.85464 × 78,140 + 0.13872 × 90,740 + 0.00664 × 94,576

= Rs 79,997

Conclusion: Do not take sample.

In order to make one indifferent between sampling and not sampling, the cost of sampling per unit can be determined as follows:

E(Cost of not sample) = E(Cost of take sample) + Cost of sampling

76,000 = 79,997 – 2 × 2,000 + 2C

or C = (76,000 – 75,997)/2 = Rs 15

Thus, if the cost of sampling a unit is greater than a bare Rs 1.50, we would choose to accept the lot without sampling and if C < Rs 1.50, then we would take a sample of two.

Calculation of EVPI

We have

Expected cost of accepting a 10% defective lot	= 1,04,000 – 4,000
	= Rs 1,00,000
Expected cost of rejecting a 10% defective lot	= 94,000 – 4,000
	= Rs 90,000
Expected cost of accepting a 4% defective lot	= 44,000 – 4,000
	= Rs 40,000
Expected cost of rejecting a 4% defective lot	= 1,00,000 – 4,000
	= Rs 96,000

Since expected cost of rejecting a 10% defective lot is lower than that of accepting it, and expected cost of accepting a 4% defective lot is lower than that of rejecting it, we would take the respective decisions of rejecting a 10% and accepting a 4% defective lot. Since these would occur with probabilities 0.6 and 0.4, we have,

Expected cost under perfect information,

= 0.6 × 90,000 + 0.4 × 40,000 = Rs 70,000

Now, EVPI = Expected cost without information *minus* Expected cost under perfect information

= Rs 76,000 – 70,000

= Rs 6,000

43. (a) (i) Using EMV Criterion,

EMV = 100,000 × 0.5 + (–60,000) × 0.5 = Rs 20,000

It being positive, he should accept the contract.

(ii) Using EU criterion, the contract is acceptable if 0.5 × U(100,000) + 0.5 × U(–60,000) > (U(0)

Here 0.5 × 0.72 + 0.5 × 0.30 works out to be 0.51, which is smaller than U(0) = 0.55. Hence, he should not accept.

(b) If the contract is offered twice, we have

Outcome	*Pay-off*	*Probability*
Two successes	200,000	0.25
One success one failure	40,000	0.50
Two failures	(1,20,000)	0.25

(i) Using EMV Criterion,

EMV = 2,00,000 × 0.25 + 40,000 × 0.50 + (–1,20,000) × 0.50

= Rs 40,000

The EMV being positive, he should accept the offer.

(ii) U(2,00,000) × 0.25 + U(40,000) × 0.50 + U(–1,20,000) × 0.25 = 1.00 × 0.25 + 0.62 × 0.50 + 0 × 0.25 = 0.56

Since it is greater than $U(0) = 0.55$, he should accept.

44. Here $U(10{,}000) = 0.5 \times 30 + 0.5 \times (-2) = 14$

Now, if p be the probability for outcome of Rs 0, we have,

$p \times U(0) + (1 - p) \times P(20{,}000) =- U(10{,}000)$

$\therefore \quad p \times 0 + (1 - p)(20) = 14$

or $\quad p = 6/20 = 0.30$

Required probabilities are 0.30 and 0.70 respectively.

45. (a) EU of the opportunity $= 0.54 \times 0 + 0.30 \times 0.80 + 0.16 \times 1.00$
$= 0.40$

Since $U(x) = 0.40$, which corresponds to Rs 2,000, the manager would be prepared to pay a maximum of Rs 2,000 for the opportunity in question.

(b) $EU(\text{Alternative } A_1) = 0.5 \times 0.80 + 0.5 \times 0.40$
$= 0.4 + 0.2 = 0.60$

$EU(\text{Alternative } A_2) = 0.5 \times 0.90 + 0.5 \times 0.25$
$= 0.45 + 0.125 = 0.575$

$EU(\text{Alternative } A_3) = 0.5 \times 1.00 + 0.5 \times 0.0$
$= 0.5 + 0.0 = 0.5$

Alternative A_1 is, therefore, the best one.

46. (a) *For a single individual:*

With $U(2{,}00{,}000) = 0.90$, $U(-1{,}00{,}000) = 0.40$ and $U(0) = 0.70$,

$EU(\text{with contract}) = 0.5 \times 0.9 + 0.5 \times 0.4$
$= 0.45 + 0.20 = 0.65$

$EU(\text{without contract}) = 0.70$

Hence, he would not be inclined to accept offer of the contract.

For four individuals:

Gain per person = Rs 2,00,000/4 = Rs 50,000, $U(50{,}000) = 0.78$

Loss per person = Rs 1,00,000/4 = Rs 25,000, $U(-25{,}000) = 0.64$

$EU(\text{with contract}) = 0.5 \times 0.78 + 0.5 \times 0.64$
$= 0.39 + 0.32 = 0.71$

$EU(\text{without contract}) = 0.70$

Hence, it is preferable for four individuals to accept the contract as a group.

(b) *For* C_1,

$EU = 0.40 \times U(-40{,}000) + 0.60 \times U(70{,}000)$
$= 0.40 \times 0.60 + 0.60 \times 0.80 = 0.72 > U(0) = 0.70$

For C_2,

$EU = 0.25 \times U(70{,}000) + 0.50 \times U(15{,}000) + 0.25 \times U(-40{,}000)$
$= 0.25 \times 0.80 + 0.50 \times 0.73 + 0.25 \times 0.60 = 0.715 > U(0) = 0.70$

Conclusion: Both are acceptable but prefer C_2. Accept C_2.

CHAPTER 14

1. **Transition Probability Matrix**

From State	*Rs 0*	*To Rs 10*	*State Rs 20*	*Rs 30*	*Rs 40*	*Rs 50*	*Rs 60*
Rs 0	1	0	0	0	0	0	0
Rs 10	0.6	0	0	0.4	0	0	0
Rs 20	0	0.6	0	0	0.4	0	0
Rs 30	0	0	0.6	0	0	0.4	0
Rs 40	0	0	0	0.6	0	0	0.4
Rs 50	0	0	0	0	0	1	0
Rs 60	0	0	0	0	0	0	1

2. The transition probability matrix is given below:

	Transition Probability Matrix		
	S_1	S_2	S_3
S_1	0.5625	0.3125	0.1250
S_2	0.3750	0.5000	0.1250
S_3	0.2000	0.0500	0.7500

The entries in the matrix are obtained as follows. Of the 800 customers in S_1 in the beginning of the year, 250 are lost to S_2 and 100 to S_3, while the remaining 450 stay with S_1. Accordingly, the transition probabilities in row 1 are obtained as 450/800, 250/800, and 100/800; or 0.5625, 0.3215, and 0.1250 respectively. Similarly, other row values are calculated.

3. (a) From the given data, $Q(1) = (0.60 \quad 0.40)$, $Q(2) = (0.64 \quad 0.36)$ and $Q(3) = (0.656 \quad 0.344)$. Let the transition probability matrix, P, be

$$P = \begin{bmatrix} x_1 & 1 - x_1 \\ x_2 & 1 - x_2 \end{bmatrix}$$

Now, $Q(2) = Q(1) \times P$ and $Q(3) = Q(2) \times P$. Thus,

$$(0.64 \quad 0.36) = (0.64 \quad 0.40) \begin{pmatrix} x_1 & 1 - x_1 \\ x_2 & 1 - x_2 \end{pmatrix}$$

or $0.60x_1 + 0.40x_2 = 0.64$ (i)

$0.60(1 - x_1) + 0.40(1 - x_2) = 0.36$ (ii)

$$(0.656 \quad 0.344) = (0.64 \quad 0.36) \begin{pmatrix} x_1 & 1 - x_1 \\ x_2 & 1 - x_2 \end{pmatrix}$$

or $0.64x_1 + 0.36x_2 = 0.656$ (iii)

$0.64(1 - x_1) + 0.36(1 - x_2) = 0.344$ (iv)

From equations (i) and (iii), $x_1 = 0.8$ and $x_2 = 0.4$. Thus,

$$P = \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix}$$

(b) Expected market share in period 4, $Q(4)$ can be had as

$$Q(4) = Q(3) \times P = (0.656 \quad 0.344)\begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix}$$

$$= (0.6624 \quad 0.3376)$$

∴ Market shares are: 66.24% and 33.76%.

(c) The actual market share of 66% and 34% are very close to the shares estimated above. Hence, there appears to be no reason to revise the transition probability matrix.

4. From the given information, we have

Transition probability Matrix

	On time	*Late*
On time	0.70	0.30
Late	0.90	0.10

If q_1 and q_2 be the long-run probabilities for being on time and late respectively, we have

$$q_1 = 0.70q_1 + 0.90q_2$$
$$q_2 = 0.30q_1 + 0.10q_2$$

Accordingly,
$$0.30q_1 - 0.90q_2 = 0 \quad \text{(i)}$$
$$-0.30q_1 + 0.90q_2 = 0 \quad \text{(ii)}$$
$$q_1 + q_2 = 1 \quad \text{(iii)}$$

Solving equations (ii) and (iii) simultaneously, we get $q_1 = 0.75$ and $q_2 = 0.25$.

Thus, in the long run, the employee is expected to be on time and late with probabilities 0.75 and 0.25 respectively.

5. (a) Evidently, brand y has more loyal customers as (i) it retains 95% of its customers, and (ii) more customers are shifting from the other brand to this (0.10) from this brand to the other (0.05).

Let q_1 and q_2 be the projected market shares of the two brands. Thus,

$$q_1 = 0.90q_1 + 0.05q_2$$
$$q_2 = 0.10q_1 + 0.95q_2$$
$$q_1 + q_2 = 1$$

Solving these, we get $q_1 = 1/3$ and $q_2 = 2/3$.

(b) Let q_1, q_2 and q_3 be the long-run market shares of brands X, Y and Z respectively. Accordingly,

$$q_1 = 0.80q_1 + 0.05q_2 + 0.40q_3 \quad \text{(i)}$$
$$q_2 = 0.10q_1 + 0.75q_2 + 0.30q_3 \quad \text{(ii)}$$
$$q_3 = 0.10q_1 + 0.20q_2 + 0.30q_3 \quad \text{(iii)}$$
$$q_1 + q_2 + q_3 = 1 \quad \text{(iv)}$$

Considering equations (i), (ii) and (iv), and re-arranging,

$$0.20q_1 - 0.05q_2 - 0.40q_3 = 0$$
$$-0.10q_1 + 0.25q_2 - 0.30q_3 = 0$$
$$q_1 + q_2 + q_3 = 1$$
$$\Delta_1 = 0.26,\ \Delta_1 = 0.115,\ \Delta_2 = 0.100,\ \Delta_3 = 0.045$$

Thus, $q_1 = 0.115/0.26 = 0.44$, $q_2 = 0.100/0.26 = 0.39$ and $q_3 = 0.045/0.26 = 0.17$.

The expected market shares are 44%, 39% and 17%. Brand Y is expected to suffer from the introduction of new brand.

6. In equilibrium, the firm C would hold the entire market. This is because this firm retains all the customers that reach it (the transition probability $C - C$ being equal to 1).

From the given transition probabilities, the equilibrium probabilities q_1, q_2, and q_3 may be stated as follows:

$q_1 = 0.80q_1 + 0.15q_2,$
$q_2 = 0.12q_1 + 0.70q_2,$
$q_3 = 0.08q_1 + 0.15q_2 + q_3$, and
$q_1 + q_2 + q_3 = 1$

Rearranging the first two of the equations and taking the last equation alongside, we get

$$0.20q_1 - 0.15q_2 = 0$$
$$-0.12q_1 + 0.30q_2 = 0$$
$$q_1 + q_2 + q_3 = 1$$

In matrix notation,

$$\begin{pmatrix} 0.20 & -0.15 & 0 \\ -0.12 & 0.30 & 0 \\ 1.00 & 1.00 & 1.00 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Here, $\Delta = 0.360$, $\Delta_1 = 0$, $\Delta_2 = 0$, and $\Delta_3 = 0.360$. Thus, $q_1 = \Delta_1/\Delta = 0/0.360 = 0$, $q_2 = \Delta_2/\Delta = 0/0.360 = 0$, and $q_3 = \Delta_3/\Delta = 0.360/0.360 = 1$. Accordingly, the shares of the firms *A* and *B* would be nil and *C* would have cent per cent share of the market. It is true that *C* is an absorbing state.

7. (a) In accordance with the given information, the transition probability matrix is given below:

Transition Probability Matrix		
	AA	*BB*
AA	0.75	0.25
BB	0.40	0.60

(b) Calculation of probabilities:

(i) The probability of a currently *AA* buyer to buy Cola *BB* in the next-to-next purchase would be given by element, 1, 2 of the matrix P^2, where

$$P = \begin{pmatrix} 0.75 & 0.25 \\ 0.40 & 0.60 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.75 & 0.25 \\ 0.40 & 0.60 \end{pmatrix}\begin{pmatrix} 0.75 & 0.25 \\ 0.40 & 0.60 \end{pmatrix} = \begin{pmatrix} 0.6625 & 0.3375 \\ 0.5400 & 0.4600 \end{pmatrix}$$

$\therefore$ Required probability = 0.3375

(ii) To get the required probability, we have Initial condition = (0.60 0.40),

$$P^3 = \begin{pmatrix} 0.75 & 0.25 \\ 0.40 & 0.60 \end{pmatrix}^3 = \begin{pmatrix} 0.631875 & 0.368125 \\ 0.589000 & 0.411000 \end{pmatrix}$$

Post-multiplying initial condition vector by column one of the above matrix, we get 0.614725. Thus, the probability that three periods from now, the customers would buy Cola *AA* is 0.61.

(iii) To determine long-run shares, q_1 and q_2, for the two Colas *AA* and *BB* respectively, we have

$q_1 = 0.75q_1 + 0.40q_2$ (i)
$q_2 = 0.25q_1 + 0.60q_2$ (ii)
Also, $1 = q_1 + q_2$ (iii)

Now, solving equations (ii) and (iii) simultaneously, we get $q_1 = 0.615$ and $q_2 = 0.385$.
The long-run market shares for the two Colas shall be 61.5% and 38.5% respectively.

8. (a) When frequent fliers make one flight in a month:

$$Q(2) = Q(0) \times P^2 = \begin{pmatrix}0.20 & 0.50 & 0.30\end{pmatrix}\begin{pmatrix}0.90 & 0.03 & 0.07\\ 0.15 & 0.80 & 0.05\\ 0.20 & 0.30 & 0.50\end{pmatrix}$$

$$= \begin{pmatrix}0.3957 & 0.4629 & 0.1414\end{pmatrix}$$

$\therefore$ Expected market shares: AA = 39.57%, BB = 46.29% and CC = 14.14%.

When frequent fliers make two flights in a month:

$$Q(4) = Q(0) \times P^4 = \begin{pmatrix}0.20 & 0.50 & 0.30\end{pmatrix}\begin{pmatrix}0.90 & 0.03 & 0.07\\ 0.15 & 0.80 & 0.05\\ 0.20 & 0.30 & 0.50\end{pmatrix}$$

$$= \begin{pmatrix}0.4965 & 0.3897 & 0.1138\end{pmatrix}$$

$\therefore$ Expected market shares: AA = 49.65%, BB = 38.97% and CC = 11.38%.

(b) When frequent fliers make one flight in a month:

If q_1, q_2 and q_3 be the respective shares of AA, BB and CC airlines in the long run, we have

$q_1 = 0.90q_1 + 0.15q_2 + 0.20q_3$ (i)

$q_2 = 0.03q_1 + 0.80q_2 + 0.30q_3$ (ii)

$q_3 = 0.07q_1 + 0.05q_2 + 0.50q_3$ (iii)

$q_1 + q_2 + q_3 = 1$ (iv)

Using equations (i), (ii) and (iv),

$$\begin{pmatrix}0.10 & -0.15 & -0.20\\ -0.03 & 0.20 & -0.30\\ 1 & 1 & 1\end{pmatrix}\begin{pmatrix}q_1\\ q_2\\ q_3\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$$

q_1 = 0.085/01365 = 0.6227 or 62.27%

q_2 = 0.036/01365 = 0.2637 26.37%

q_3 = 0.0155/01365 = 01136 11.36%

When frequent fliers make two flights in a month, the answer is same.

9. From the given information, we may derive transition probability matrix as follows:

	State		
	X	*Y*	*Z*
X	0	0	1
Y	2/3	0	1/3
Z	2/3	1/3	0

If the long-run proportionate visits to cities X, Y, and Z be q_1, q_2, and q_3 respectively, we can write

$q_1 = 0q_1 + 0.667q_2 + 0.667q_3$

$q_2 = 0q_1 + \quad 0q_2 + 0.333q_3$

$q_3 = \quad q_1 + 0.333q_2 + 0q_3$

Also, $q_1 + q_2 + q_3 = 1$

Rearranging the above equations, we get

$$q_1 - 0.667q_2 - 0.667q_3 = 0 \quad \text{(i)}$$
$$0q_1 + q_2 - 0.333q_3 = 0 \quad \text{(ii)}$$
$$-q_1 - 0.333q_2 + q_3 = 0 \quad \text{(iii)}$$
$$q_1 + q_2 + q_3 = 0 \quad \text{(iv)}$$

Taking equations (i), (ii) and (iv) and arranging in matrix notation, we get

$$\begin{pmatrix} 1 & -0.667 & -0.667 \\ 0 & 1 & -0.333 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

To solve for q_1, q_2 and q_3, we get

$$\Delta = 2.2222$$
$$\Delta_1 = 0.8888$$
$$\Delta_2 = 0.3333$$
$$\Delta_3 = 1.0000$$

Accordingly, $q_1 = \Delta_1/\Delta = 0.8888/2.2222 = 0.40$

$q_2 = \Delta_2/\Delta = 0.3333/2.2222 = 0.15$

$q_3 = \Delta_3/\Delta = 1.0000/2.2222 = 0.45$

Hence, the salesman would visit the three cities 40%, 15%, and 45% times respectively.

10. Using the given information, we may express the initial condition $Q(0)$ and transition probability matrix P as follows:

$$Q(0) = \begin{pmatrix} 0.40 & 0.30 & 0.30 \end{pmatrix}$$

$$P = \begin{pmatrix} 0.88 & 0.07 & 0.05 \\ 0.12 & 0.85 & 0.03 \\ 0.08 & 0.10 & 0.82 \end{pmatrix}$$

Both, the row- and column-wise, the values for *Business Today*, *Business Line*, and *Business Life* are shown.

From the above, we may determine the likely share of the *Business Today* by multiplying the row vector $Q(0)$ with first column of the matrix P.

Accordingly, expected share of *Business Today* in the next year = $0.40 \times 0.88 + 0.30 \times 0.12 + 0.30 \times 0.08 = 0.412$

Now, we may assess the desirability of the policy as follows:

At present:

Profit for *Business Today* = 0.15 (40% of Rs 50,00,000)
= Rs 3,00,000

Proposed Policy:

Expected profit for *Business Today*
= 0.15(41.2% of Rs 50,00,000) – 50,000
= Rs 2,59,000

Clearly it is not advisable to adopt the proposed policy.

11. From the given data, we have

$$Q(0) = \begin{pmatrix}0.3 & 0.4 & 0.3\end{pmatrix} \quad \text{and} \quad P = \begin{bmatrix} 0.85 & 0.08 & 0.07 \\ 0.05 & 0.90 & 0.05 \\ 0.15 & 0.07 & 0.78 \end{bmatrix}$$

$$Q(1) = Q(0) \times P = \begin{pmatrix}0.32 & 0.405 & 0.275\end{pmatrix}$$

$$Q(2) = Q(0) \times P^2 = \begin{pmatrix}0.3335 & 0.40935 & 0.25715\end{pmatrix}$$

Accordingly,

Market shares next year, Jan. 1 are 32%, 40.5% and 27.5% respectively.

Market shares next to next year, Jan. 1 would be 33.35%, 40.93% and 25.72% respectively.

Market shares in equilibrium:

If q_1, q_2 and q_3 be the respective shares in equilibrium,

We have

$$\begin{aligned} 0.85q_1 + 0.05q_2 + 0.15q_3 &= q_1 \Rightarrow \;\; 0.15q_1 - 0.05q_2 - 0.15q_3 = 0 \\ 0.08q_1 + 0.90q_2 + 0.07q_3 &= q_2 \Rightarrow -0.08q_1 + 0.10q_2 - 0.07q_3 = 0 \\ 0.07q_1 + 0.05q_2 + 0.78q_3 &= q_3 \Rightarrow -0.07q_1 - 0.05q_2 + 0.22q_3 = 0 \\ q_1 + q_2 + q_3 &= 1 \Rightarrow \;\; q_1 + q_2 + q_3 = 1 \end{aligned}$$

Using above equations except the third one,

$$\begin{bmatrix} 0.15 & -0.05 & -0.15 \\ -0.08 & 0.10 & -0.07 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Delta = 0.052$, $\Delta_1 = 0.0185$, $\Delta_2 = 0.0225$ and $\Delta_3 = 0.011$

Thus,
$$q_1 = \Delta_1/\Delta = 0.0185/0.052 = 0.3558$$
$$q_2 = \Delta_2/\Delta = 0.0225/0.052 = 0.4327$$
$$q_3 = \Delta_3/\Delta = 0.011/0.052 = 0.2115$$

Thus, long-run shares of the three are expected to be 35.58%, 43.27% and 21.15% respectively for *A*, *B* and *C*.

12. (a) Given, initial condition Q(0) = (0.40 0.40 0.20), and transition probability matrix,

	ABC	*PQR*	*XYZ*
ABC	0.80	0.16	0.04
PQR	0.12	0.84	0.04
XYZ	0.18	0.06	0.76

The share of market expected to be held by different firms on January 1, 2004, may be obtained by $Q(0)P^2$. Accordingly,

$$Q(0)P^2 = \begin{pmatrix}0.40 & 0.40 & 0.20\end{pmatrix} \begin{bmatrix} 0.80 & 0.16 & 0.04 \\ 0.12 & 0.84 & 0.04 \\ 0.18 & 0.06 & 0.76 \end{bmatrix}^2$$

$$= (0.40576 \quad 0.42176 \quad 0.17248)$$

$\therefore$ Expected shares on the desired date are:

ABC = 40.6%; *PQR* = 42.2% and *XYZ* = 17.2%

(b) *Determination of equilibrium market shares:* Let q_1, q_2, and q_3 be the respective shares of *ABC*, *PQR* and *XYZ*. Accordingly,

$$q_1 = 0.80q_1 + 0.12q_2 + 0.18q_3,$$
$$q_2 = 0.16q_1 + 0.84q_2 + 0.06q_3,$$
$$q_3 = 0.04q_1 + 0.04q_2 + 0.76q_3, \text{ and}$$
$$q_1 + q_2 + q_3 = 1$$

Rearranging the above equations,

$$0.20q_1 - 0.12q_2 - 0.18q_3 = 0 \quad \text{(i)}$$
$$-0.16q_1 + 0.16q_2 - 0.06q_3 = 0 \quad \text{(ii)}$$
$$-0.04q_1 - 0.04q_2 + 0.24q_3 = 0 \quad \text{(iii)}$$
$$q + q_2 + q_3 = 1 \quad \text{(iv)}$$

Putting equation (i), (ii) and (iv) in matrix notation.

$$\begin{pmatrix} 0.20 & -0.12 & -0.18 \\ -0.16 & 0.16 & -0.06 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Now, we may solve for q_1, q_2, and q_3 to get,

$$\Delta = 0.0896,\ \Delta_1 = 0.0360,\ \Delta_2 = 0.0408,\ \Delta_3 = 0.0128$$

$$q_1 = \frac{0.0360}{0.0896} = 40.18\%$$

$$q_2 = \frac{0.0408}{0.0896} = 45.54\%$$

$$q_3 = \frac{0.0128}{0.0896} = 14.28\%$$

13. (a) Market shares expected after two months, $Q(2)$, can be obtained as $Q(0) \times P^2$.

$$\therefore \qquad Q(2) = \begin{pmatrix} 0.45 & 0.25 & 0.30 \end{pmatrix} \begin{bmatrix} 0.80 & 0.10 & 0.10 \\ 0.03 & 0.95 & 0.02 \\ 0.20 & 0.05 & 0.75 \end{bmatrix}^2$$

$$= \begin{pmatrix} 0.4059 & 0.3391 & 0.2550 \end{pmatrix}$$

$\therefore$ Expected market shares after two months are:

$$X = 40.59\%,\ Y = 33.91\% \text{ and } Z = 25.50\%$$

(b) Let the long run market shares be q_1, q_2 and q_3 respectively for *X*, *Y* and *Z*. Thus,

$$q_1 = 0.80q_1 + 0.03q_2 + 0.20q_3 \quad \text{(i)}$$
$$q_2 = 0.10q_1 + 0.95q_2 + 0.05q_3 \quad \text{(ii)}$$
$$q_3 = 0.10q_1 + 0.02q_2 + 0.75q_3 \quad \text{(iii)}$$
$$q_1 + q_2 + q_3 = 1 \quad \text{(iv)}$$

Using equations (i), (ii) and (iv), in matrix notation

$$\begin{bmatrix} 0.20 & -0.03 & -0.20 \\ -0.10 & 0.05 & -0.05 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$q_1 = 0.0115/0.0485 = 0.2371$ or 23.71%
$q_2 = 0.0300/0.0485 = 0.6186$ or 61.86%
$q_3 = 0.0070/0.0485 = 0.1443$ or 14.43%

(c) The actual market shares in the long run are not likely to be close to the expected market shares because changing circumstances like consumer preferences, introduction of other brands, etc. may render transition probability matrix invalid.

14. Here,

$$Q(0) = \begin{pmatrix}0.45 & 0.30 & 0.25\end{pmatrix} \text{ and } P = \begin{bmatrix}0.80 & 0.14 & 0.06 \\ 0.03 & 0.90 & 0.07 \\ 0.06 & 0.09 & 0.85\end{bmatrix}$$

$$Q(1) = Q(0) \times P = \begin{pmatrix}0.3840 & 0.3555 & 0.2605\end{pmatrix}$$

Expected market shares of *ABC*, *XYZ* and *PQR* on Jan. 1 next year are 38.40%, 35.55% and 20.05% respectively. Let q_1, q_2 and q_3 be the equilibrium market shares of the firms.

$$0.80q_1 + 0.03q_2 + 0.06q_3 = q_1 \Rightarrow 0.20q_1 - 0.03q_2 - 0.06q_3 = 0 \quad \text{(i)}$$
$$0.14q_1 + 0.90q_2 + 0.09q_3 = q_2 \Rightarrow -0.14q_1 + 0.10q_2 - 0.09q_3 = 0 \quad \text{(ii)}$$
$$0.06q_1 + 0.07q_2 + 0.85q_3 = q_3 \Rightarrow -0.06q_1 - 0.07q_2 + 0.15q_3 = 0 \quad \text{(iii)}$$
$$q_1 + q_2 + q_3 = 1 \quad \text{(iv)}$$

From equations (i), (ii) and (iv), we have

$$\begin{bmatrix}0.20 & -0.03 & -0.06 \\ -0.14 & 0.10 & -0.09 \\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}q_1 \\ q_2 \\ q_3\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ 1\end{bmatrix}$$

$\Delta = 0.0509$, $\Delta_1 = 0.0087$, $\Delta_2 = 0.0264$, $\Delta_3 = 0.0158$

$\therefore$ $q_1 = 0.0087/0.0509 = 0.1709$
$q_2 = 0.0264/0.0509 = 0.5187$
$q_3 = 0.158/0.0509 = 0.3104$

Thus, expected shares of the firms in equilibrium are 17.09%, 51.87% and 31.04% respectively.

15. *Policy 1:*
From the given information, it is evident that all the bearings will be replaced at each inspection and none will be worn out. Thus, in such a situation, cost per bus per inspection = Rs 250.
Policy 2:
For this, let the states be redefined as *G* (that is, new) and *A* (alternative) and states 3 and 4 go immediately to state *G*. The transition probability matrix shall be as follows:

	G	*A*
G	0.15	0.85
A	0.3 + 0.2	0.50

From this matrix, we have

$$q_1 = 0.15q_1 + 0.50q_2$$
$$q_2 = 0.85q_1 + 0.50q_2$$
$$q_1 + q_2 = 1$$

Solving these, we get $q_1 = 10/27$ and $q_2 = 17/27$.
Thus, at next inspection, (10/27) × (15/100) + (17/27) × (30/100) = 11/45 is the fraction needing replacement since it is in state 3 and (17/27) × (20/100) = 17/135 is the fraction worn out. Accordingly, the cost per bus per inspection = (11/45) × 250 + (17/35) × 1200 = Rs 212.22.
Policy 2 is better, therefore.

16. Let W, F and P indicate, respectively, the states of running well, running fairly well, and running poorly. Also, let q_1, q_2 and q_3 be the equilibrium probabilities of the three states respectively.

Policy 1:

The transition probability matrix is:

	W	F	P
W	0.7	0.2	0.1
F	0	0.6	0.4
P	1	0	0

Further,
$$q_1 = 0.7q_1 + 0q_2 + q_3$$
$$q_2 = 0.2q_1 + 0.6q_2 + 0q_3$$
$$q_3 = 0.1q_1 + 0.4q_2 + 0q_3$$
$$q_1 + q_2 + q_3 = 1$$

Solving these, we get $q_1 = 0.556$, $q_2 = 0.278$, $q_3 = 0.167$.

Policy 2:

The transition probability matrix is

	W	F	P
W	0.7	0.2	0.1
F	1	0	0
P	1	0	0

Here
$$q_1 = 0.7q_1 + q_2 + q_3$$
$$q_2 = 0.2q_1$$
$$q_3 = 0.1q_1$$
$$q_1 + q_2 + q_3 = 1$$

Solving these, we get $q_1 = 0.769$, $q_2 = 0.154$, and $q_3 = 0.077$. Thus, downtime percentage is:

Policy 1: 16.7%, *Policy 2:* 15.4 + 7.7 = 23.1%.

17. As a first step, we calculate steady-state probabilities. Let q_1, q_2, q_3 and q_4 be the steady-state probabilities for the states 1, 2, 3 and 4 respectively. From the given information,

$$q_1 = 0q_1 + 0q_2 + 0q_3 + 0q_4$$
$$q_2 = 0.75q_1 + 0.50q_2 + 0q_3 + 0q_4$$
$$q_3 = 0.25q_1 + 0.50q_2 + 0.50q_3 + 0q_4$$
$$q_4 = 0q_1 + 0q_2 + 0.50q_3 + 0q_4$$
$$q_1 + q_2 + q_3 + q_4 = 1$$

The solution of above equations yields $q_1 = 0.182$, $q_2 = 0.273$, $q_3 = 0.364$, and $q_4 = 0.182$.

The probabilities indicate that on an average 18.2% of the days the machine will be overhauled, for 27.3% days it will be in good condition, and in 36.4% days it will be in fair condition. Similarly, of the total, in 18.2% days, it will be inoperative at the day-end. Using this information,

$$\text{Average cost of maintenance per day} = 0.182 \times 125 + 0.182 \times 75$$
$$= \text{Rs } 36.36$$

18. The transition probability matrix, considering the service departments as transient states and the production departments as absorbing states, may be expressed as follows:

Transition Probability Matrix

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ P_1 \\ P_2 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & P_1 & P_2 \end{array} \\ \begin{bmatrix} 0 & 0.25 & 0.05 & 0.40 & 0.30 \\ 0.10 & 0 & 0.25 & 0.35 & 0.30 \\ 0.30 & 0.15 & 0 & 0.15 & 0.40 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

From this,

$$Q = \begin{pmatrix} 0.00 & 0.25 & 0.05 \\ 0.10 & 0.00 & 0.25 \\ 0.30 & 0.15 & 0.00 \end{pmatrix} \text{ and } R = \begin{pmatrix} 0.40 & 0.30 \\ 0.35 & 0.30 \\ 0.15 & 0.40 \end{pmatrix}$$

Now,

$$(I - Q) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.00 & 0.25 & 0.05 \\ 0.10 & 0.00 & 0.25 \\ 0.30 & 0.15 & 0.00 \end{pmatrix}$$

$$= \begin{pmatrix} 1.00 & -0.25 & -0.05 \\ -0.10 & 1.00 & -0.25 \\ -0.30 & -0.15 & 1.00 \end{pmatrix}$$

Taking inverse of $(I - Q)$,

$$(I - Q)^{-1} = \frac{1}{0.903}\begin{pmatrix} 0.9625 & 0.2575 & 0.1125 \\ 0.1750 & 0.9850 & 0.2550 \\ 0.3150 & 0.2250 & 0.9750 \end{pmatrix}$$

Further,

$$(I - Q)^{-1} R = \frac{1}{903}\begin{pmatrix} 492 & 411 \\ 453 & 450 \\ 351 & 552 \end{pmatrix}$$

Now, the direct overhead matrix D, for S_1, S_2, and S_3 is given to be (60,000 25,500 60,500). Thus,

$$D(I - Q)^{-1} R = \begin{pmatrix} 60{,}000 & 25{,}500 & 60{,}500 \end{pmatrix}\begin{pmatrix} 492/903 & 411/903 \\ 453/903 & 450/903 \\ 351/903 & 552/903 \end{pmatrix}$$

$$= \begin{pmatrix} 69{,}000 & 77{,}000 \end{pmatrix}$$

Thus, the total overhead to be allocated to P_1 and P_2 would be Rs 69,000 and Rs 77,000 respectively.

19. From the given data,

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ P_1 \\ P_2 \end{array}\begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & P_1 & P_2 \end{array} \\ \left[\begin{array}{ccc:cc} 0 & 0.15 & 0.25 & 0.40 & 0.20 \\ 0.20 & 0 & 0.05 & 0.35 & 0.40 \\ 0.35 & 0.20 & 0 & 0.25 & 0.20 \\ \hdashline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

So,

$$Q = \begin{pmatrix} 0 & 0.15 & 0.25 \\ 0.20 & 0 & 0.05 \\ 0.35 & 0.20 & 0 \end{pmatrix} \quad R = \begin{pmatrix} 0.40 & 0.20 \\ 0.35 & 0.40 \\ 0.25 & 0.20 \end{pmatrix}$$

$$I - Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0.15 & 0.25 \\ 0.20 & 0 & 0.05 \\ 0.35 & 0.20 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -0.15 & -0.25 \\ -0.20 & 1 & -0.05 \\ -0.35 & -0.20 & 1 \end{pmatrix}$$

Allocated expenses, $E = K \times (I - Q)^{-1} \times R$

where $k = (6{,}000 \quad 8{,}000 \quad 68{,}500)$, the vector of direct expenses

$$\therefore \qquad E = (6{,}000 \quad 8{,}000 \quad 68{,}500) \begin{pmatrix} 1.15133 & 0.23259 & 0.29946 \\ 0.25294 & 1.06120 & 0.11630 \\ 0.45355 & 0.29365 & 1.12807 \end{pmatrix} \begin{pmatrix} 0.40 & 0.20 \\ 0.35 & 0.40 \\ 0.25 & 0.20 \end{pmatrix}$$

$$= (46{,}500 \quad 36{,}000)$$

$$\therefore \qquad \text{Total cost} = (60{,}000 \quad 74{,}000) + (46{,}500 \quad 36{,}000)$$

$$= (1{,}06{,}500 \quad 1{,}10{,}000)$$

20. To consider this problem as absorbing chains, we express the holding company and subsidiary companies as transient states H_1, S_1, and S_2 respectively, and the outside shareholders of these, O_1, O_2, O_3, as the absorbing states. We first obtain the transition probability matrix as follows:

States

$$P = \begin{array}{c} \\ H_1 \\ S_1 \\ S_2 \\ O_1 \\ O_2 \\ O_3 \end{array} \begin{array}{c} \begin{array}{cccccc} H_1 & S_1 & S_2 & O_1 & O_2 & O_3 \end{array} \\ \begin{bmatrix} 0 & 0.03 & 0.06 & 0.91 & 0 & 0 \\ 0.60 & 0 & 0.10 & 0 & 0.30 & 0 \\ 0.80 & 0.10 & 0 & 0 & 0 & 0.10 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Here,

$$Q = \begin{pmatrix} 0.00 & 0.03 & 0.06 \\ 0.60 & 0.00 & 0.10 \\ 0.80 & 0.10 & 0.00 \end{pmatrix} \text{ and } R = \begin{pmatrix} 0.91 & 0.00 & 0.00 \\ 0.00 & 0.30 & 0.00 \\ 0.00 & 0.00 & 0.10 \end{pmatrix}$$

Now,

$$(I-Q) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.00 & 0.03 & 0.06 \\ 0.60 & 0.00 & 0.10 \\ 0.80 & 0.10 & 0.00 \end{pmatrix} = \begin{pmatrix} 1 & -0.03 & -0.06 \\ -0.60 & 1 & -0.10 \\ -0.80 & -0.10 & 1 \end{pmatrix}$$

Taking inverse of $(I - Q)$, we get

$$(I-Q)^{-1} = \frac{1000}{918}\begin{pmatrix} 0.990 & 0.036 & 0.063 \\ 0.680 & 0.952 & 0.136 \\ 0.860 & 0.124 & 0.982 \end{pmatrix}$$

Now,

$$(I-Q)^{-1} R = \frac{1}{918}\begin{pmatrix} 900.9 & 10.8 & 6.3 \\ 618.8 & 285.6 & 13.6 \\ 782.6 & 37.2 & 98.2 \end{pmatrix}$$

With the net profits matrix N, for H_1, S_1 and S_2 given as $N = (30{,}000 \quad 17{,}500 \quad 5{,}000)$, the matrix of the amount of profits going to outside shareholders D is given as

$$D = N(I-Q)^{-1} R = (45{,}500 \quad 6{,}000 \quad 1{,}000)$$

Evidently, the total profit to the outside shareholders is 45,500 + 6,000 + 1,000 = 52,500, which is equal to total net profit earned by three companies separately, which works out to be 30,000 + 17,500 + 5,000 = Rs 52,500.

21. From the given data, we have

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\overset{\textit{Category}}{\begin{array}{ccc:cc} 1 & 2 & 3 & 4 & 5 \\ \hline 0.4 & 0.2 & 0.1 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.1 & 0.1 & 0.2 \\ \hdashline 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{array}}$$

Thus,

$$Q = \begin{pmatrix} 0.4 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.1 \end{pmatrix} \text{ and } I - Q = \begin{pmatrix} 0.6 & -0.2 & -0.1 \\ -0.3 & 0.6 & -0.1 \\ -0.2 & -0.4 & 0.9 \end{pmatrix}$$

$$(I-Q)^{-1} = \begin{bmatrix} 2.29358 & 1.00917 & 0.36697 \\ 1.33028 & 2.38532 & 0.41284 \\ 1.10092 & 1.28440 & 1.37615 \end{bmatrix}$$

Expected amounts that may be eventually (i) collected and (ii) defaulted,

$$E = (20{,}00{,}000 \quad 40{,}00{,}000 \quad 30{,}000)\ (I-Q)^{-1}\begin{pmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.2 \end{pmatrix}$$

$$= (48{,}34{,}862 \quad 41{,}65{,}138).$$

CHAPTER 15

1. (a)

	b_1	b_2	*Row Minima*	
a_1	3	7	3*	Saddle point = a_1b_1
a_2	–5	5	–5	
Column Maxima	3*	7		

(b)

	b_1	b_2	b_3	b_4	*Row Minima*	
a_1	5	8	2	4	2*	
a_2	2	6	1	3	1	Saddle point = a_1b_3
Column Maxima	5	8	2*	4		

2.

	b_1	b_2	b_3	b_4	*Row Minima*	
a_1	5	–4	5	8	–4	
a_2	6	2	0	–5	–5	Saddle point = a_3b_1
a_3	7	12	8	7	7*	
a_4	2	8	–6	5	–6	
Column Maxima	7*	12	8	8		

Optimal strategies: for $A = a_3$, for $B = b_1$; Value of game = 7

3.

	b_1	b_2	b_3	b_4	b_5	b_6	*Row Minima*
a_1	18	8	18	8	18	8	8*
a_2	15	6	15	6	15	6	6
a_3	18	8	18	8	18	8	8*
a_4	–15	–6	–15	–6	–15	–6	–15
Column Maxima	18	8*	18	8*	18	8*	

The game is strictly determinable. It has multiple saddle points. They are: a_1b_2, a_1b_4, a_1b_6, a_3b_2, a_3b_4 and a_3b_6. Value of game = 8. It is not fair since $V \neq 0$.

4.

	B_1	B_2	B_3	B_4	B_5	*Row Minima*
A_1	8	10	–3	–8	–12	–12
A_2	3	6	0	6	12	0*
A_3	7	5	–2	–8	17	–8
A_4	–11	12	–10	10	20	–11
A_5	–7	0	0	6	2	–7
Column Max.	8	12	0*	10	20	

(a) Maximum strategy = A_2, Minimum strategy = B_3
(b) Yes, since a saddle point exists
(c) $V = 0$
(d) Yes, since in the game value = 0

5.

	B_1	B_2	B_3	*Row Minima*
A_1	5	9	3	3
A_2	6	–12	–11	–12
A_3	8	16	10	8*
Column Maxima	8*	16	10	

The saddle point is evidently given by A_3B_1. Thus, optimal strategies for A and B are A_3 and B_1 respectively. Game value = 8.

Principle of dominance

Step 1: $R3$ dominates $R1$ and $R2$ both. Delete rows 1 and 2.

Step 2: $C1$ dominates $C2$ and $C3$ both. Delete columns 2 and 3 of the reduced matrix. This leaves only a single value = 8, which is the saddle point.

6.

		XYZ		
		Major Change	*No Major Change*	*Row Minima*
ABC	Major Change	0	*a*	0*
	No Major Change	–*b*	0	–*b*
	Column Maxima	0*	*a*	

Optimal strategies: Major change by each of the companies. $V = 0$

7. Strategies available to Kumar:
   1. One plane on installation I and five on installation II
   2. Two planes on installation I and four on installation II
   3. Three planes on installation I and three on installation II
   4. Four planes on installation I and two on installation II
   5. Five planes on installation I and one on installation II

   Enemies' strategies:
   1. One plane on installation I and four on installation II
   2. Two planes on installation I and two on installation II
   3. Three planes on installation I and two on installation II
   4. Four planes on installation I and one installation II

Kumar's strategies	*Enemy's strategies* 1	2	3	4	*Row Minima*
1	4	2	1	0	0
2	1	3	0	–1	–1
3	–2	2	2	–2	–2
4	–1	0	3	1	–1
5	0	1	2	4	0
Column Maxima	4	3	3	4	

Evidently, the game has no saddle point.

8.

Godrej & Boyce	*Hindustan Level Ltd* *No advertising (1)*	*Med. Advertising (2)*	*Heavy adv. (3)*	*Row Minima*
No advertising (1)	50	40	28	28
Medium advertising (2)	70	50	45	45
Heavy advertising (3)	75	47.5	50	47.5
Column maxima	75	50	50	

It is clear that the game has no saddle point. Thus, the players need to play mixed strategies.

Row 3 dominates Row 1. Delete Row 1.

Column 2 dominates column 1 in the reduced matrix, so delete column 1. The reduced matrix is:

50	45
47.5	50

Let Godrej and Boyce play strategies 2 and 3 with probabilities p and $1 - p$ respectively; and Hindustan Lever Ltd plays strategies 2 and 3 with probabilities y and $1 - y$ respectively.

$$x = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{50 - 47.5}{(50 + 50) - (47.5 + 45)} = 0.33$$

$$y = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{50 - 45}{(50 + 50) - (45 + 47.5)} = 0.67$$

$$v = \frac{(a_{11} \times a_{22}) - (a_{12} \times a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{(50 \times 50) - (45 \times 47.5)}{(50 + 50) - (45 + 47.5)} = 48.33$$

∴ Optimal strategies: for Godrej & Boyce (0, 0.33, 0.67)
for Hindustan Lever (0, 0.67,0.33)

Market share for Godrej & Boyce = 48.33

9. From the given information, the pay-off matrix is drawn here:

First partner's strategy	*Second partner's strategy* 0	*1*	*2*	*3*	*4*	*Row Minima*
0	0	–1	0	1	2	–1
1	1	0	0	1	2	0*
2	0	0	0	1	2	0*
3	–1	–1	–1	0	2	–1
4	–2	–2	–2	–2	0	–2
Column Maxima	1	0*	0*	1	2	

There are four saddle points here. Thus, there are four pairs of optimal strategies: 1-1, 1-2, 2-1 and 2-2. The game is indeed a fair one as the game value is zero.

10. On the basis of the given information, the pay-off matrix can be stated as follows:

Firm 1's strategy	*Firm 2's strategy* *A*	*B*	*C*	*Row Minima*
A	0	1	–1	–1
B	–1	0	–1	–1
C	1	1	0	0*
Column Maxima	1	1	0*	

Since a saddle point exists corresponding to $C - C$, both the firms should open their branches in city C.

11. Let a_1, b_1: 5 paise coin; a_2, b_2: 10 paise coin and a_3, b_3: 20 paise coin.

		Player B b_1	b_2	b_3	*Row Minima*
Player A	a_1	–5	10	20	–5
	a_2	5	–10	–10	–10
	a_3	5	–20	–20	–20
Column Maxima		5	10	20	

The game has no saddle point. We attempt to reduce the size of the given matrix.
Row 2 dominates Row 3. Delete Row 3.
In the reduced matrix, column 2 dominates column 3. Delete column 3. The revised matrix is:

	b_1	b_2
a_1	–5	10
a_2	5	–10

With usual notations, $x = \dfrac{-10-5}{(-5-10)-(5+10)} = \dfrac{1}{2}$

$$y = \frac{-10-10}{(-5-10)-(5+10)} = \frac{2}{3}$$

$$v = \frac{(-5)(-10)-(5\times 10)}{(-5-10)-(5+10)} = 0 \;=0$$

∴ Optimal strategy: $A(1/2, 1/2, 0)$, $B(2/3, 1/3, 0)$, $V = 0$

12.

Firm A's strategy	*Firm B's strategy*			*Row Minima*
	No promo.	*Mod. Promo.*	*High promo.*	
No Prmotion	0	2	–15	–15
Mod. Promotion	12	18	–4	–4
Price cut	20	15	6	6*
Col. Maxima	20	15	6*	

(a) The game has a saddle point. Optimal strategies are: Firm A: Price cut, Firm B: High promotion.
(b) Value of the game, $V = 6$. The game is strictly determinable. It is not fair since $V \neq 0$.
(c) Yes. Given the situation, there is no better option.
(d) The strategies need not maximize profits for either of the firms but none can obtain higher profits in the given circumstances.

13. The row minima are –2, 12 and 10 respectively, while column maxima values are 16, 14 and 13 respectively. The game has no saddle point since maximin and minimax values are not equal.
To check for dominance, we find that column 2 dominates column 1. Hence, column 1 can be deleted. Further, row 3 dominates row 1. So row 1 is deleted. The game is now reduced to the size 2 × 2.
With usual notations,

$$x = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{10-14}{(12+10)-(14+13)} = \frac{-4}{-5} = 0.8$$

$$y = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{10-13}{(12+10)-(14+13)} = \frac{-3}{-5} = 0.6$$

$$v = \frac{(a_{11}\times a_{22})-(a_{21}\times a_{12})}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{12\times 10-14\times 13}{(12+10)-(14+13)} = \frac{-62}{-5} = 12.4$$

Thus, optimal policy is:
For A (0, 0.8, 0.2), For B (0, 0.6, 0.4); and game value = 12.4.

14. From the given information, the pay-off matrix may be derived as shown here:

A's strategy	*P's strategy*		
	Do nothing	*Price cut*	*Row Minina*
Advertise	7	–2	–2*
Do nothing	0	–5	–5
Column Maxima	7	–2*	

Evidently, a saddle point exists. The optimal strategy for firm A is to advertise, and for firm P to go for price cut.

15. With row minima values as –4, 9 and –5, and the column maxima values as 10, 20 and 16, the game has no saddle point. Using rule of dominance,
Step 1: A_2 dominates A_1. Delete A_1.
Step 2: B_1 dominates B_2 in the reduced matrix. Delete B_2. This leaves the matrix as:

	B_1	B_3
A_2	9	16
A_3	10	–5

Now, let x be the probability with which A plays A_2 and y be probability that B plays B_1. We have

$$x = \frac{-5-10}{(9-5)-(16+10)} = \frac{15}{22}$$

$$y = \frac{-5-16}{(9-5)-(16+10)} = \frac{21}{22}$$

$$v = \frac{(9)(-5)-(16\times 10)}{(9-5)-(16+10)} = \frac{205}{22}$$

Accordingly,
Optimal strategy for A: (0, 15/22, 7/22)
Optimal strategy for B: (21/22, 0, 1/22)
Values of the game = 205/22

16. The given game has no saddle point. We observe,
Row 3 dominates Row 1. So we delete Row 1.
Column 2 dominates column 1 in the reduced matrix. Deletion of this column leads to the following matrix:

	Med. advt.	*Small advt.*
Med. advt.	60	95
Small advt.	90	65

With usual notations,

$$x = \frac{65-90}{(60+65)-(90+95)} = \frac{5}{12}$$

$$y = \frac{65-95}{(60+65)-(90+95)} = \frac{1}{2}$$

$$v = \frac{(60\times 65)-(90\times 95)}{(60+65)-(90+95)} = 77\frac{1}{2}$$

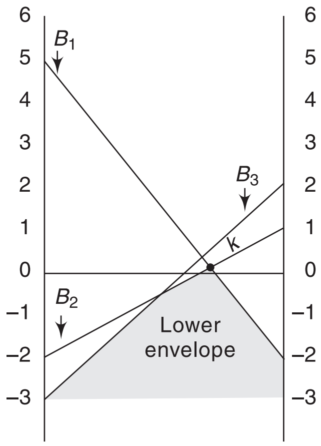
$\therefore$ Optimal strategy for A = (0, 5/12, 7/12)
Optimal strategy for B = (0, 1/2, 1/2)

Value of game = $77\frac{1}{2}$

17. This problem does not have a saddle point. Both the parties have to play mixed strategies. We can attempt to reduce it to a 2 × 2 problem for solution. The graph is shown here.
The highest point in the lower envelope is k, given by the intersection of B_1 and B_2. Thus, the 2 × 2 problem is:

	B_1	B_2
A_1	–2	1
A_2	5	–2

If x is the probability that management plays strategy A_1 and y is the probability that union plays B_1, we have



$$x = \frac{-2-5}{(-2-2)-(5+1)} = \frac{7}{10}$$

$$y = \frac{-2-1}{(-2-2)-(5+1)} = \frac{3}{10}$$

$$v = \frac{(-2)(-2)-(5)(1)}{(-2-2)-(5+1)} = \frac{1}{10}$$

Thus, optimal strategy for the management is (7/10, 3/10), for the union it is (3/10, 7/10, 0) and the game value = 1/10.

18. For F_1, the strategies are:

a_1: make 300 colour sets
a_2: make 300 black and white sets

For F_2, the strategies are:

b_1: make 600 colour sets
b_2: make 300 colour and 300 black and white sets
b_3: make 600 black and white sets

Foe the combination of a_1b_1, the profit to F_1 would be $\frac{300}{300+600} \times 300 \times 200$ = Rs 20,000

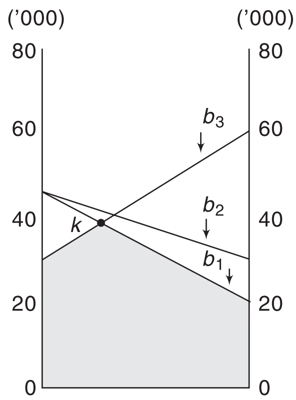
wherein (300/(300 + 600)) represents share of market for F_1, 300 is the total market for colour television sets and Rs 200 is the profit per set.

In a similar say, other profit figures may be obtained. They are shown in the matrix below.

		F_2's strategy		
		b_1	b_2	b_3
F_1's strategy	a_1	20,000	30,000	60,000
	a_2	45,000	45,000	30,000

Since no saddle point exists, determine optimal mixed strategy. From the graph, we find that maximum point in the lower envelope is given by strategies b_1 and b_3 of F_2.

With usual notations,



$$x = \frac{a_{22} - a_{21}}{(a_{11} - a_{22}) - (a_{12} - a_{21})}$$

$$= \frac{30{,}000 - 45{,}000}{(20{,}000 + 30{,}000) - (60{,}000 + 45{,}000)} = \frac{3}{11}$$

$$y = \frac{a_{22} - a_{12}}{(a_{11} - a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{30{,}000 - 60{,}000}{(20{,}000 + 30{,}000) - (60{,}000 + 45{,}000)} = \frac{6}{11}$$

$$v = \frac{(a_{11} \times a_{22}) - (a_{12} \times a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{20{,}000 \times 30{,}000 - 60{,}000 \times 45{,}000}{(20{,}000 + 30{,}000) - (60{,}000 + 45{,}000)} = 38{,}182.$$

Thus, optimal for F_1 is (3/11, 8/11), for F_2 is (6/11, 0, 5/11) and the game value is Rs 38,182.

19. With row minima values as –5, –70, –5 and –80, and column maxima values as 20, 16, 60 and 15, the game does not have a saddle point. Using the rule of dominance,
  (i) Row X_1 dominates X_3. Delete X_3.
  (ii) Column Y_1 dominates column Y_3 while column Y_4 dominates Y_2. Delete columns Y_2 and Y_3.
  (iii) Row X_2 dominates X_4. Deletion of X_4 leads to the following matrix:

	Y_1	Y_4
X_1	–5	15
X_2	20	–70

If x be the probability for player X to play X_1 and y be the probability for Y to Y_1, we have

$$x = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{-70 - 20}{(-5 - 70) - (15 + 20)} = \frac{9}{11}$$

$$y = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{-70 - 15}{(-5 - 70) - (15 + 20)} = \frac{17}{22}$$

$$v = \frac{(a_{11} \times a_{22}) - (a_{12} \times a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{(-5)(-70) - (15 \times 20)}{(-5 - 70) - (15 + 20)} = \frac{-5}{11}$$

Thus, optimal strategy for X: (9/11, 2/11, 0, 0), optimal strategy for Y: (17/22, 0, 0, 5/22), and game value, v = –5/11.

20. The game does not have a saddle point. Applying dominance rule,
  (i) Row A_1 dominates row A_3. Delete A_3.
  (ii) Column B_3 dominates column B_4. Delete B_4.
  (iii) Column B_3 is dominated by $0.5B_1 + 0.5B_2$. The reduced problem is:

	B_1	B_2
A_1	150	–18
A_2	6	102

With usual notations,

$$x = \frac{102 - 6}{(150 + 102) - (6 - 18)} = \frac{96}{264} = \frac{4}{11}$$

$$y = \frac{102 + 18}{(150 + 102) - (6 - 18)} = \frac{5}{11}$$

$$v = \frac{(150 - 102) - (6)(-18)}{(150 + 102) - (6 - 18)} = \frac{642}{11}$$

Accordingly,
Optimal strategy for A: (4/11, 7/11, 0), optimal strategy for B: (5/11, 6/11, 0, 0) and value of game = 642/11.

21. The game has not saddle point. By rule of dominance, we attempt to reduce it to a 2 × 2 game.
  1. Row 3 dominates row 2. Delete the second row.
  2. Column 3 dominates column 1, which is also deleted. With usual notations, we have

$$x = \frac{50 - 20}{(40 + 50) - (-80 + 20)} = \frac{1}{5}$$

$$y = \frac{50 + 80}{(40 + 50) - (-80 + 20)} = \frac{13}{15}$$

$$v = \frac{(40 \times 50) - (-80)(20)}{(40 + 50) - (-80 + 20)} = 24$$

From these results, the optimal strategies are: A(1/5, 0, 4/5) and B(0, 13/15, 2/15). The game value = 24.

22. Here row minima are –1, –5 and –4, while column maxima are 0, 2, 4 and 5. Hence, there is no saddle point. So obtain solution to this problem, we attempt to reduce its order.
    (i) Delete column 4, since it is dominated by column 3.
    (ii) In the reduced matrix, row 1 dominates row 3. So delete row 3.
    (iii) Column 3 is deleted next, since it is seen to be dominated by columns 1 and 2. This leads to a 2 × 2 game with strategies A_1 and A_2 for player A and B_1 for B_2 and player B. Accordingly,

$$x = \frac{2+5}{(0+2)-(-1-5)} = \frac{7}{8}$$

$$y = \frac{2+1}{(0+2)-(-1-5)} = \frac{3}{8}$$

$$v = \frac{(0\times 2)-(-1)(-5)}{(0+2)-(-1-5)} = \frac{-5}{8}$$

Thus, optimal strategy for A:(7/8, 1/8, 0); for B: (3/8, 5/8, 0, 0); and value of game = –5/8.

23. There is no saddle point. Column b_2 dominates b_3. Deleting b_3 reduces the game to a 2 × 2 game. Accordingly,

$$x = \frac{3-7}{(2+3)-(5+7)} = \frac{4}{7}$$

$$y = \frac{3-5}{(2+3)-(5+7)} = \frac{2}{7}$$

$$v = \frac{(2\times 3)-(5\times 7)}{(2+3)-(5+7)} = \frac{29}{7}$$

Optimal strategies are: A(4/7, 3/7), B(2/7, 5/7, 0) and the game value = 29/7.

24. The row minima are 13, 8, 8 and 18 while column maxima are 63, 68, 33 and 23 respectively. The maximin value is 18 while the minimax value is 23. Hence, the game has no saddle point and, therefore, no pure strategies.
Next, we check for dominance.
A_1 dominates A_2. So delete A_2.
B_3 dominates both B_1 and B_2. Hence, delete B_1 and B_2. Finally, delete A_3 as it is doninated by A_4.
With no further dominance seen, the game is reduced to 2 × 2. With usual notations,

$$x = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{23-18}{(33+23)-(18+13)} = \frac{5}{25} = 0.2$$

$$y = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{23-13}{(33+23)-(18+13)} = \frac{10}{25} = 0.4$$

$$v = \frac{(a_{11}\times a_{22})-(a_{21}\times a_{12})}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{33\times 23-18\times 13}{(33+23)-(18+13)} = \frac{525}{25} = 21$$

The optimal strategies are: for A (0.2, 0, 0, 0.8); for B (0, 0, 0.4, 0.6); with game value = 21.

25. (i) The row minima are –2, –1 and 2 while the column maxima are 3, 4 and 6. Thus, the game has no saddle point. Accordingly, pure strategies do not exist for the players.
    (ii) Deletion of column 3, which is dominated by column 1 entries, reduces the game to a 3 × 2 game and the rule of dominance is not seen to work further. We can proceed graphically to solve the problem. From the graph, P and Q are seen to be the two minimax points.

Evaluation of P and Q

For P:

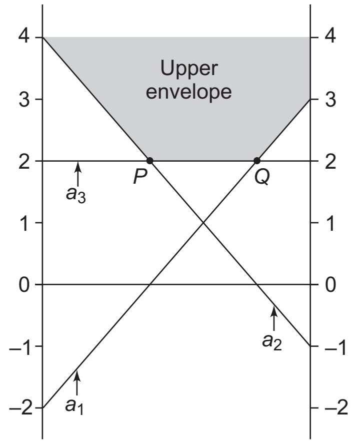
	b_1	b_2
a_2	–1	4
a_3	2	2

$$y = \frac{2-4}{(-1+2)-(2+4)} = \frac{2}{5} = 0.4$$

For Q:

	b_1	b_2
a_1	2	–2
a_3	2	2

$$y = \frac{2+2}{(3+2)-(-2+2)} = \frac{4}{5} = 0.8$$



Thus, an optimal strategy for A is (0, 0, 1) while for B it is any pair of $(y, 1 - y)$ where $0.4 \le y \le 0.8$. The game value, $v = 2$.

26. The game does not have saddle point. It is observed that row 3 strategy dominates row 2 strategy and, in the revised matrix, column 3 strategy dominates column 1 strategy. This leaves the game as a 2×2 game as follows:

	Radio	TV
Newspaper	50	–17
TV	30	60

Accordingly,

$$x = \frac{60-30}{(50+60)-(-17+30)} = \frac{30}{97}$$

$$y = \frac{60+17}{(50+60)-(-17+30)} = \frac{77}{97},$$

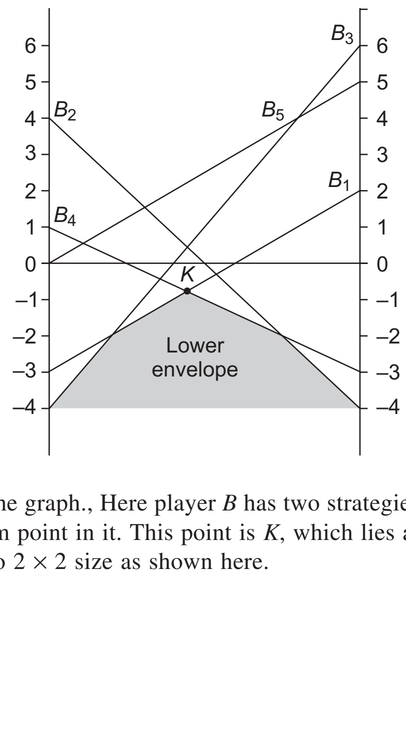
$$v = \frac{(50 \times 60)-(-17)(30)}{(50+60)-(-17+30)} = \frac{3510}{97}$$

Thus, optimal strategy for A: (30/97, 0, 67/97), for B: (0, 77/97, 20/97), and game value, v = 3510/97.

27. The game has no saddle point. We solve the game graphically.

The highest point in the lower envelope is K, which is determined by strategies B_1 and B_4. Thus, the game is reduced to the order 2×2 as follows:

	B_1	B_4
A_1	2	–3
A_2	–3	1



It is solved analytically, with usual notations as:

$$x = \frac{1-(-3)}{(2+1)-(-3-3)} = \frac{4}{9}$$

$$y = \frac{1-(-3)}{(2+1)-(-3-3)} = \frac{4}{9}$$

$$v = \frac{(2 \times 1)-(-3)(-3)}{(2+1)-(-3-3)} = \frac{-7}{9}$$

Therefore, optimal strategy for A: (4/9, 5/9), for B: (4/9, 0, 0, 5/9, 0); and game value = –7/9.

28 The pay-off corresponding to various strategies are presented on the graph., Here player B has two strategies available, we consider the upper envelope and locate the minimum point in it. This point is K, which lies at the intersection of A_1 and A_3. Accordingly, the game is reduced to 2×2 size as shown here.

	B_1	B_2
A_1	3	4
A_3	6	–2

With usual notations,

$$x = \frac{-2-6}{(3-2)-(4+6)} = \frac{8}{9}$$

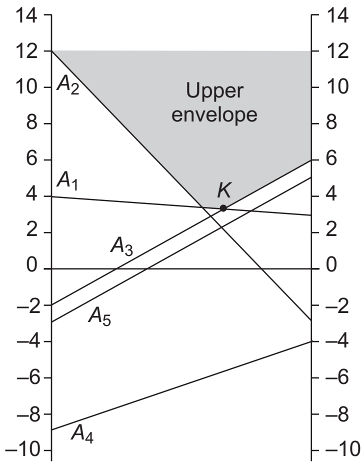
$$y = \frac{-2-4}{(3-2)-(4+6)} = \frac{2}{3}$$

$$v = \frac{(3)(-2)-(4\times 6)}{(3-2)-(4+6)} = \frac{10}{3}$$

Thus, optimal strategy for A: (8/9, 0, 1/9, 0, 0); for B: (2/3, 1/3); and game value = 10/3.

29. With row minima values as 6, 5 and 7, and the column maxima values as 9, 11, 9 there is evidently no saddle point. It may be observed that 0.5 A_1 + 0.5 A_2 dominates A_3. After deleting A_3, B_1 is seem to dominate B_3. Its detetion leads to the following 2 × 2 game.

	B_1	B_2
A_1	6	11
A_2	9	5



With usual notations,

$$x = \frac{5-9}{(6+5)-(9+11)} = \frac{4}{9}$$

$$y = \frac{5-11}{(6+5)-(9+11)} = \frac{6}{9}$$

$$v = \frac{(6\times 5)-(11\times 9)}{(6+5)-(11+9)} = \frac{23}{3}$$

Accordingly, optimal strategy for A: (4/9, 5/9, 0), for B: (2/3, 1/3, 0) and value of the game, v = 23/3.

30. Let x_i be the probability that firm X would play i th strategy. If U be value of game, we define $X_i = x_i/U$. Similarly, let y_i be the probability that firm Y would play jth strategy. If V be the game value, we define $Y_i = y_i/V$. The prblem is:

From X's point of view

Minimise $\frac{1}{U} = X_1 + X_2 + X_3$

Subject to

$90X_1 + 110X_2 + 120X_3 \geq 1$
$80X_1 + 100X_2 + 70X_3 \geq 1$
$110X_1 + 90X_2 + 80X_3 \geq 1$
$X_1, X_2, X_3 \geq 0$

From Y's point of view

Maximise $\frac{1}{V} = Y_1 + Y_2 + Y_3$

Subject to

$90Y_1 + 80Y_2 + 110Y_3 \leq 1$
$110Y_1 + 100Y_2 + 90Y_3 \geq 1$
$120Y_1 + 70Y_2 + 80Y_3 \leq 1$
$Y_1, Y_2, Y_3 \geq 0$

Solution to the game: Column 2 dominates column 1. Delete the dominated column. In the reduced matrix, the third row is seen to be dominated by the other rows. Hence, it is deleted. The resulting matrix is:

	Do not change	Reduce price
Increase price	80	110
Do not change	100	90

With usual notations,

$$x = \frac{90 - 100}{(80 + 90) - (110 + 100)} = \frac{1}{4}$$

$$y = \frac{90 - 110}{(80 + 90) - (110 + 100)} = \frac{1}{2}$$

$$v = \frac{(80 \times 90) - (110 \times 100)}{(80 + 90) - (110 + 100)} = 95$$

∴ Optimal strategy for *X*: (1/4, 3/4, 0); for *Y*: (0, 1/2, 1/2) and game value = 95.

31. Let x_i be the probability of player *A* to play the *i*th strategy and *U* be the value of the game. We define $X_i = x_i/U$. Similarly, let y_j be the probability of player *B* to play the *j*th strategy, and *V* be the value of the game. We define $Y_j = y_j/V$. The LPP is:

From A's point of view

Minimise $\frac{1}{U} = X_1 + X_2 + X_3$

Subject to

$5X_1 + 10X_2 + 6X_3 \geq 1$
$7X_1 + 4X_2 + 2X_3 \geq 1$
$2X_1 + 9X_2 \geq 0$
$X_1, X_2, X_3 \geq 0$

From B's point of view

Maximise $\frac{1}{V} = Y_1 + Y_2 + Y_3$

$5Y_1 + 7Y_2 + 2Y_3 \leq 1$
$10Y_1 + 4Y_2 + 9Y_3 \leq 1$
$6Y_1 + 2Y_2 \leq 1$
$Y_1, Y_2, Y_3 \geq 0$

32. Since some of the entries in the matrix are negative, we add constant (say 10) as will make all values to be non-negative. The resulting matrix is:

	Player *B*			
	18	30	7	11
Player *A*	16	35	14	12
	10	2	22	19
	26	19	31	10

The linear programming formulation of the game is:

For Player A

Minimise $1/U = X_1 + X_2 + X_3 + X_4$

Subject to

$18X_1 + 16X_2 + 10X_3 + 26X_4 \geq 1$
$30X_1 + 35X_2 + 2X_3 + 19X_4 \geq 1$
$7X_1 + 14X_2 + 22X_3 + 31X_4 \geq 1$
$11X_1 + 12X_2 + 19X_3 + 10X_4 \geq 1$
$X_1, X_2, X_3, X_4 \geq 0$

For Player B

Maximise $1/V = Y_1 + Y_2 + Y_3 + Y_4$

Subject to

$18Y_1 + 30Y_2 + 7Y_3 + 11Y_4 \leq 1$
$16Y_1 + 35Y_2 + 14Y_3 + 12Y_4 \leq 1$
$10Y_1 + 2Y_2 + 22Y_3 + 19Y_4 \leq 1$
$26Y_1 + 19Y_2 + 31Y_3 + 10Y_4 \leq 1$
$Y_1, Y_2, Y_3, Y_4 \geq 0$

Where $X_i = x_i/U$, x_i is the probability that *A* plays *i*th strategy, *U* is the game value; $Y_j = y_j/V$, y_j is the probability that *B* plays *j*th strategy and *V* is the game value. The true game value = *U* (or *V*) *minus* 10.

33. (a) Here the maximum value is – 2 while the minimax is 1. So the game has no saddle point.
(b) Apparently, none of the strategies is seen to dominate another. So the game cannot be reduced in size.
(c) The LPP is:

Maximise $1/V = Y_1 + Y_2 + Y_3$

Subject to

$8Y_1 + Y_2 + Y_3 \leq 1$
$Y_1 + Y_2 + 5Y_3 \leq 1$
$Y_1 + 4Y_2 + Y_3 \leq 1$
$Y_1, Y_2, Y_3 \geq 0$

Notes: $Y_1 = y_1/V$, $Y_2 = y_2/V$ and $Y_3 = y_3/V$ where y_1, y_2 and y_3 are the respective probabilities with which the three strategies are played by player Y, and V is the game value.

For formulating the game as LPP, a constant (=3) has been added to all values in the matrix so that no negative values appear.

The solution follows.

Simplex Tableau 1: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1 0	8	1	1	1	0	0	1	1
S_2 0	1	1	5	0	1	0	1	1/5←
S_3 0	1	4	1	0	0	1	1	1
C_j	1	1	1	0	0	0		
Solution	0	0	0	1	1	1		
Δ_j	1	1	1 ↑	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1 0	39/5	4/5	0	1	–1/5	0	4/5	1
Y_3 1	1/5	1/5	1	0	1/5	0	1/5	1
S_3 0	4/5	19/5	0	0	–1/5	1	4/5	4/19 ←
C_j	1	1	1	0	0	0		
Solution	0	0	1/5	4/5	0	4/5		
Δ_j	4/5	4/5 ↑	0	0	–1/5	0		

Simplex Tableau 3: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1 0	145/19	0	0	1	–3/19	–4/19	12/19	12/145 ←
Y_3 1	3/19	0	1	0	4/19	–1/19	3/19	1
Y_2 1	4/19	1	0	0	–1/19	5/19	4/19	1
C_j	1	1	1	0	0	0		
Solution	0	4/19	3/19	12/19	0	0		
Δ_j	12/19 ↑	0	0	0	–3/19	–4/19		

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i
Y_1 1	1	0	0	19/145	–3/145	–4/145	12/145
Y_3 1	0	0	1	–3/145	31/145	–7/145	21/145
Y_2 1	0	1	0	–4/145	–7/145	39/145	28/145
C_j	1	1	0	0	0		
Solution	12/145	28/145	21/145	0	0	0	61/145
Δ_j	0	0	0	–12/145	–21/145	–28/145	

Thus, $Y_1 = 12/145$, $Y_2 = 28/145$, $Y_3 = 21/145$ and V = Rec 61/145 or 145/61. Accordingly, $y_1 = 12/61$, $Y_2 = 28/61$ and $y_3 = 21/61$.

(d) Game Value = $V - 3 = 145/61 - 3 = -38/61$.

34. *From A's point of view:*
Minimise $1/U = X_1 + X_2 + X_3$
Subject to
$5X_1 + 5X_2 + 8X_3 \geq 1$
$4X_1 + 8X_2 + 5X_3 \geq 1$
$7X_1 + 4X_2 + 6X_3 \geq 1$
$X_1, X_2, X_3 \geq 0$

From B's point of view:
Maximise $1/V = Y_1 + Y_2 + Y_3$
Subject to
$5Y_1 + 4Y_2 + 7Y_3 \leq 1$
$5Y_1 + 8Y_2 + 4Y_3 \leq 1$
$8Y_1 + 5Y_2 + 6Y_3 \leq 1$
$Y_1, Y_2, Y_3 \geq 0$

Where $X_i = x_i/U$ and x_i(for i = 1, 2, 3) is the probability of A to play A_1, A_2 and A_3 respectively; and where $Y_i = y_i/V$ and y_i(for i = 1, 2, 3) is the probability to play B_1, B_2 and B_3 respectively by B. The solution to the game from B's point of view is given here.

Simplex Tableau 1: Non-optimal Solution

Basis		Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	5	4	7	1	0	0	1	1/5
S_2	0	5	8	4	0	1	0	1	1/5
S_3	0	8*	5	6	0	0	1	1	1/8
C_j		1	1	1	0	0	0	0	
Solution		0	0	0	1	1	1		
Δ_j		1 ↑	1	1	0	0	0		

Simplex Tableau 2: Non-optimal Solution

Basis		Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	7/8	13/4	1	0	–5/8	3/8	3/7
S_2	0	0	39/8*	1/4	0	1	–5/8	3/8	3/39 ←
Y_1	1	1	5/8	3/4	0	0	1/8	1/8	1/5
C_j		1	1	1	0	0	0		
Solution		1/8	0	0	3/8	3/8	0		
Δ_j		0	3/8 ↑	2/8	0	0	–1/8		

Simplex Tableau 3: Non-optimal Solution

Basis		Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1	0	0	0	125/39*	1	–7/39	–20/39	4/13	12/125 ←
Y_2	1	0	1	2/39	0	8/39	–5/39	1/13	3/2
Y_1	1	1	0	28/39	0	–5/39	8/39	1/13	3/28
C_j		1	1	1	0	0	0		
Solution		1/13	1/13	0	4/13	0	0		
Δ_j		0	0	3/13	0	–1/13	–1/13		

Simplex Tableau 4: Optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i
Y_3 1	0	0	1	39/125	–7/125	–20/125	12/125
Y_2 1	0	1	0	–2/125	26/125	–15/125	9/125
Y_1 1	1	0	0	–28/125	–11/125	40/125	1/125
C_j	1	1	1	0	0	0	
Solution	1/125	9/125	12/125	0	0	0	
Δ_j	0	0	0	–9/125	–8/125	–5/125	

With $Y_1 = \frac{1}{125}$, $Y_2 = \frac{9}{125}$, and $Y_3 = \frac{12}{125}$, $\frac{1}{N} = \frac{1}{125} + \frac{9}{125} + \frac{12}{125} = \frac{22}{125}$

Thus, $V = \frac{125}{22}$ and $y_1 = \frac{1}{125} \times \frac{125}{22} = \frac{1}{22}$, $y_2 = \frac{9}{125} \times \frac{125}{22} = \frac{9}{22}$, $y_3 = \frac{12}{125} \times \frac{125}{22} = \frac{12}{22}$

Similarly, for player A: $\frac{1}{U} = \frac{9}{125} + \frac{8}{125} + \frac{5}{125} = \frac{22}{125}$ and $U = \frac{125}{22}$

Thus, $x_1 = \frac{9}{125} \times \frac{125}{22} = \frac{9}{22}$, $x_2 = \frac{8}{125} \times \frac{125}{22} = \frac{8}{22}$, $x_3 = \frac{5}{125} \times \frac{125}{22} = \frac{5}{22}$

35. (a) Since some of the entries are negative, we add a constant (= 2) to each of the values in the matrix. Next, let y_1, y_2 and y_3 be the respective probabilities of player Y playing the three strategies and V be the value of the game. We define $Y_i = y_i/V$. The LP model is

Maximise $1/V = Y_1 + Y_2 + Y_3$

Subject to

$3Y_1 + Y_2 + Y_3 \leq 1$

$Y_1 + Y_2 + 5Y_3 \leq 1$

$Y_1 + 6Y_2 + Y_3 \leq 1$

$Y_1, Y_2, Y_3 \geq 0$

Simplex Tableau 1: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
S_1 0	3	1	1	1	0	0	1	1/3 ←
S_2 0	1	1	5	0	1	0	1	1
S_3 0	1	6	1	0	0	1	1	1
C_j	1	1	1	0	0	0		
Solution	0	0	0	1	1	1		
Δ_j	1	1	1	0	0	0		
	↑							

Simplex Tableau 2: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
Y_1 1	1	1/3	1/3	1/3	0	0	1/3	1
S_2 0	0	2/3	14/3	–1/3	1	0	2/3	1
S_3 0	0	17/3	2/3	–1/3	0	1	2/3	2/17 ←
C_j	1	1	1	0	0	0		
Solution	1/3	0	0	0	2/3	2/3		
Δ_j	0	2/3	2/3	–1/3	0	0		
		↑						

Simplex Tableau 3: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i	b_i/a_{ij}
y_1 1	1	0	5/17	6/17	0	–1/17	5/17	1
S_2 0	0	0	78/17*	–5/17	1	–2/17	10/17	5/39 ←
Y_2 1	0	1	2/17	–1/17	1	3/17	2/17	1
C_j	1	1	1	0	0	0		
Solution	5/17	2/17	0	0	10/17	0		
Δ_j	0	0	10/17	–5/17	0	–2/17		
		↑						

Simple Tableau 4: Optimal Solution

Basis	Y_1	Y_2	Y_3	S_1	S_2	S_3	b_i
Y_1 1	1	0	0	29/78	–5/78	–2/39	10/39
Y_3 1	0	0	1	–5/78	17/78	–1/39	5/39
Y_2 1	0	1	0	–2/39	–1/39	7/39	4/39
C_j	1	1	1		0	0	
Solution	10/39	4/39	5/39	0	0	0	
Δ_j	0	0	0	–10/39	–5/39	–4/39	

Here $Y_1 = \frac{10}{39}$, $Y_2 = \frac{4}{39}$ and $Y_3 = \frac{5}{39}$ and $\frac{1}{V} = \frac{19}{39}$ or $V = \frac{39}{19}$

$\therefore \quad y_1 = \frac{10}{39} \times \frac{39}{19} = \frac{10}{19}$, $y_2 = \frac{4}{39} \times \frac{39}{19} = \frac{4}{19}$, $y_3 = \frac{5}{39} \times \frac{39}{19} = \frac{5}{19}$ and the game value = 39/19 – 2 = 1/19.

(b) Dual Problem:

Minimise $\frac{1}{U} = X_1 + X_2 + X_3$

Subject to

$$3X_1 + X_2 + X_3 \geq 1$$
$$X_1 + X_2 + 6X_3 \geq 1$$
$$X_1 + 5X_2 + X_3 \geq 1$$
$$X_1, X_2, X_3 \geq 0$$

From the Simplex Tableau 4, we have X_1 = 10./39, X_2 = 5/39 and X_3 = 4/39, and $1/U$ = 19/39 or U = 39/19. Accordingly, $x_1 = \frac{10}{39} \times \frac{39}{19} = \frac{10}{19}$, $x_2 = \frac{5}{39} \times \frac{39}{19} = \frac{5}{19}$ and $x_3 = \frac{4}{39} \times \frac{39}{19} = \frac{4}{19}$. Game value = 39/19 – 2 = 1/19.

36. The given problem does not have saddle point. Accordingly, we may formulate and solve it as an LPP.

Let x_i be the probability that Hindustan Motor Co. would play ith strategy. If U be the value of the game, we define $X_i = x_i/U$. Similarly, let y_j be the probability that jth strategy would be played by India Motor Co. If V be the game value, we define $Y_j = y_j/V$. Accordingly, the problem is stated below.

From Hindustan Motor Co's point of view:

Minimise $\frac{1}{U} = X_1 + X_2 + X_3 + X_4 + X_5$

Subject to

$$3X_1 + 5X_2 + 2X_3 + 6X_4 + 2X_5 \geq 1$$
$$4X_1 + 6X_2 + X_3 + 4X_4 + X_5 \geq 1$$
$$2X_1 + 7X_2 + 4X_3 + 2X_4 + 9X_5 \geq 1$$
$$8X_1 + 4X_2 + 5X_3 + 3X_4 + 4X_5 \geq 1$$
$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

From India Motor Co's point of view:

$$\text{Maximise} \quad \frac{1}{V} = Y_1 + Y_2 + Y_3 + Y_4$$

Subject to

$$\begin{aligned}
3Y_1 + 4Y_2 + 2Y_3 + 8Y_4 &\le 1 \\
5Y_1 + 6Y_2 + 7Y_3 + 4Y_4 &\le 1 \\
2Y_1 + Y_2 + 4Y_3 + 5Y_4 &\le 1 \\
6Y_1 + 4Y_2 + 2Y_3 + 3Y_4 &\le 1 \\
2Y_1 + Y_2 + 9Y_3 + 4Y_4 &\le 1 \\
Y_1, Y_2, Y_3, Y_4 &\ge 0
\end{aligned}$$

Solution from India Motor Co's point of view follows. It is obtained in tables below.

Simplex Tableau 1: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	Y_4	S_1	S_2	S_3	S_4	S_5	b_i	b_i/a_{ij}
S_1 0	2	4	2	8	1	0	0	0	0	1	1/3
S_2 0	5	6	7	7	0	1	0	0	0	1	1/5
S_3 0	2	1	4	5	0	0	1	0	0	1	1/2
S_4 0	6*	4	2	3	0	0	0	1	0	1	1/6 ←
S_5 0	2	1	9	4	0	0	0	0	1	1	1/2
C_j	1	1	1	1	0	0	0	0	0		
Solution	0	0	0	0	1	1	1	1	1		
Δ_j	1	1	1	1	0	0	0	0	0		
	↑										

Simplex Tableau 2: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	Y_4	S_1	S_2	S_3	S_4	S_5	b_i	b_i/a_{ij}
S_1 0	0	2	1	13/2	1	0	0	–1/2	0	1/2	1/2
S_2 0	0	8/3	16/3*	3/2	0	1	0	–5/6	0	1/6	1/32 ←
S_3 0	0	–1/3	10/3	4	0	0	1	–1/3	0	2/3	1/5
Y_1 1	1	2/3	1/3	1/2	0	0	0	1/6	0	1/6	1/2
S_5 0	0	–1/3	25/3	3	0	0	0	–1/3	1	2/3	2/25
C_j	1	1	1	1	0	0	0	0	0		
Solution	1/6	0	0	0	1/2	1/6	2/3	0	2/3		
Δ_j	0	1/3	2/3	1/2	0	0	0	–1/6	0		
			↑								

Simplex Tableau 3: Non-optimal Solution

Basis	Y_1	Y_2	Y_3	Y_4	S_1	S_2	S_3	S_4	S_5	b_i	b_i/a_{ij}
S_1 0	0	3/2	0	199/32*	1	–3/16	0	–11/32	0	15/32	15/199 ←
Y_3 1	0	1/2	1	9/32	0	3/16	0	–5/32	0	1/32	1/9
S_3 0	0	–2	0	49/16	0	–5/8	1	3/16	0	9/16	9/49
Y_1 1	1	1/2	0	13/32	0	–1/16	0	7/32	0	5/32	5/13
S_5 0	0	–9/2	0	21/32	0	–25/16	0	31/32	1	13/32	13/21
C_j	1	1	1	1	0	0	0	0	0		
Solution	5/32	0	1/32	0	15/32	0	9/16	0	13/32		
Δ_j	0	0	0	5/16	0	–1/8	0	–1/32	0		
				↑							

Simplex Tableau 4: Non-optimal Solution

Basis		Y_1	Y_2	Y_3	Y_4	S_1	S_2	S_3	S_4	S_5	b_i
Y_4	1	0	48/199	0	1	32/199	–6/199	0	–11/199	0	15/199
Y_3	1	0	86/199	1	0	–9/199	39/199	0	–28/199	0	2/199
S_3	0	0	–545/199	0	0	–98/199	–106/199	1	71/199	0	66/199
Y_1	1	1	359/398	0	0	–13/199	–10/199	0	48/199	0	25/199
S_5	0	0	–927/199	0	0	–21/199	–307/199	0	200/199	1	71/199
C_j		1	1	1	1	0	0	0	0	0	
Solution		2/199	0	2/199	15/199	0	0	66/199	0	71/199	
Δ_j		0	–229/398	0	0	–10/199	–23/199	0	–9/199	0	

Objective function value = $\frac{1}{V}$

$$= \frac{25}{199} + 0 + \frac{2}{199} + \frac{15}{199} = \frac{42}{199}$$

Thus, game value, $V = 199/42$.

Now, since $y_j = Y_j \times V$, we have

$$y_1 = Y_1 \times V = \frac{25}{199} \times \frac{199}{42} = \frac{25}{42};\ y_2 = Y_2 \times V = 0 \times \frac{199}{42} = 0;$$

$$y_3 = Y_3 \times V = \frac{2}{199} \times \frac{199}{42} = \frac{2}{42};\ \text{and } y_4 = Y_4 \times V = \frac{15}{199} \times \frac{199}{42} = \frac{15}{42}.$$

Similarly, we can derive the values of x_1, x_2, x_3, x_4 and x_5 from the Δ_j values of the Simplex Tableau 4.

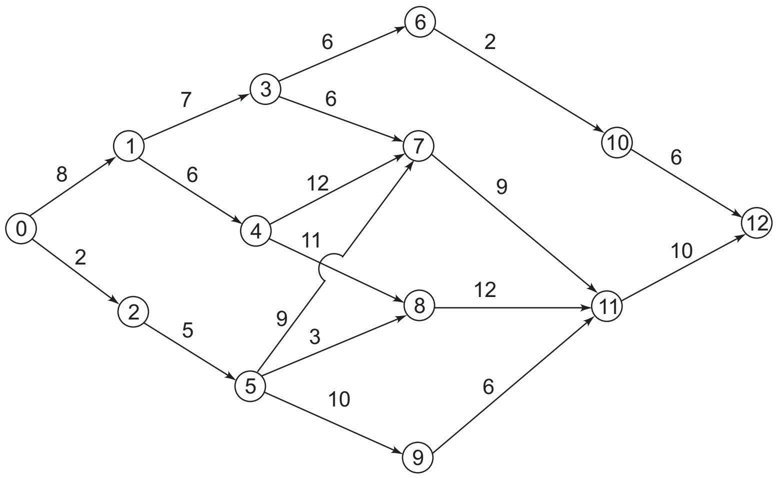
Accordingly, $x_1 = \frac{10}{199} \times \frac{199}{42} = \frac{10}{42}$, $x_2 = \frac{23}{199} \times \frac{199}{42} = \frac{23}{42}$, $x_3 = 0$, $x_4 = \frac{9}{199} \times \frac{199}{42} = \frac{9}{42}$ and $x_5 = 0$.

Thus, optimal strategy for Hindustan Motor Co.: (10/42, 23/42, 0, 9/42, 0); for India Motor Co.: (25/42, 0, 2/42, 15/42), and game value = 199/42.

If it were known that India Motor Co. would produce model K_4 only, Hindustan Motor Co. would produce model J_1 because it will entail the highest pay-off.

CHAPTER 16

1.



↑ Stage 1 ↑ Stage 2 ↑ Stage 3 ↑ Stage 4 ↑ Stage 5

Recursive relationship: $f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$

Stage 5

State, s_5 Node	$f_5^*(s_5)$	x_5^*
10	6	10-12
11	10	11-12

Stage 4

State, s_4 Node	$f_4(s_4) = c_4(s_4, x_4) + f_5^*(s_5)$		$f_4^* s_4$	x_4^*
6	x_4 = 6-10	2 + 6 = 8	8	6-10
7	x_4 = 7-11	9 + 10 = 19	19	7-11
8	x_4 = 8-11	12 + 10 = 22	22	8-11
9	x_4 = 9-11	6 + 10 = 16	16	9-11

Stage 3

State, s_3 Node	$f_3(s_3) = c_3(s_3, x_3) = f_4^*(s_4)$		$f_3^*(s_3)$	x_3^*
3	x_3 = 3-6	4 + 8 = 12		
	x_3 = 3-7	6 + 19 = 25	25	3-7
4	x_3 = 4-7	12 + 19 = 31		
	x_3 = 4-8	11 + 22 = 33	33	4-8
5	x_3 = 5-7	9 + 19 = 28	28	5-7
	x_3 = 5-8	3 + 22 = 25		
	x_3 = 5-9	10 + 16 = 26		

Stage 2

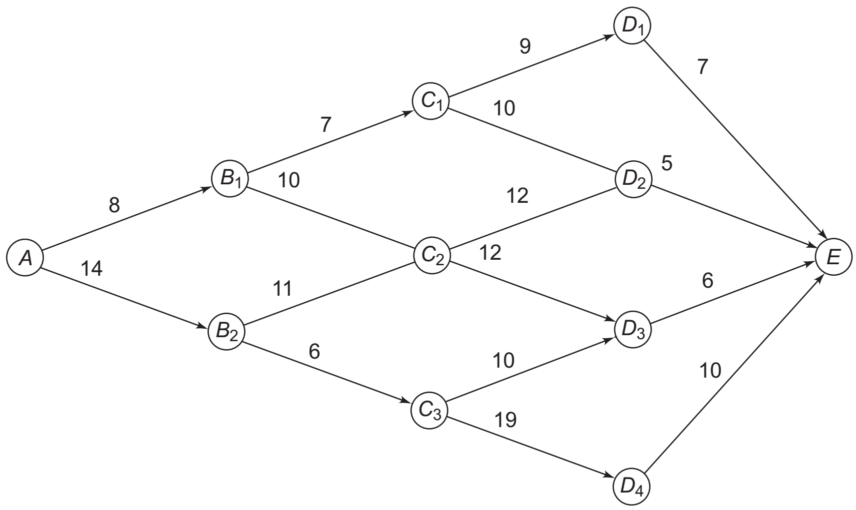
State, s_2 Node	$f_2(s_2) = c_2(s_2, x_2) + f_3^*(s_3)$		$f_2^*(s_2)$	x_2^*
1	1-3	7 + 25 = 32		
	1-4	6 + 33 = 39	39	1-4
2	2-5	5 + 28 = 33	33	2-5

Stage 1

Stage, s_1 Node	$f_1(s_1) = c_1(s_1, x_1) + f_2^*(s_2)$		$f_1^*(s_1)$	x_1^*
0	0-1	8 + 39 = 47	47	0-1
	0-2	2 + 33 = 35		

Scanning through Stages 1 through 5 decisions, we observe that the longest path is 0-1-4-8-11-12 with length of 47.

2.



↑ Stage 1 ↑ Stage 2 ↑ Stage 3 ↑ Stage 4

Recursive relationship: $f_n^*(s_n) = \max_{x_n}\left\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\right\}$

(i) Stage 4

State, s_5 Node	$f_4^*(s_4)$		x_4^*
D_1	$x_4 = D_1$	7	D_1-E
D_2	$x_4 = D_2$	5	D_2-E
D_3	$x_4 = D_3$	6	D_3-E
D_4	$x_4 = D_4$	10	D_4-E

Stage 3

State, s_3 Node	$f_3(s_3) = c_3(s_3, x_3) = f_4^*(s_4)$		$f_3^*(s_3)$	x_3^*
c_1	$x_3 = C_1\text{-}D_1$	9 + 7 = 16		
	$x_3 = C_1\text{-}D_2$	10 + 5 = 15	15	$C_1\text{-}D_2$
c_2	$x_3 = C_2\text{-}D_2$	12 + 5 = 17	17	$C_2\text{-}D_2$
	$x_3 = C_2\text{-}D_3$	12 + 6 = 18		
c_3	$x_3 = C_3\text{-}D_3$	10 + 6 = 16	16	$C_3\text{-}D_3$
	$x_3 = C_3\text{-}D_4$	19 + 10 = 29		

Stage 2

State, s_2 Node	$f_2(s_2) = c_2(s_2, x_2) + f_3^*(s_3)$		$f_2^*(s_2)$	x_2^*
B_1	$x_2 = B_1\text{-}C_1$	7 + 15 = 22	22	$B_1\text{-}C_1$
	$x_2 = B_1\text{-}C_2$	10 + 17 = 27		
B_2	$x_2 = B_2\text{-}C_2$	11 + 17 = 28		
	$x_2 = B_2\text{-}C_3$	6 + 16 = 22	22	$B_2\text{-}C_3$

Stage 1

Stage, s_1 Node	$f_1(s_1) = c_1(s_1, x_1) + f_2^*(s_2)$		$f_1^*(s_1)$	x_1^*
A	$x_1 = A\text{-}B_1$	8 + 22 = 30	30	$A\text{-}B_1$
	$x_1 = A\text{-}B_2$	14 + 22 = 36		

From the above calculations, it is evident that the shortest route from A to E is: $A\text{-}B_1\text{-}C_1\text{-}D_2\text{-}E$ with a length equal to 30.

(ii) Stage 3

State, s_3 Node	$f_3(s_3)$		$f_3^*(s_3)$	x_3^*
C_1	$x_3 = C_1\text{-}D_1$	9	9	$C_1\text{-}D_1$
	$x_3 = C_1\text{-}D_2$	10		
C_2	$x_3 = C_2\text{-}D_2$	12	12	$C_2\text{-}D_2$ or
	$x_3 = C_2\text{-}D_3$	12	12	$C_2\text{-}D_3$
C_3	$x_3 = C_3\text{-}D_3$	10	10	$C_3\text{-}D_3$
	$x_3 = C_3\text{-}D_4$	19		

Stage 2

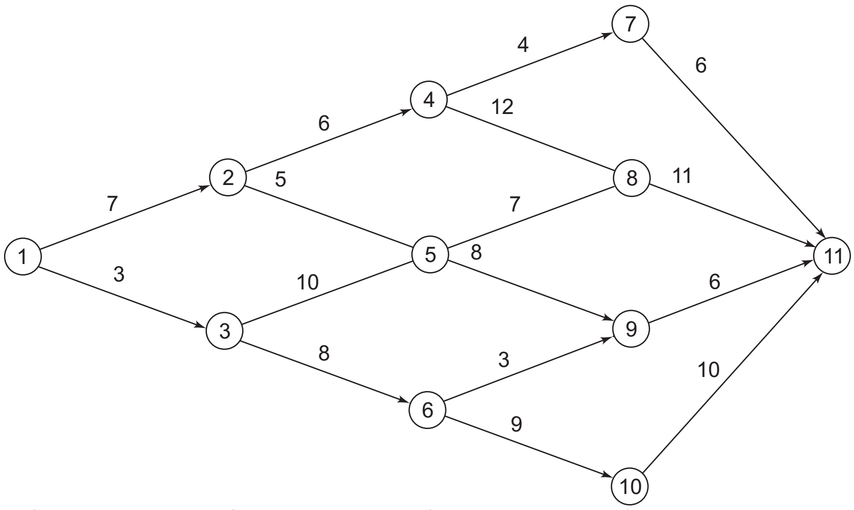
State, s_2 Node	$f_2(s_2)$	$c_2(s_2, x_2) + f_2^*(s_2)$	$f_1^*(s_1)$	x_1^*
B_1	$B_1\text{-}C_1$	7 + 9 = 16	16	$B_1\text{-}C_1$
	$B_1\text{-}C_2$	10 + 12 = 22		
B_2	$B_2\text{-}C_2$	11 + 12 = 23		
	$B_2\text{-}C_3$	6 + 10 = 16	16	$B_2\text{-}C_3$

Stage 1

State, s_1 Node	$f_1(s_1) = c_1(s_1, x_1) + f_2^*(s_2)$		$f_1^*(s_1)$	x_1^*
A	A-B_1	8 + 16 = 24	24	A-B_1
	A-B_2	14 + 16 = 30		

From the preceding analysis, it is evident that the shortes path from A to any point on D is A-B_1-C_1-D_1 with a length = 24.

3. (a)



↑ Stage 1 ↑ Stage 2 ↑ Stage 3 ↑ Stage 4

Stage 4

State, s_4 Node	$f_4^*(s_4)$		x_4^*
7	x_4 = 7-11	6	7-11
8	x_4 = 8-11	11	8-11
9	x_4 = 9-11	6	9-11
10	x_4 = 10-11	10	10-11

Stage 3

State, s_3 Node	$f_3(s_3)\ c(s_3, x_3) + f_4^*(s_4)$		$f_3^*(s_3)$	x_3^*
4	x_3 = 4-7	4 + 6 = 10	10	4-7
	x_3 = 4-8	12 + 11 = 23		
5	x_3 = 5-8	7 + 11 = 18		
	x_3 = 5-9	8 + 6 = 14	14	5-9
6	x_3 = 6-9	3 + 6 = 9	9	6-9
	x_3 = 6-10	9 + 10 = 19		

Stage 2

State, s_2 Node	$f_2(s_2) = c(s_2, x_2) + f_3^*(s_3)$		$f_2^*(s_3)$	x_2^*
2	x_2 = 2-4	6 + 10 = 16	16	2-4
	x_2 = 2-5	5 + 14 = 19		
3	x_2 = 3-5	10 + 14 = 24		
	x_2 = 3-6	8 + 9 = 17	17	3-6

Stage 1

State, s_1 Node	$f_1(s_1) = c(s_1, x_1) + f_2^*(s_2)$		$f_1^*(s_1)$	x_1^*
1	x_1 = 1-2	7 + 16 = 23		
	x_1 = 1-3	3 + 17 = 20	20	1-3

It is clear that shortest route from initial node to the final node is 1-3-6-9-11, with a distance of 20 km.

(b) *When 6-9 is not available:* Stage 4, as above

Stage 3

State, s_3 Node	$f_3(s_3) = c(s_3, x_3) = f_4^*(s_4)$		$f_3^*(s_3)$	x_3^*
4	x_3 = 4-7	4 + 6 = 10	10	4-7
	x_3 = 4-8	12 + 11 = 23		
5	x_3 = 5-8	7 + 11 = 18		
	x_3 = 5-9	8 + 6 = 14	14	5-9
6	x_3 = 6-10	9 + 10 = 19	19	6-10

Stage 2

State, s_2 Node	$f_2(s_2) = c(s_2, x_2) + f_3^*(s_3)$		$f_2^*(s_2)$	x_2^*
2	x_2 = 2-4	6 + 10 = 16	16	2-4
	x_2 = 2-5	5 + 14 = 19		
3	x_2 = 3-5	10 + 14 = 24	24	3-5
	x_2 = 3-6	8 + 19 = 27		

Stage 1

State, s_1 Node	$f_1(s_1) = c(s_1, x_1) + f_2^*(s_2)$		$f_1^*(s_1)$	x_1^*
1	x_1 = 1-2	7 + 16 = 23	23	1-2
	x_1 = 1-3	3 + 24 = 27		

Thus, if 6-9 is not available, then the shortest route would be 1-2-4-7-11 and the distance would be 23 km.

4. There are two stages here: $n = 1$ and 2. Let x_n be the decision variables at stage n. Further, the state S_n indicates the amount of resources R_1 and R_2 at least. Let $S_\text{n} = (R_1, R_2)$. Thus,

$$S_1 = (4{,}800,\ 7{,}200)$$
$$S_2 = (4{,}800 - 20x_1,\ 7{,}200 - 80x_1)$$

Recursive relationship:

$$f^*(R_1, R_2) = \min_{x_n} (R_1, R_2, x_\text{n})$$
$$f_2^*(R_1, R_2) = \min_{\substack{50\, x_2 \geq 4{,}800 \\ 50\, x_2 \geq 7{,}200}} (24x_2)$$

$$f_1(4{,}800,\ 7{,}200) = 40x_1 + f_2^*(4{,}800 - 20x_1,\ 7{,}200 - 80x_1)$$

$$f_1^*(4{,}800,\ 7{,}200) = \min_{\substack{20x_1 \geq 4800 \\ 80x_1 \geq 7200}} \{40x_1 + f_1^*(4{,}800 - 20x_1,\ 7{,}200 - 80x_1)$$

Stage 2	$f_2^*(R_1, R_2)$	x_2^*
State(R_1, R_2)	24 max (R_1/50, R_2/50)	Max (R_1/50, R_2/50)
$R_1 \geq 0$, $R_2 \geq 0$		

Stage 1

We have, $f_2^*(4{,}800 - 20x_1,\ 7{,}200 - 80x_1) = 24\ \max(R_1/50,\ R_2/50)$

$$= 24 \max \left[\frac{4{,}800 - 20x_1}{50}, \frac{7{,}200 - 80x_1}{50}\right]$$

$$= 24 \max\ (96 - 0.4x_1,\ 144 - 1.6x_1)$$

$$f_1^*(4{,}800,\ 7{,}200) = \min_{x_1 \geq 0} \{40x_1 + 24 \max(96 - 0.4x_1,\ 144 - 1.6x_1)\}$$

The minimum value that x_1 can take is 0, which yields $f_1 = 24 \times 144 = 3{,}456$. Similarly, the maximum value that x_1 can take (here $R_1, R_2 \geq = 0$) is 240 which yields f_1 equal to $40 \times 240 = 9{,}600$. To know, if there is any value of x_1 between 0 and 240 which may yield value of f_1 lower than the values obtained so far, we solve the two constraint equations simultaneously to get $x_1 = 40$ (and $x_2 = 80$) which yields $f_1 = 3{,}520$. From the results, it is evident that the optimal solution is $x_1 = 0$, $s_2 = 144$ and $Z = 3{,}456$.

5. There are two stages involved here, $n = 1, 2$. Here x_n is the decision variable at stage n.
The states S_n indicate the amounts of resources, R_1 and R_2, available for allocation. Let $s_n = (R_1, R_2)$. Thus.

$$S_1 = (12,\ 40)$$
$$S_2 = (12 - 2x_1,\ 40 - 15x_1)$$

Recursive relationship:

$$f^*(R_1, R_2) = \max_{x_n}(R_1, R_2, x_n)$$

$$f_2^*(R_1, R_2) = \max_{\substack{4x_2 \leq R_1, \\ 5x_2 \leq R_2,\ x_2 \geq 0}} (20x_2)$$

$$f_1(12,\ 40) = 30x_1 + f_2^*(12 - 2x_1,\ 40 - 15x_1)$$

Also,
$$f_1^*(12,\ 40) = \max_{\substack{2x_1 \leq 12, \\ 15x_1 \leq 40,\ x_1 \geq 0}} \{30x_1 + f_2^*(12 - 2x_1,\ 40 - 15x_1)\}$$

Stage 2		
State (R_1,R_2)	$f_2^*(R_1, R_2)$	x_2^*
$R_1 \geq 0$ $R_2 \geq 0$	20 min{R_1/4, R_2/5}	min{R_1/4, R_2/5}

Stage 1

We have $f_2^*(12 - 2x_1,\ 40 - 15x_1) = 20\ \min\{R_1/4,\ R_2/5\}$

$$= 20 \min \left[\frac{12 - 2x_1}{4}, \frac{40 - 15x_1}{5}\right]$$

$$= 20 \min\ \{3 - 0.5x,\ 8 - 3x_1\}$$

Further,

$$f_1^*(12,\ 40) = \max_{x_1 \geq 0} [30x_1 + 20 \min\{3 - 0.5x_1,\ 8 - 3x_1\}]$$

To solve, we first set $x_1 = 0$. For this, $f_1(12{,}40) = 30 \times 0 + 20(3) = 60$. Further, the maximum value that x_1 can assume is 8/3. This gives $f_1(12,\ 40) = 80$. Next, we see if any value between 0 and 8/3 yields a higher value of the function. For this, we set $R_1 = R_2$, so that $3 - 0.5\ x_1 = 8 - 3x_1$, implying $x_1 = 2$. This gives f_1 (12, 40) = $30 \times 2 + 20 \times 2 = 100$, which indeed is higher than the previously obtained values. Thus, optimal solution is : $x_1 = 2$, $x_2 = 2$, and $Z = 100$.

6. Here there are two stages: n = 1 and 2, and x_n represents the decision variable at stage n. The states s_n indicate the amounts of resources R_1, R_2 and R_3 available for distribution.

$$S_1 = (8, 36, 36)$$
$$S_2 = (8 - 2x_1, 36, 36 - 6x_1)$$

Since x_1 is not known, let $S_2 = (R_1, R_2, R_3)$

Recursive relationship:

$$f^*(R_1, R_2, R_3) = \max_{x_n}(R_1, R_2, R_3; x_n)$$

$$f_2^*(R_1, R_2, R_3) = \max_{\substack{6x_2 \le R_2,\\ 4x_2, \le R_3}} (20x_2)$$

$$f_1(8, 36, 36,) = 12x_1 + f_2^*(8 - 2x_1, 36, 36 - 6x_1)$$

Also $$f_1^*(8, 36, 36) = \max_{\substack{2x \le 8\\ 6x_1 \le 36}}\{12x_1 + f_2^*(8 - 2x_1, 36, 36 - 6x_1)$$

Stage 2

State (R_1, R_2, R_3)	$f_2^*(R_1, R_2, R_3)$	x_n^*
$R_1 \ge 0$ $R_2 \ge R_3 \ge 0$	$20 \min \left\{\dfrac{R_2}{6}, \dfrac{R_3}{4}\right\}$	$\min\{R_2/6, R_3/4\}$

Stage 1

We have, $$f_2^*(R_1, R_2, R_3) = 20 \min \{R_2/6, R_3/4\}$$
$$= 20 \min \left\{\frac{36}{6}, \frac{36 - 6x_1}{4}\right\}$$
$$= 20 \min \{6{,}9 - 1.5x_1\}$$

Further,

$$f_1^*(8, 36, 36) = \max_{\substack{2x_1, \le 8,\\ 6x_1 \le 36}} \{12x_1 + 20 \min (6, 9 - 1.5x_1)\}$$

Here the minimum value for $x_1 = 0$ (leaving $x_2 = 6$) while the maximum value it can take is 4 (with $x_2 = 3$). When $x_1 = 0$, f_1 works out to be 120 and when $x_1 = 4$ f_1 equals 108. Now, to consider if there are any values which may yield the function value greater than 120, we consider equating R_1, R_2, and R_3 pairwise. Setting $R_1 = R_2$, we get $x_1 = 4$ and $x_2 = 6$ which is not feasible. When we set $R_1 = R_3$, we get $x_1 = 4$ and $x_2 = 3$ which we have alrady considered. Finally, considering R_2 and R_3 together, we obtain $x_1 = 2$ (and $x_2 = 6$) This yields f_1 equal to 144. This is higher than the values obtained earlier. Thus, optimal solution to the problem is: $x_1 = 2$, $x_2 = 6$, and $Z = 144$.

7. Here stage are the stores 1, 2, and 3 while states are the number of loads available for allocation. The recursive relationship is:

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 3

State, s_3	$f_3(s_3) = c_3(s_3, x_3)$ $x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$x_3 = 5$	$f_3^*(s_3)$	x_3^*
0	0						0	0
1	0	1,600					1,600	1
2	0	1,600	3,600				3,600	2
3	0	1,600	3,600	5,200			5,200	3
4	0	1,600	3,600	5,200	7,200		7,200	4
5	0	1,600	3,600	5,200	7,200	8,000	8,000	5

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$ $x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$x_2 = 5$	$f_2^*(s_3)$	x_2^*
0	0						0	0
1	1,600	2,400					2,400	1
2	3,600	4,00	4,400				4,400	2
3	5,200	6,000	6,000	6,000			6,000	1, 2, 3
4	7,200	7,600	8,000	7,600	7,600		8,000	2
5	8,000	9,600	9,600	9,600	9,200	8,800	9,600	1, 2, 3

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$ $x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$x_1 = 5$	$f_1^*(s_4)$	x_1^*
5	9,600	10,000	9,600	10,000	9,200	8,400	10,000	1,3

Scanning through the calculations, the optimal solution is: store 1 : 1 load, store 2 : 2 loads, store 3 : 2; loads; or store 1 : 3 loads, store 2 : 2 loads and store 3: nil. In each case the profit obtainable = Rs 10,000.

8. The Stages here are the regions 1, 2, 3 and 4 while the states are the number of salespersons available for assignment. The recursive relationship is:

$$f_n^*(s_n) = \max_{x_n} \{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 4

State, s_4	$f_4(s_4) = c_4(s_4, x_4)$ $x_4 = 0$	$x_4 = 1$	$x_4 = 2$	$x_4 = 3$	$x_4 = 4$	$x_4 = 5$	$x_4 = 6$	$f_4^*(s_4)$	x_4^*
0	0							0	0
1	0	12						12	1
2	0	12	22					22	2
3	0	12	22	28				28	3
4	0	12	22	28	32			32	4
5	0	12	22	28	32	34		34	5
6	0	12	22	28	32	34	36	36	6

Stage 3

State, s_3	$f_3(s_3) = \{c_3(s_3, x_3) + f_4^*(s_4)\}$ $x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$x_3 = 5$	$x_3 = 6$	$f_3^*(s_3)$	x_3^*
0	0							0	0
1	12	10						12	0
2	22	22	20					22	0, 1
3	28	32	32	30				32	1, 2
4	32	38	42	42	36			42	2, 3
5	34	42	48	52	48	42		52	3
6	36	44	52	58	58	54	44	58	3, 4

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$ $x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$x_2 = 5$	$x_2 = 6$	$f_2^*(s_2)$	x_2^*
0	0							0	0
1	12	14						14	1
2	22	26	22					26	1
3	32	36	34	32				36	1
4	42	46	44	44	36			46	1
5	52	56	54	54	48	42		56	1
6	58	66	64	64	58	54	44	66	1

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$ $x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$x_1 = 5$	$x_1 = 6$	$f_1^*(s_1)$	x_1^*
6	66	64	64	66	62	58	48		

An analysis of optimal solutions at various stages, we obtain the following multiple optimal solution to the problem:

Region:	1	2	3	4	Increase in sales (Rs lakh)
Sales persons:	3	1	0	2	66
	3	1	1	1	66
	0	1	3	2	66

9. The four areas, *A*, *B*, *C*, and *D* are the four stages while the states are the number of commercial ads available to be inserted. The recursive relationship is:

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 4

State, s_4	$f_4(s_4) = c_4(s_4, x_4)$ $x_4 = 0$	$x_4 = 1$	$x_4 = 2$	$x_4 = 3$	$x_4 = 4$	$x_4 = 5$	$f_4^*(s_4)$	x_4^*
0	0						0	0
1	0	7					7	1
2	0	7	15				15	2
3	0	7	15	25			25	3
4	0	7	15	25	29		29	4
5	0	7	15	25	29	33	33	5

Stage 3

State, s_1	$f_3(s_3) = \{c_3(s_3, x_3) + f_4^*(s_4)\}$ $x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$x_3 = 5$	$f_3^*(s_3)$	x_3^*
0	0						0	0
1	7	11					11	1
2	15	18	19				19	2
3	25	26	26	23			26	1, 2
4	29	36	34	30	21		36	1
5	33	40	44	45	28	27	45	3

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)$						$f_2^*(s_2)$	x_2^*
	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$x_2 = 5$		
0	0						0	
1	11	13					13	1
2	19	24	17				24	1
3	26	32	28	21			32	1
4	36	39	36	32	23		39	1
5	44	49	43	40	34	25	49	1

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$						$f_1^*(s_1)$	x_1^*
	$x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$x_1 = 5$		
5	49	48	47	43	38	31	49	0

The optimal solution is:

Area:	*A*	*B*	*C*	*D*	No. of additionl votes
No. of ads:	0	1	1	3	0 + 3 + 11 + 25 = 49 thousand

10. In this case, the stages are the four districts *A*, *B*, *C* and *D*, while the states are the number of workers available for employment. The recursive relationship is:

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 4

State, s_4 Node	$f_4^*(s_4)$	x_4^*
0	0	0
1	13	1
2	24	2
3	32	3
4	39	4
5	45	5
6	50	6

Stage 3

State, s_3	$f_3(s_3) = \{c_3(s_3, x_3) + f_4^*(s_4)\}$							$f_3^*(s_3)$	x_3^*
	$x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$x_3 = 5$	$x_3 = 6$		
0	0							0	0
1	13	23						33	1
2	24	46	43					46	1
3	32	57	56	47				57	1
4	39	65	67	60	50			67	2
5	45	72	75	71	63	52		75	2
6	50	78	82	79	74	65	53	82	2, 6

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$ $x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$x_3 = 5$	$x_3 = 6$	$f_2^*(s_2)$	x_2^*
0	0							0	0
1	33	20						33	0
2	46	53	38					53	1
3	57	66	71	54				71	2
4	67	77	84	87	65			87	3
5	75	87	95	100	98	73		100	3
6	82	95	105	111	111	106	80	111	3, 4

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$ $x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$x_1 = 5$	$x_1 = 6$	$f_1^*(s_1)$	x_1^*
6	111	125	129	126	116	102	72	129	2

Thus, optimal solution is:
A: 2 workers, *B*: 3 workers, *C*: 1 worker, *D*: none. Estimated increase in the number of votes = 42 + 54 + 33 = 129.

11. The stages here are represented by the containers *A*, *B*, *C* and *D* while the states are indicated by the capacity short of 15 tons. The recursive relationship is:

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 4

State, s_5	$f_4^*(s_4)$	x_4^*
0	0	0
1	840	1
2	840	1
3	840	1
4	840	1
5	840	1
6	840	1
7	1,680	2
8	1,680	2
9	1,680	2
10	1,680	2
11	1,680	2
12	1,680	2
13	2,520	3
14	2,520	3
15	2,520	3

Stage 3

State, s_3	$f_3(s_3) = \{c_3(s_3, x_3) + f_4^*(s_4)\}$					$f_3^*(s_3)$	x_3^*
	$x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$		
0	0					0	0
1	840	720				720	1
2	840	720				720	1
3	840	720				720	1
4	840	720				720	1
5	840	1,560	1,440			840	0
6	840	1,560	1,440			840	0
7	1680	1,560	1,440			1,440	2
8	1,680	1,560	1,440			1,440	2
9	1,680	1,560	2,280	2,160		1,560	1
10	1,680	1,560	2,280	2,160		1,560	1
11	1,680	2,400	2,280	2,160		1,680	0
12	1,680	2,400	2,280	2,160		1,680	0
13	2,520	2,400	2,280	3,000	2,880	2,280	2
14	2,520	2,400	2,280	3,000	2,880	2,280	2
15	2,520	2,400	3,120	3,000	2,880	2,400	1

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$						$f_2^*(s_2)$	x_2^*
	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$x_2 = 5$		
0	0						0	0
1	720	600					600	1
2	720	600					600	1
3	720	600					600	1
4	720	1,320	1,200				720	0
5	840	1,320	1,200				840	0
6	840	1,320	1,200				840	0
7	1,440	1,320	1,920	1,800			1,320	1
8	1,440	1,440	1,920	1,800			1,440	0, 1
9	1,560	1,440	1,920	1,800			1,440	1
10	1,560	2,040	1,920	2,520	2,400		1,560	0
11	1,680	2,040	2,040	2,520	2,400		1,680	0
12	1,680	2,160	2,040	2,520	2,400		1,680	0
13	2,280	2,160	2,640	2,520	3,120	3,000	2,160	1
14	2,280	2,280	2,640	2,640	3,120	3,000	2,280	0, 1
15	2,400	2,280	2,760	2,640	3,120	3,000	2,280	1

Stage 1

State, s_1	$f_1(s_1) = c_1(s_1, x_1) + f_2^*(s_2)$ $x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$f_1^*(s_1)$	x_1^*
15	2,280	2,340	2,400	3,000	2,280	0

The optimal solution is: *A*: none; *B*: 1; *C*: none; *D*: 2. Total cost = Rs 2,280.

12. Let the four products represent the four stages and the amount of budget remaining to be allocated to be s_n. The recursive relationship is:

$$f_n^*(s_n) = \max_{1 \le x_n \le s_n} \{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

where x_n is the advertising amount (in lakhs of Rs) spent on product n.

Stage 4

State, s_4	$f_4^*(s_4)$	x_4^*
1	9	1
2	13	2
3	19	3
4	25	4

Stage 3

State, s_3	$f_3(s_3) = \{c_3(s_3, x_3) + f_4^*(s_4)\}$ $x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$f_3^*(s_3)$	x_3^*
2	15				15	1
3	19	21			21	2
4	25	25	27		27	3
5	31	31	31	33	33	4

Stage 2

State s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$ $x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$f_2^*(s_2)$	x_2^*
3	30				30	1
4	36	32			36	1
5	42	38	20		42	1
6	48	44	35	42	48	1

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$ $x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$f_1^*(s_1)$	x_1^*
7	57	57	54	51	57	1, 2

From the analysis, we obtain the following optimal solutions:
Product *A*: 1 lakh, *B*: 1 lakh, *C*: 4 lakh, *D*: 1 lakh or
Product *A*: 2 lakh, *B*: 1 lakh, *C*: 3 lakh, *D*: 1 lakh,
Increase in sales = Rs 57 lakh.

13. There are three stages here, $n = 1, 2, 3$, represented by plants *X*, *Y* and *Z*, while the amount (in lakh Rs) available for allocation represents the states. If x_n be the amount allocated to plant *n*, the recursive relationship is:

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 3

State, s_3	$f_3^*(s_3)$	x_3^*
0	0	0
20	8	20
40	18	40
60	22	60
80	32	80

Stage 2

State,	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$					$f_2^*(s_2)$	x_2^*
s_2	$x_2 = 0$	$x_2 = 20$	$x_2 = 40$	$x_2 = 60$	$x_2 = 80$		
0	0					0	0
20	8	6				8	0
40	18	14	12			18	0
60	22	24	20	24		24	20, 60
80	32	28	30	32	30	32	0, 60

Stage 1

State,	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$					$f_1^*(s_1)$	x_1^*
s_1	$x_1 = 1$	$x_1 = 20$	$x_1 = 40$	$x_1 = 60$	$x_1 = 80$		
80	32	28	28	38	28	38	60

The optimal solution is: plant *X*: Rs 60 lakh, Plant *Y*: nil, Plant *Z*: Rs 20 lakh. Total return = Rs 38 lakh.

The second-best solution is: Plants *X* and *Y*: nil, Plant *Z*: Rs 80 lakh; or Plant *X*: nil, Plant *Y*: Rs 60 lakh and plant *Z*: Rs 20 lakh. Return = Rs 32 lakh.

14. There are three stages ($n = 1, 2, 3$) represented by contractors *A*, *B* and *C*. The states are the number of sub-stations available for allocation, while the recursive relationship is

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 3

State, s_3	$f_3^*(s_3)$	x_3^*
0	0	0
1	65	1
2	120	2
3	200	3
4	270	4
5	340	5
6	400	6

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$ $x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$x_2 = 5$	$x_2 = 6$	$f_2^*(s_2)$	x_2^*
0	0							0	0
1	65	60						60	1
2	120	125	120					120	0, 2
3	200	180	185	170				170	3
4	270	260	240	235	250			235	3
5	340	330	320	290	315	320		290	3
6	400	400	390	370	370	385	410	370	3, 4

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)$ $x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$x_1 = 5$	$x_1 = 6$	$f_1^*(s_1)$	x_1^*
6	370	360	375	370	380	390	400	360	1

Optimal solution: *A*: 1; *B*: 3 and *C*: 2. Total cost = Rs 360

15. Let the four items I_1, I_2, I_3 and I_4 represent the four stages, n = 1, 2, 3, 4. The states are the number of tons available for loading. If x_n be the number of units of the items loaded, the recursive relationship is

$$f_n^*(s_n) = \max_{x_n}\{c_n(s_n, x_n) + f_{n+1}^*(s_{n+1})\}$$

Stage 4

State, s_4	$f_4^*(s_4)$	x_4^*
0	0	0
1	0	0
2	36	1
3	36	1
4	72	2
5	72	2
6	108	3
7	108	3
8	144	4
9	144	4
10	180	5
11	180	5
12	216	6
13	216	6
14	252	7

Stage 3

State, s_3	$f_3(s_3) = \{c_3(s_3, x_3) + f_4^*(s_4)\}$ $x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$f_3^*(s_3)$	x_3^*
0	0			0	0
1	0			0	0
2	36			36	0
3	36			36	0
4	72			72	0
5	72	60		72	0
6	108	60		108	0
7	108	96		108	0
8	144	96		144	0
9	144	132		144	0
10	180	132	120	180	0
11	180	168	120	180	0
12	216	168	156	216	0
13	216	204	156	216	0
14	252	204	192	252	0

Stage 2

State, s_2	$f_2(s_2) = \{c_2(s_2, x_2) + f_3^*(s_3)\}$ $x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$f_2^*(s_2)$	x_2^*
0	0				0	0
1	0				0	0
2	36				36	0
3	36				36	0
4	72	50			72	0
5	72	50			72	0
6	108	86			108	0
7	108	86			108	0
8	144	122	100		144	0
9	144	122	100		144	0
10	180	158	136		180	0
11	180	158	136		180	0
12	216	194	172	150	216	0
13	216	194	172	150	216	0
14	252	230	208	186	252	0

Stage 1

State, s_1	$f_1(s_1) = \{c_1(s_1, x_1) + f_2^*(s_2)\}$ $x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$x_1 = 4$	$f_1^*(s_1)$	x_1^*
14	252	220	224	192	196	252	0

A scan of the optimal decisions at various stages leads to the following overall optimal decision:
$I_1 = I_2 = I_3$ = nil, I_4 = 7 units; Total value = 252 units.

16. *Stages:* Each day is considered as a stage. There are five stages, therefore, Monday: 1, Tuesday: 2; Wednesday: 3; Thursday: 4 and Friday: 5.
States: In each stage, there are two states: to sell or to hold (except in stage 5, where shares can only be sold).
Payoff: The payoff is the expected price realised.
Solution:
Stage 5: If shares are not sold by this time, then
Expected payoff = 0.25 × 40 + 0.30 × 41 + 0.45 × 42 = Rs 41.2. At each other stage, the investor can either sell or wait. She should sell the shares if the prevailing price is greater than the expected price (payoff) in the next stage and wait if the price is lower than that.
Stage 4: There is a 70% of chance of waiting and a 30% chance of selling (since Rs 42 > Rs 41.2)
Expected payoff = 0.30 × 4.2 + 0.70 × 41.2 = Rs 41.44
Stage 3: Expected payoff = 0.30 × 42 + 0.70 × 41.44 = Rs 41.608
Stage 2: Expected payoff = 0.30 × 42 + 0.70 × 41.608 = Rs 41.7256
Stage 1: Expected payoff = 0.30 × 42 + 0.70 × 41.7256 = Rs 41.80792
Thus, following optimal policy, the investor can sell the shares for an expected price of Rs 41.81. For various days, the decision rule is as follows:
Days: Monday through Thursday
Sell if the price of shares is Rs 22, else wait
Day: Friday
Sell at any price
Expected receipt = 5,000 (41.81 – 0.20) = Rs 2,08,050.

17. *Stages:* Each week may be considered as a stage. Thus, there are four stages.
States: There are two states in every stage: to buy or to wait.
Payoff: The payoff function is given by the expected price at a given stage.
Solution:
Stage 4: Here, the manager has no choice so that the scrap must be bought if it has not been purchased earlier.
Expected price = 0.2 × 1,000 + 0.5 × 1,100 + 0.3 × 1,200 = Rs 1,110
Other stages: At each other stage, the manager can either buy at the prevailing price or he can wait until next week. The optimal policy dictates that if the price in the current week is higher than the expected price in the next week (stage), he can wait until next stage, while if the price is lower than that, he should buy in the current stage. Accordingly, analysis for other stages follows.
Stage 3: There is a 20% of buying at Rs 1,000 and a 50% chance of buying at Rs 1,110. Similarly, there is a 30% chance of waiting, in which case the payoff would be Rs 1,110 (from stage 4). Thus,
Expected payoff = 0.20 × 1,000 + 0.50 × 1,100 + 0.30 × 1,110 = Rs 1,083
Stage 2: Expected payoff = 0.20 × 1,000 + 0.80 × 1,083 = Rs 1,066.4
Stage 1: Expected payoff = 0.20 × 1,000 + 0.80 × 1,066.4 = Rs 1,053.1
Accordingly, following optimal policy, the manager will pay an expected price of Rs 1,053.1. For each week, the decision rule is:
First week: Buy if the price is Rs 1,000
Wait if the price is Rs 1,100 or Rs 1,200
Second week: Buy if the price is Rs 1,000
Wait if the price is Rs 1,100 or Rs 1,200

Third week: Buy if the price is Rs 1,000 or Rs 1,100
Wait if the price is Rs 1,200
Fourth week: Buy at the prevailing price.

18. Assuming that the salesman starts from city *A*, this is the last city he would visit. When he has only one city left to visit, his problem is trivial: he goes from city he is in to *A*. Next, we can work backward to a problem in which he is in some city and left only two cities to visit, and finally, we can determine the shortest tour that originates in *A* and has four cities to visit. Accordingly, we let the stages be represented by the number of cities the salesman has already visited. At any stage, the city to be next visited would be determined by the current location of the salesman and cities already visited by him. We define $f_t\,(i, s)$ to be the minimum distance to be travelled to complete a tour when $t - 1$ cities in the set *S* have been visited and city *i* is the last city visited. Further, let C_{ij} represents the distance between cities *i* and *j*. In the solution, we represent cities *A*, *B*, *C* and *D* be represented as 1, 2, 3 and 4 respectively.

Stage 4: Here $S = \{2, 3, 4\}$ and possible states are (2, {2, 3, 4}), (3, {2, 3, 4}) and (4, {2, 3, 4}). Thus,

$f_4(2, \{2, 3, 4\}) = C_{21} = 1{,}297^*$ (2 to 1)
$f_4(3, \{2, 3, 4\}) = C_{31} = 1{,}522^*$ (3 to 1)
$f_4(4, \{2, 3, 4\}) = C_{41} = 772^*$ (4 to 1)

Stage 3

$f_3(2, \{2, 3\}) = C_{24} + f_4(4, \{2, 3, 4\}) = 1{,}360 + 772 = 2{,}132^*$ (2 to 4)
$f_3(3, \{2, 3\}) = C_{34} + f_4(4, \{2, 3, 4\}) = 884 + 772 = 1{,}656^*$ (3 to 4)
$f_3(2, \{2, 4\}) = C_{23} + f_4(3, \{2, 3, 4\}) = 1{,}306 + 1{,}522 = 2{,}828^*$ (2 to 3)
$f_3(4, \{2, 4\}) = C_{43} + f_4(3, \{2, 3, 4\}) = 884 + 1{,}522 = 2{,}406^*$ (4 to 3)
$f_3(3, \{3, 4\}) = C_{32} + f_4(2, \{2, 3, 4\}) = 1{,}306 + 1{,}297 = 2{,}603^*$ (3 to 2)
$f_3(4, \{3, 4\}) = C_{42} + f_4(2, \{2, 3, 4\}) = 1{,}360 + 1{,}297 = 2{,}657^*$ (4 to 2)

Stage 2

$$f_2\,(2, \{2\}) = \min \left[\begin{array}{ll} C_{23} + f_3\,(3, \{2, 3\}) = 1{,}306 + 1{,}656 = 2{,}962^* & (2 \text{ to } 3) \\ C_{24} + f_3(4, \{2, 4\}) = 1{,}306 + 2{,}406 = 3{,}766 & (2 \text{ to } 4) \end{array} \right.$$

$$f_2(3, \{3\}) = \min \left[\begin{array}{ll} C_{34} + f_3(4, \{3, 4\}) = 884 + 2{,}657 = 3{,}541 & (3 \text{ to } 4) \\ C_{32} + f_3(2, \{2, 3\}) = 1{,}306 + 2{,}132 = 3{,}438^* & (3 \text{ to } 2) \end{array} \right.$$

$$f_2\,(4, \{4\}) = \min \left[\begin{array}{ll} C_{42} + f_3(2, \{2, 4\}) = 1{,}360 + 2{,}828 = 4{,}188 & (4 \text{ to } 2) \\ C_{43} + f_3(3, \{3, 4\}) = 884 + 2{,}603 = 3{,}487^* & (4 \text{ to } 3) \end{array} \right.$$

Stage 1

At this stage, the salesman has not visited any cities and he happens to be in city *A*.

$$f_1\,(1, \{\cdot\}) = \min \left[\begin{array}{l} C_{12} + f_2(2, \{2\}) = 1{,}297 + 2{,}962 = 4{,}259^* \\ C_{13} + f_2(3, \{3\}) = 1{,}522 + 3{,}438 = 4{,}960 \\ C_{14} + f_2(4, \{4\}) = 772 + 3{,}487 = 4{,}259^* \end{array} \right.$$

Optimal decision: There are two optimal decisions indicated here: Go from city 1(*A*) to city 2(*B*), from city 2(*B*) to city 3(*C*) and, finally, from city *C* to city 4(*D*) and then to city 1(*A*). Else, go from city 1(*A*) to city 4(*D*), then to city 3(*C*), next to city 2(*B*) and, finally, to city 1(*A*). Each of these involves a total distance of 4,259 units of distance. Of course, one of the tours here is simply a reversal of other.

CHAPTER 17

1. **Simulation Worksheet**

S. No.	*Arrival time of customer (t)*	*Customer Service*		*Waiting time*	*Idle time*
		Begins	*Ends*		
1.	0	0	4	0	0
2.	1.8	4	8	2.2	0
3.	3.6	8	12	4.4	0
4.	5.4	12	16	6.6	0
5.	7.2	16	20	8.8	0
6.	9.0	20	24	11.0	0
7.	10.8	24	28	13.2	0
8.	12.6	28	32	15.4	0
			Total	61.6	

Average waiting time per customer = 61.6/8 = 7.7 units. Percentage idle time of the facility = nil.

2. Determination of Random Number Intervals

Daily demand	*Probability*	*Cumulative probability*	*R. No. interval*
0	0.01	0.01	00
10	0.20	0.21	01–20
20	0.15	0.36	21–35
30	0.50	0.86	36–85
40	0.12	0.98	86–97
50	0.02	1.00	98–99

The simulation of demand is given here.

Simulation Worksheet

Day	*R. No.*	*Demand*	*Day*	*R. No.*	*Demand*
1	25	20	6	05	10
2	39	30	7	73	30
3	65	30	8	89	40
4	76	30	9	19	10
5	12	10	10	49	30

Average demand per day = 240/10 = 24 units

3. **Determination of Random Number Intervals**

Demand	*Frequency*	*Probability*	*R. No. Interval*
0	2	0.04	00–03
5	11	0.22	04–25
10	8	0.16	26–41
15	21	0.42	42–83
20	5	0.10	84–93
25	3	0.06	94–99
Total	50		

Demand Simulation:

Week:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R. No.:	35	52	90	13	23	73	34	57	35	83	94	56	67	66	60
Demand:	10	15	5	5	5	15	10	15	10	15	25	15	15	15	15

Average demand = 12.6 units

4. *Step 1:* Determine random number intervals

Demand (units)	*Probability*	*Cumulative probability*	*Random number interval*
15	0.05	0.05	00–04
16	0.08	0.13	05–12
17	0.20	0.33	13–32
18	0.45	0.78	33–77
19	0.10	0.88	78–87
20	0.07	0.95	88–94
21	0.03	0.98	95–97
22	0.02	1.00	98–99

Step 2: Simulation of demand for 20 years would be 16, 15, 20, 22, 17, 18, 18, 17, 16, 16, 20, 17, 15, 18, 18, 17, 17, 16, 20, and 18.

It is given in the form of a frequency distribution:

Demand :	15	16	17	18	19	20	21	22
No. of Years :	2	4	5	5	0	3	0	1

Step 3: Obtain conditional pay-off matrix as follows.

Conditional Pay-off Matrix

Demand	*Course of Action*							
	15	*16*	*17*	*18*	*19*	*20*	*21*	*22*
15	300	270	240	210	180	150	120	90
16	300	320	290	260	230	200	170	140
17	300	320	340	310	280	250	220	190
18	300	320	340	360	330	300	270	240
19	300	320	340	360	380	350	320	290
20	300	320	340	360	380	400	370	340
21	300	320	340	360	380	400	420	390
22	300	320	340	360	380	400	420	440

Step 4: Using frequency distribution of demand simulated, we may calculate the expected pay-offs having reference to the pay-off matrix. This is shown in the table here.

Calculation of Expected Pay-off

Course of Action	*Expected Pay-off*	*Total*
15	20 × 300	6,000
16	2 × 270 + 18 × 320	6,300
17	2 × 240 + 4 × 290 + 14 × 340	6,400
18	2 × 210 + 4 × 260 + 5 × 310 + 9 × 360	6,250
19	2 × 180 + 4 × 230 + 5 × 280 + 5 × 330 + 4 × 380	5,850
20	2 × 150 + 4 × 200 + 5 × 250 + 5 × 300 + 4 × 400	5,200
21	2 × 120 + 4 × 170 + 5 × 220 + 5 × 270 + 3 × 370 + 1 × 420	4,900
22	2 × 90 + 4 × 140 + 5 × 190 + 5 × 240 + 3 × 340 + 1 × 440	4,350

The optimal policy is to buy 17 copies every year since it will entail the highest expected profit.

5. From the given information, the conditional probability distributions may be expressed as follows. Alongside, random number intervals are given.

When help is needed the previous day: (Distribution D_1)

Whether help needed today	*Probability*	*Random number interval*
Yes	0.60	00–59
No	0.40	60–99

When help is not needed the previous day: (Distribution D_2)

Whether help needed today	*Probability*	*Random number interval*
Yes	0.30	00–29
No	0.70	30–99

The simulation worksheet is given here with the random numbers taken from second row in table of random numbers (B7).

Simulation Worksheet

Day	*R. No.*	*Help?*	*Distribution*	*Day*	*R. No.*	*Help?*	*Distribution*
1	19	Yes	D_2	11	85	No	D_2
2	36	Yes	D_1	12	52	No	D_2
3	27	Yes	D_1	13	05	Yes	D_2
4	59	Yes	D_1	14	30	Yes	D_1
5	46	Yes	D_1	15	62	No	D_1
6	13	Yes	D_1	16	39	No	D_2
7	79	No	D_1	17	77	No	D_2
8	93	No	D_2	18	32	No	D_2
9	37	No	D_2	19	77	No	D_2
10	55	No	D_2	20	09	Yes	D_2

Thus, proportion of days when extra help is needed = 9/20.

6. In the following solution, the random numbers used are taken from the first three columns of the table of random numbers (Table B7). The three columns are used respectively for price, yield and cost.

Simulation Worksheet

Run	*Price/Quintal*		*Yield(Q/acre)*		*Cost/acre*		*Profit/acre*
	R. No.	*Price (Rs)*	*R. No.*	*Yield*	*R. No.*	*Cost (Rs)*	*(Rs)*
1	22	250	17	70	68	16,000	1,500
2	19	250	36	75	27	12,000	6,750
3	16	250	77	80	23	12,000	8,000
4	78	260	43	75	76	16,000	3,500
5	03	240	28	75	28	12,000	6,000
6	93	280	22	75	53	14,000	7,000
7	78	260	76	80	58	14,000	6,800
8	23	250	68	80	35	14,000	6,000

(*Contd.*)

(*Contd.*)

9	15	250	39	75	25	12,000	6,750
10	58	260	71	80	96	18,000	2,800
11	57	260	35	75	27	12,000	7,500
12	48	260	50	75	86	18,000	1,500
13	61	260	96	85	48	14,000	8,100
14	36	260	93	85	89	18,000	4,100
15	18	250	89	80	00	12,000	8,000
16	88	270	56	75	53	14,000	6,250
17	09	240	72	80	95	18,000	1,200
18	12	250	96	85	88	18,000	3,250
19	85	270	94	85	57	14,000	8,950
20	38	260	64	80	43	14,000	6,800

Expected profit = Rs 5,537.50

7. *Step 1:* Determination of Random Number Intervals

Receipts (Rs)			*Payments (Rs)*		
Amount	*Prob.*	*R. No. Interval*	*Amount*	*Prob.*	*R. No. Interval*
3,000	0.20	00–19	4,000	0.30	00–29
5,000	0.30	20–49	6,000	0.40	30–69
7,000	0.40	50–89	8,000	0.20	70–89
12,000	0.10	90–99	10,000	0.10	90–99

Step 2: Simulation of Receipts and Payments ('000 Rs)

Week:	1	2	3	4	5	6	7	8	9	10	11	12
Receipts												
R. No.:	03	91	38	55	17	46	32	43	69	72	24	22
Amount:	3	12	5	7	3	5	5	5	7	7	5	5
Payments												
R. No.:	61	96	30	32	03	88	48	28	88	18	71	99
Amount :	6	10	6	6	4	8	6	4	8	4	8	10

Step 3:

Receipts and Payments Statement

Week	*Opening Balance*	*Receipts*	*Payments*	*Closing Balance*
1	8,000	3,000	6,000	5,000
2	5,000	12,000	10,000	7,000
3	7,000	5,000	6,000	6,000
4	6,000	7,000	6,000	7,000
5	7,000	3,000	4,000	6,000
6	6,000	5,000	8,000	3,000
7	3,000	5,000	6,000	2,000
8	2,000	5,000	4,000	3,000
9	3,000	7,000	8,000	2,000
10	2,000	7,000	4,000	5,000
11	5,000	5,000	8,000	2,000
12	2,000	5,000	10,000	(3,000)

Estimated balance at the end = (Rs 3,000)
Highest weekly balance = Rs 7,000
Average weekly balance = Rs 3,750

8. Hint: Let '0' indicate head and '1' indicate tail. Assign probability of 0.5 to each. Scan the random number is some order and locate 0 and 1, until the difference between heads and tails is equal to 5. Proceed to find the gain.
9. (a) Assuming that the system is initially empty, we can record the arrival and service of the customers as shown in the simulate worksheet.
Average time in queue = 71/10 = 7.1 or 142 seconds
Average time in system = 506/10 = 50.6 or 1,012 seconds

Average number in the queue = $\frac{71-9}{622}$ = 0.10

Average number in the system = $\frac{506-26}{622}$ = 0.77

Simulation Worksheet

Arrival	*Arrival Time*	*Service Start*	*Departure Time*	*Time in Queue*	*Time in System*
1	41	41	111	0	70
2	87	111	123	24	36
3	125	125	148	0	23
4	182	182	218	0	36
5	269	269	405	0	136
6	490	490	545	0	55
7	510	545	610	35	100
8	609	610	614	1	5
9	612	614	631	2	19
10	622	631	648	9	26
			Total	71	506

(b) From the given data,
Average inter-arrival time = 622/10 = 62.2 or 1,244 seconds
Average service time = 435/10 = 43.5 or 870 seconds
Accordingly,

Arrival rate, $\lambda = \frac{1}{1,244}$ per second

Service rate, $\mu = \frac{1}{870}$ per second

Substituting these values in the formulae given,
Average time in queue = 2,024 seconds
Average time in system = 2,894 seconds
Average number in the queue = 1.63
Average number in the system = 2.33
Obviously, there are differences in the two sets of results. There are primarily two reasons for this:
(i) The formulae are based on the assumptions of Poisson arrivals and negatively exponentially distributed service times. The greater the departures from these assumptions, the more variation in the results.

(ii) The formulae also assume steady state while the simulation here is based on the assumption of an empty system.

10. **Simulation Worksheet**

Customer	*Arrivals*			*Service*				*Waiting*	*Idle*
	R. No.	*IAT*	*AT*	*R. No.*	*ST*	*SS*	*SF*	*time*	*time*
1	19	04	04	08	01	04	05	0	4
2	32	04	08	27	03	08	11	0	3
3	59	06	14	74	07	14	21	0	1
4	81	08	22	96	09	22	31	0	0
5	27	04	26	48	05	31	36	5	0
6	45	06	32	07	01	36	37	4	0
7	26	04	36	65	05	37	42	1	0
8	52	06	42	78	07	42	49	0	1
9	77	08	50	92	09	50	59	0	0
10	46	06	56	49	05	59	64	3	
							Total	13	9

Average waiting time = 13/10 = 1.3 minutes
Probability of idle time = 9/64
Notes:

1. In the given four-digit random numbers, the first two digits are used for inter-arrival times while the other two are used for service times.
2. According to the inter-arrival times simulated, only 10 customers arrive within the stipulated 60 minutes.

IAT: inter-arrival time; AT: arrival time (*t*); ST: service time; SS: service starts; SF: service finish.

11. The probabilities of having *A*, *B*, and *C* defects are given as 0.15, 0.20, and 0.10 respectively. Thus, chances of not having these would be 0.85, 0.80, and 0.90 respectively. Accordingly, we may determine the random number intervals for each of these as follows:

Defect A			*Defect B*			*Defect C*		
Presence	*Prob.*	*R. No.*	*Presence*	*Prob.*	*R. No.*	*Presence*	*Prob.*	*R. No.*
Yes	0.15	00–14	Yes	0.20	00–19	Yes	0.10	00–09
Yes	0.85	15–99	No	0.80	20–99	No	0.90	10–99

The results of simulation are given in the simulation worksheet.

Simulation Worksheet (Defects and Rework)

Item	*Random number for defect*			*Presence of defect/s*	*Rework time*	*Remarks*
	A	*B*	*C*			
1	48	47	82	None	—	
2	55	36	95	None	—	
3	91	57	18	None	—	
4	40	04	96	*B*	15	
5	93	79	20	None	—	
6	01	55	84	*A*	—	Scrap
7	83	10	56	*B*	15	
8	63	13	11	*B*	15	
9	47	57	52	None	—	
10	52	09	03	*B*, *C*	15 + 30 = 45	

Thus, for the simulated runs, five out of ten items were found to have no defects, one item was scrapped and a total of 90 minutes of rework time was required by four items.

12. Using the given data, we first obtain the arrival times of the patients and state the service times required by them.

Patient	*Time since last Arrival (R. No. 00–80)*	*Arrival (clock) time*	*Service time (R. No. 15–14)*
1	07	007	23
2	21	0:28	37
3	12	0:40	16
4	80	2:00	28
5	08	2:08	30
6	03	2:00	18
7	32	2:43	25
8	65	3:48	34
9	43	4:31	19
10	74	5:45	21

(a) The required calculations are shown in Simulation Worksheet 1.

Simulation Worksheet 1

Patient	*Arrival Time*	*Service Time*	*Service*		*Patients in Queue*	*Time in Queue*	*Idle Time of Doctor*
			Begins	*Ends*			
1	0:07	23	0:07	0:30	0	0	7
2	0:28	37	0:30	1:07	1	2	0
3	0:40	16	1:07	1:23	1	27	0
4	2:00	28	2:00	2:28	0	0	37
5	2:08	30	2:28	2:58	1	20	0
6	2:11	18	2:58	3:16	2	47	0
7	2:43	25	3:16	3:41	2	32	0
8	3:48	34	3:48	4:22	0	0	7
9	4:31	19	4:31	4:50	0	0	9
10	5:45	21	5:45	6:06	0	0	55
					Total	128	115

Average patients' queue time = 128/10 = 12.8 minutes.
Percentage of time the doctor is idle = 115/366 = 31%.

(b) The calculations are presented in Simulation Worksheet 2.

Simulation Worksheet 2

Patient	*Arrival Time*	*Service Time*	*Doctor 1 or 2*	*Service*		*Waiting in Queue*	*Doctor's Idle Time*	
				Begins	*Ends*		*1*	*2*
1	0:07	23	1	0:07	0:30	0	7	—
2	0:28	37	2	0:28	1:05	0	—	28
3	0:40	16	1	0:40	0:56	0	10	—
4	2:00	28	2	2:00	2:28	0	—	55
5	2:08	30	1	2:08	2:38	0	12	—
6	2:11	18	2	2:28	2:46	17	—	—
7	2:43	25	1	2:43	3:08	0	5	—
8	3:48	34	2	3:48	4:22	0	—	62
9	4:31	19	1	4:31	4:50	0	83	—
10	5:45	21	2	5:45	6:06	0	—	83
					Total	17	117	228

Average patient queue time = 17/10 = 1.7 minutes.
Percentage idle time for doctor 1 = 117/366 = 32%.
Percentage idle time for doctor 2 = 228/366 = 62%.

13. *Step 1:* Determine random number intervals.
The random number intervals are determined both for supply and demand, as a first step. This is given below:

Supply	*Prob.*	*R. No. interval*	*Demand*	*Prob.*	*R. No. interval*
10	0.08	00–07	10	0.10	00–09
20	0.10	08–17	20	0.22	10–31
30	0.38	18–55	30	0.40	32–71
40	0.30	56–85	40	0.20	72–91
50	0.14	86–99	50	0.08	92–99

The probabilities for various supply/demand values are obtained by dividing the given frequencies by their respective totals.

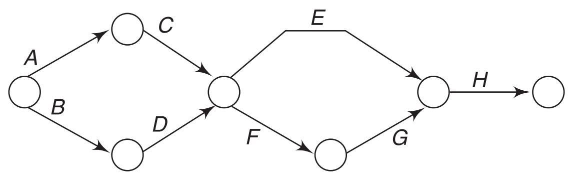
Step 2: Simulate supply/demand using given random numbers. Calculate profit/loss.
The simulation using random numbers is shown here. Further, profit has been calculated as:
Sale revenue – Cost – Loss on unsatisfied demand.

Simulation Worksheet

Day	*Supply*		*Demand*		*Profit/Loss*
	R. No.	*Amount (kg)*	*R. No.*	*Amount (kg)*	
1	31	30	18	20	600 – 600 = 0
2	63	40	84	40	1,200 – 800 = 400
3	15	20	79	40	600 – 400 – 160 = 40
4	07	10	32	30	300 – 200 – 160 = (60)
5	43	30	75	40	900 – 600 – 80 = 220
6	81	40	27	20	600 – 800 = (200)

Note: This question does not appear to be properly worded. The loss on unsatisfied demand should not be considered in determining the profit since unearned profit is actually opportunity loss and non out-of-pocket loss. It can be adjusted with profit only when a penalty is required to be paid for not satisfying some demand.

14. Based on given precedence relationships, the network is drawn here:



Network

It is evident that length of the critical path may be determined as:

Greater of times of *A* and *C*, and *B* and *D*	*plus*	Greater of times of *E*, and *F* and *G*	*plus*	Time of *H*

To simulate the activity durations, we first obtain random number intervals for each of the activities as shown in the following table.

Determination of Random Number Intervals

Activity	*Duration (Days)*	*Prob.*	*R. No. Interval*	*Activity*	*Duration (Days)*	*Prob.*	*R. No. Interval*
A	2	0.20	00–19	*E*	4	0.60	00–59
	3	0.40	20–59		5	0.40	60–99
	4	0.40	60–99	*F*	2	0.80	00–79
B	4	0.30	00–29		3	0.20	80–99
	6	0.70	30–99	*G*	1	0.30	00–29
C	3	0.30	00–29		3	0.50	30–79
	4	0.30	30–59		4	0.20	80–99
	5	0.40	60–99	*H*	2	0.10	00–09
D	2	0.20	00–19		3	0.20	10–29
	3	0.60	20–79		4	0.30	30–59
	5	0.20	80–99		5	0.40	60–99

Using random numbers, the activity times are obtained as shown in the worksheet. The random numbers are read column-wise, beginning with north-east corner of the Table B7.

Simulation Worksheet

Activity	*Simulation Run*											
	1		*2*		*3*		*4*		*5*		*6*	
	R. No.	*Time*	*R. No.*	*Time*	*R. No.*	*Time*	*R. No.*	*Time*	*R. No.*	*Time*	*R. No.*	*Time*
A	22	3	17	2	68	4	65	4	84	4	68	4
B	19	4	36	6	27	4	59	6	46	6	13	4
C	16	3	77	5	23	3	02	3	77	5	09	3
D	78	3	43	3	76	5	71	3	61	3	20	3
E	03	4	28	4	28	4	26	4	08	4	73	5
F	93	3	22	2	53	2	64	2	39	2	07	2
G	78	3	76	3	58	3	54	3	74	3	92	4
H	23	3	68	5	35	4	26	3	00	2	99	5

From the given information, the critical path for each of the runs and the project duration may be obtained as shown below:

Simulation run	*Critical path(s)*	*Duration*
1	*B–D–F–G–H*	16
2	*B–D–F–G–H*	19
3	*B–D–F–G–H*	18
4	*B–D–F–G–H*	17
5	*A–C–F–G–H/B–D–F–G–H*	16
6	*A–C–F–G–H/B–D–F–G–H*	18

15. To solve the problem, we first determine random number intervals in accordance with the given probabilities.

Determination of Random Number Intervals

Demand ('000 units)	*Prob.*	*R.No. Interval*	*Profit (Rs)*	*Prob.*	*R. No. Interval*	*Investment ('000 Rs)*	*Prob.*	*R. No. Interval*
25	0.05	00–04	3	0.10	00–09	2,750	0.25	00–24
30	0.10	05–14	5	0.20	10–29	3,000	0.50	25–74
35	0.20	15–34	7	0.40	30–69	3,500	0.25	75–99
40	0.30	35–64	9	0.20	70–89			
45	0.20	65–84	10	0.10	90–99			
50	0.10	85–94						
55	0.05	95–99						

Based on the given random numbers and the random number intervals, the simulated values are given in the simulation worksheet where return on investment is also shown in the last column. The return is calculated as the ratio of total profit to total investment, expressed as a percentage. From the values calculated, the only value seen to repeat itself is 5.83 per cent, which is the most likely return therefore.

Simulation Worksheet

Run	*Demand*		*Unit Profit*		*Investment*		*Total Profit ('000 Rs)*	*Return (%)*
	R. No.	*'000 units*	*R. No.*	*Rs*	*R. No.*	*('000 Rs)*		
1	30	35	12	5	16	2,750	175	6.34
2	59	40	09	3	69	3,000	120	4.00
3	63	40	94	10	26	3,000	400	13.33
4	27	35	08	3	74	3,000	105	3.50
5	64	40	60	7	61	3,000	280	9.33
6	28	35	28	5	72	3,000	175	5.83
7	31	35	23	5	57	3,000	175	5.83
8	54	40	85	9	20	2,750	360	13.09
9	64	40	68	7	18	2,750	280	10.18
10	32	35	31	7	87	3,500	245	7.00

16. As a first step, we determine random number intervals for each of the three variables, in keeping with the probabilities of various values thereof.

Determination of Random Number Intervals

(a) *Contribution:*

Contribution Per unit (Rs)	*Prob.*	*Cumulative probability*	*Random Number interval*
3	0.10	0.10	00–09
5	0.20	0.30	10–29
7	0.40	0.70	30–69
9	0.20	0.90	70–89
10	0.10	1.00	90–99

(b) *Demand:*

Annual demand (in '000 units)	*Prob.*	*Cumulative probability*	*Random number interval*
20	0.05	0.05	00–04
25	0.10	0.15	05–14
30	0.20	0.35	15–34
35	0.30	0.65	35–64
40	0.20	0.85	65–84
45	0.10	0.95	85–94
50	0.05	1.00	95–99

(c) *Investment:*

Investment ('000 Rs)	*Prob.*	*Cumulative probability*	*Random number interval*
1,750	0.25	0.25	00–24
2,000	0.50	0.75	25–74
2,500	0.25	1.00	75–99

We may now simulate the output of 10 runs using the given random numbers in order to find the percentage of return on investment (ROI%) defined as:

$$\text{ROI} = \frac{\text{Cash inflow}}{\text{Cash outflow}} \times 100$$

Where cash inflow = Contribution per unit × Demand.

Simulation Worksheet (ROI)

S. No.	*Random Number*	*Contribution per unit (Rs)*	*Demand ('000 units)*	*Investment ('000 Rs)*	*ROI%*
1	93	10	45	2,500	18.00
2	03	3	20	1,750	3.45
3	51	7	35	2,000	12.25
4	59	7	35	2,000	12.25
5	77	9	40	2,500	14.40
6	61	7	35	2,500	12.25
7	71	9	40	2,000	18.00
8	62	7	35	2,000	12.25
9	99	10	50	2,500	20.00
10	15	5	30	1,750	8.57

The ROI in the last column is the ratio of the product of contribution per unit and demand to the investment. For example, the first value is obtained as:

$$\frac{10 \times 45{,}000}{25{,}00{,}000} \times 100 = 18\%$$

Since the modal value of ROI% is 12.25, the optimal investment strategy is to invest Rs 20,00,000.

17. In accordance with the probabilities given for each input variable, the random number intervals are determined first. This is shown below.

Selling price			*Sales offtake (A)*		
Price (Rs)	*Prob.*	*R. No. interval*	*Units*	*Prob.*	*R. No. interval*
25	0.55	00–54	45,000	0.20	00–19
30	0.45	55–99	50,000	0.35	20–54
			55,000	0.45	55–99

Sales offtake (B)			*Variable cost*		
Units	*Prob.*	*R. No. interval*	*Units cost (Rs)*	*Prob.*	*R. No. interval*
40,000	0.35	00–34	10	0.25	00–24
45,000	0.40	35–74	12	0.35	25–59
50,000	0.25	75–99	14	0.40	60–99

Fixed cost		
Cost (Rs lakh)	*Prob.*	*R. No. interval*
3	0.35	00–34
4	0.45	35–79
5	0.20	80–99

From the given information, it is evident that if the estimated selling price (in accordance with the chosen value of random number) is Rs 25, then distribution *A* of the sales offtake would be referred to. On the other hand, if the selling price is Rs 30, then distribution *B* would be used. The simulation is shown in the worksheet.

Simulation Worksheet: Profit Estimation

Iteration	Selling Price (Rs)		Sales ('000 units)		Variable Cost per unit (Rs)		Fixed Cost (lakh Rs)		Profit/Loss (lakh Rs)
	R. No.	*Amt.*	*R. No.*	*Amt.*	*R. No.*	*Amt.*	*R. No.*	*Amt.*	
1	12	25	09	45	33	12	65	4	1.85
2	87	30	79	50	15	10	43	4	6.00
3	28	25	46	50	72	14	11	3	2.50
4	98	30	92	50	05	10	13	3	7.00
5	25	25	67	55	54	12	90	5	2.15
6	42	25	38	50	76	14	45	4	1.50
7	98	30	06	40	33	12	28	3	4.20
8	64	30	10	40	74	14	92	5	1.40
9	71	30	27	40	53	12	04	3	4.20
10	01	25	06	45	67	14	96	5	(0.05)
11	48	25	52	50	37	12	45	4	2.50
12	80	30	33	40	12	10	67	4	4.00

From the values given in the last column of the table, the required probabilities are obtained as follows:

(i) Probability that the company will not break-even = 1/12

(ii) Probability that volume would exceed Rs 3 lakh = 5/12

(iii) Probability that profit would not be over Rs 4 lakh = 2/3

18. As a first step, we determine random number intervals to simulate demand for 10 days, in accordance with the random numbers given. This is done below.

Demand	*Probability*	*Cumulative probability*	*Random number Interval*
0	0.05	0.05	00–04
1	0.10	0.15	05–14
2	0.30	0.45	15–44
3	0.45	0.90	45–89
4	0.10	1.00	90–99

The demand is estimated below:

Day :	1	2	3	4	5	6	7	8	9	10
R. No. :	89	34	78	63	61	81	39	16	13	73
Demand :	3	2	3	3	3	3	2	2	1	3

Now, each of the two policies may be evaluated. This is shown in the simulation worksheets.

Each of these is drawn on the basis of the assumption that the demand for a given day can be met out of the stock in hand and the units receivable, if any, at the end of that day.

Policy 1: Inventory at the beginning + Orders outstanding < 8, Order 5 books.

Simulation Worksheet: Policy 1

Day	*Opening Stock*	*Demand*	*Receipts*	*Closing Stock*	*Outstanding Orders*		
					Opening	*Orders*	*Closing*
1	8	3	—	5	6	—	6
2	5	2	6	9	6	—	—
3	9	3	—	6	—	—	—
4	6	3	—	3	—	5	5
5	3	3	—	0	5	—	5
6	0	3	5	2	5	5	5
7	2	2	—	0	5	5	10
8	0	2	5	3	10	—	5
9	3	1	5	7	5	—	—
10	7	3	—	4	—	5	5

Carrying cost = 39 × 0.50 = Rs 19.50
Ordering cost = 4 × 10 = Rs 40.00
Total Cost = Carrying cost + Ordering cost
= Rs 19.50 + Rs 40.00 = Rs 59.50

Policy 2: Inventory at the beginning + Orders outstanding < 8, Order 8 books.

Simulation Worksheet: Policy 2

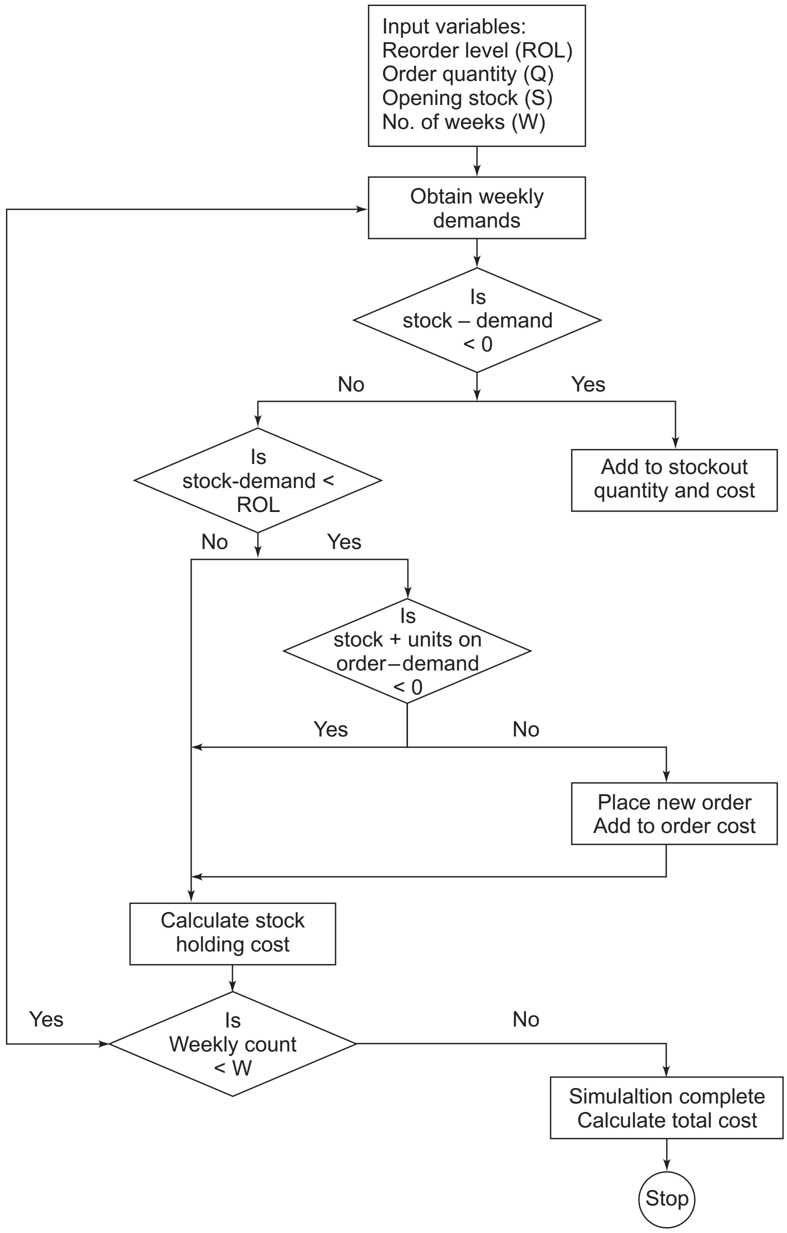
Day	*Opening Stock*	*Demand*	*Receipts*	*Closing Stock*	*Outstanding Orders*		
					Opening	*Orders*	*Closing*
1	8	3	—	5	6	—	6
2	5	2	6	9	6	—	—
3	9	3	—	6	—	—	—
4	6	3	—	3	—	8	8
5	3	3	—	0	8	—	8
6	0	3	8	5	8	—	—
7	5	2	—	3	—	8	8
8	3	2	—	1	8	—	8
9	1	1	8	8	8	—	—
10	8	3	—	5	—	—	—

Carrying cost = 45 × 0.50 = Rs 22.50
Ordering cost = 2 × 10 = Rs 20.00
Total cost = Rs 22.50 + Rs 20.00 = Rs 42.50

Conclusion: Adopt Policy 2.

19. (a) The weekly demand can be simulated using random numbers, on the basis of random number intervals in accordance with the given frequencies.
A flow chart depicting simulation is given here.
(b) It is evident from the given summary that the stockout cost is relatively very high in comparison to the carrying and the ordering costs. This indicates the need to adjust the recorder level and order quantity to reduce the number of stockouts. This will obviously raise the carrying cost and ordering cost. Further simulations are needed to determine the optimal levels of these two parameters. More simula-

tions will be required if the lead time is allowed to vary. The lead time has been assumed to be fixed in the above analysis.



Flowchart Showing Inventory Simulation

20. (a) **Simulation Worksheet**

Run	*Cost*		*Sales Revenue*		*Net Revenue*
	R. No.	*Amount (Rs)*	*R. No.*	*Amount (Rs)*	*(Rs)*
1	82	21,000	39	21,000	0
2	84	21,000	72	22,000	1,000
3	28	19,000	38	21,000	2,000
4	82	21,000	29	21,000	0
5	36	19,000	71	22,000	3,000
6	92	21,000	83	23,000	2,000
7	73	20,000	19	20,000	0
8	91	21,000	72	22,000	1,000
9	63	20,000	92	23,000	3,000
10	29	19,000	59	22,000	3,000
11	27	19,000	49	22,000	3,000
12	26	19,000	39	21,000	2,000
13	92	21,000	72	22,000	1,000
14	63	20,000	94	23,000	3,000
15	83	21,000	01	19,000	(2,000)
16	03	17,000	92	23,000	6,000
17	10	18,000	72	22,000	4,000
18	39	19,000	18	20,000	1,000
19	10	18,000	09	19,000	1,000
20	10	18,000	00	19,000	1,000

Probability distribution:

Net Reveneue (Rs)	*Prob.*
(2,000)	0.05
(1,000)	0.00
0	0.15
1,000	0.30
2,000	0.15
3,000	0.25
4,000	0.05
5,000	0.00
6,000	0.05

(b) **Simulation Worksheet**

Run	*Cost*		*Sales Revenue*		*Net Revenue*
	R. No.	*Amount (Rs)*	*R. No.*	*Amount (Rs)*	*(Rs)*
1	20	19,000	23	21,000	2,000
2	63	20,000	57	22,000	2,000
3	46	19,000	99	24,000	5,000
4	16	18,000	84	23,000	5,000

(Contd.)

(*Contd.*)

5	45	19,000	51	22,000	3,000
6	41	19,000	29	21,000	2,000
7	44	19,000	41	22,000	3,000
8	66	20,000	11	20,000	0
9	87	21,000	66	22,000	1,000
10	26	19,000	30	21,000	2,000
11	78	20,000	41	22,000	2,000
12	40	19,000	80	23,000	4,000
13	29	19,000	62	22,000	3,000
14	92	21,000	74	22,000	1,000
15	21	19,000	64	22,000	3,000
16	36	19,000	26	21,000	2,000
17	57	19,000	41	22,000	3,000
18	03	17,000	40	22,000	5,000
19	28	19,000	97	24,000	5,000
20	08	17,000	15	20,000	3,000

Probability distribution:

Net Revenue (Rs)	*Prob.*
0	0.05
1,000	0.10
2,000	0.30
3,000	0.30
4,000	0.05
5,000	0.20

21. As a first step, we assign random number intervals for each of the three variables.

Assignment of Random Numbers

Selling Price (Rs)	*Prob.*	*R. No.*	*Variable Cost (Rs)*	*Prob.*	*R. No.*	*Sales (Units)*	*Prob.*	*R. No.*
3	0.20	00–19	1	0.30	00–29	2,000	0.30	00–29
4	0.50	20–69	2	0.60	30–89	3,000	0.30	30–59
5	0.30	70–99	3	0.10	90–99	5,000	0.40	60–99

Using the given random numbers, we simulate the output of 10 trials to obtain the average profit for the project. The selling price, variable cost, and sales are obtained as a first step. This is given in the following table. The profit is calculated as follows:

Profit = (Selling price – Variable cost) × Sales volume – Fixed cost

Estimation of Selling Price, Variable Cost and Sales

S. No.	*R. No.*	*Selling Price (Rs)*	*R. No.*	*Variable Cost (Rs)*	*R. No.*	*Sales ('000 units)*
1	81	5	32	2	60	5
2	04	3	46	2	31	3
3	67	4	25	1	24	2
4	10	3	40	2	02	2
5	39	4	68	2	08	2
6	59	4	66	2	90	5
7	12	3	64	2	79	5
8	31	4	86	2	68	5
9	82	5	89	2	25	2
10	11	3	98	3	16	2

Simulated profit for the ten trials is as follows:

S. No.	*Profit/Loss*
1	Rs (5 – 2) × 5,000 – Rs 4,000 = Rs 11,000
2	Rs (3 – 2) × 3,000 – Rs 4,000 = (Rs 1,000)
3	Rs (4 – 1) × 2,000 – Rs 4,000 = Rs 2,000
4	Rs (3 – 2) × 2,000 – Rs 4,000 = (Rs 2,000)
5	Rs (4 – 2) × 2,000 – Rs 4,000 = 0
6	Rs (4 – 2) × 5,000 – Rs 4,000 = Rs 6,000
7	Rs (3 – 2) × 5,000 – Rs 4,000 = Rs 1,000
8	Rs (4 – 2) × 5,000 – Rs 4,000 = Rs 6,000
9	Rs (5 – 2) × 2,000 – Rs 4,000 = Rs 2,000
10	Rs (3 – 2) × 2,000 – Rs 4,000 = (Rs 4,000)
	Total Rs 21,000

Thus, average profit per trial = Rs 21,000/10 = Rs 2,100.

22. To begin with, we allocate random numbers 00–99 to each of the variables given, in proportion to the probabilities of various categories of each one.

Determination of Random Number Intervals

Cost (Rs)	*Prob.*	*R. No.*	*Life (Years)*	*Prob.*	*R. No.*	*Annual Cash Flow*	*Prob.*	*R. No.*
60,000	0.30	00–29	5	0.40	00–39	10,000	0.10	00–09
70,000	0.60	30–89	6	0.40	40–79	15,000	0.30	10–39
90,000	0.10	90–99	7	0.20	80–99	20,000	0.40	40–79
						25,000	0.20	80–99

Using the given random numbers, five simulation runs are performed and the results are given in the simulation worksheet.

Simulation Worksheet (Cost, Life, and Cash Flows)

Run	*R. No.*	*Cost (Rs)*	*R. No*	*Life (Years)*	*R. No.*	*Annual Cash Flows (Rs)*
1	09	60,000	24	5	07	10,000
2	84	70,000	38	5	48	20,000
3	41	70,000	73	6	57	20,000
4	92	90,000	07	5	64	20,000
5	65	70,000	04	5	72	20,000

We may now calculate NPV for each of the runs using discount rate of six per cent assuming that the required rate of return is six percent for the risk-free investment projects of the company. For this, we have
Present value of annuity with n = 5 and r = 6%: 4.212, and
Present value of annuity with n = 6 and r = 6%: 4.917
Accordingly,

Run 1: *NPV* = 10,000 × 4.212 – 60,000 = (Rs 17,880)
Run 2: *NPV* = 20,000 × 4.212 – 70,000 = Rs 14,240
Run 3: *NPV* = 20,000 × 4.917 – 70,000 = Rs 28,340
Run 4: *NPV* = 20,000 × 4.212 – 90,000 = (Rs 5,760)
Run 5: *NPV* = 20,000 × 4.212 – 70,000 = Rs 14,240

Payback Period

Run 1: Inflows @ Rs 10,000 p.a. and outflow Rs 70,000, payback = 6.0 years
Run 2: Inflows @ Rs 20,000 p.a. and outflow Rs 70,000, payback = 3.5 years
Run 3: Inflows @ Rs 20,000 p.a. and outflow Rs 70,000, payback = 3.5 years
Run 4: Inflows @ Rs 20,000 p.a. and outflow Rs 90,000, payback = 4.5 years
Run 5: Inflows @ Rs 20,000 p.a. and outflow Rs 70,000, payback = 3.5 years

CHAPTER 18

1. *Buyer A:*
   PV factor for $i = 8\%$ and $n = 8$ is equal to 0.5403 (Table B3). Thus, present value of the payment
   = 50,000 × 0.5403
   = Rs 27,015
   Buyer B:
   PV factor for $i = 8\%$ and $n = 6$ is equal to 0.6302 (Table B3). Thus, present value of the payments
   = 14,000 + 25,000 × 0.6302
   = Rs 29,755
   Buyer C:
   Present value of the payment = Rs 29,000
   Hence, ignoring risk, if any, the best offer to the company is from Buyer *B*, the second buyer.

2. $$M = \frac{120\,(1.05^{20} - 1)}{(1.05 - 1)}$$
   = 120 × 33.06595 = Rs 3,967.90

3. $$3{,}00{,}000 = \frac{A(1 - 1.08^{-10})}{1.08 - 1}$$
   $$\therefore \quad A = \frac{300{,}000}{6.7100814} = \text{Rs } 44{,}708.85$$

4. Here, Total Cost = Installation Cost + Electricity Charges
   TC (heater) = Rs 160 + Rs 200 × 5 = Rs 1,160
   TC (gas boiler) = Rs 760 + Rs 80 × 5 = Rs 1,160
   On the basis of total cost, either the two be chosen. We may now calculate and compare the present value for each of the alternatives.

Calculation of Present Value

Year	*PV Factor @9%*	*Electric Immersion Heater*		*Gas Boiler*	
		Operating Cost	*Present Value*	*Operating Cost*	*Present Value*
0	1.0000	160	160.00	760	760.00
1	0.9174	200	183.48	80	73.39
2	0.8417	200	168.34	80	67.33
3	0.7722	200	154.44	80	61.78
4	0.7484	200	141.68	80	56.67
5	0.6499	200	129.98	80	51.99
		Total	937.92		1,071.16

On the basis of present value calculations for a five year base, the housewife is advised to buy electric immersion heater.

When the gadgets are to be compared for an eight-year life:

TC (heater) = Rs 160 + Rs 200 × 8 = Rs 1,760
TC (gas boiler) = Rs 760 + Rs 80 × 8 = Rs 1,400

On the basis of total expenditure, gas boiler is a better choice. Now, we may compute the present value of the total expenditure. Since the present value calculations for the first five years are already available, we calculate the values for the remaining three years. This is shown here.

Calculation of Present Values

Year	*PV Factor @9%*	*Electric Immersion Heater*		*Gas Boiler*	
		Operating Cost	*Present Value*	*Operating Cost*	*Present Value*
6	0.5963	200	119.26	80	47.70
7	0.5470	200	109.40	80	43.76
8	0.5019	200	100.38	80	40.15
		Total	329.04		131.61

Present value (immersion heater) = Rs 937.92 + Rs 329.04
= Rs 1,266.96
Present value (gas boiler) = Rs 1,071.16 + Rs 131.61
= Rs 1,202.77

Thus, over an eight-year period, the present value of gas boiler is less. The housewife is, accordingly, advised to buy the gas boiler.

5. Present value of annuity,

$$V = \frac{500(1-1.01^{-30})}{1.01-1}$$

$$= 500 \times 25.8077082 = 12,904$$

∴ Cost of TV set = 2,000 + 12,904 = Rs 14,904

6. (a) Value offerred = Rs 5,00,000
   (b) Present value of the offer,

$$V = (1.08)^{-2}\left[1,24,000\left(\frac{1-1.08^{-5})}{1.08-1}\right)\right]$$

$$= (0.85734)(1,24,000)\ (3.99271)$$
$$= \text{Rs } 4,24,465$$

Thus, offer (a) is more attractive.

7. Here,

$$75,000 = 3,750\left[\frac{1-1.035^{-n}}{1.035-1}\right]$$

or $$1.035^{-n} = \frac{75,000 \times 0.035}{3,750} = 0.3$$

∴ $$n = -(\log 0.3/\log 1.035) = \frac{0.5229}{0.0149} = 35$$

8. Here M = Rs 12,00,000, i = 8% (so $R = 1 + 0.08$), and $n = 8$. Now,

$$A = \frac{M(R-1)}{R^n - 1}$$

$$= \frac{12,00,000(1.08-1)}{1.08^8 - 1}$$

$$= \text{Rs } 1,12,817.74$$

Thus, Rs 1,12,817.74 per annum should be paid in the fund.

9. Rent = 30,00,000 × 0.10 = Rs 3,00,000 p.a.

10. (a) Here $k = 20$. Thus, $i = 1/k = 1/20$ or 0.05, and $R = 1 + i = 1.05$. With $A = 1{,}000$, $R = 1.05$ and $n = 2$, we have

$$M = \frac{M(R^n - 1)}{R - 1}$$

$$= \frac{1{,}000(1.05^2 - 1)}{1.05 - 1}$$

$$= \frac{102.5}{0.05} \text{ or Rs } 2{,}050$$

(b) Given

$$20 = \frac{1 - R^{-k}}{R - 1} \quad \text{(i)} \qquad \text{and} \qquad 25 = \frac{1 - R^{-2k}}{R - 1} \quad \text{(ii)}$$

Taking the ratio of these two,

$$\frac{20}{25} = \frac{1 - R^{-k}}{R - 1} \Big/ \frac{1 - R^{-2k}}{R - 1}$$

On simplification,

$$\frac{4}{5} = \frac{1 - R^{-k}}{1 - R^{-2k}}$$

or $4R^{-2k} - 5R^{-k} + 1 = 0$

or $4R^{-2k} - 4R^{-k} - R^{-k} + 1 = 0$

or $(4R^{-k} - 1)(R^{-k} - 1) = 0$

Thus, either $4R^{-k} - 1 = 0$, i.e. $R^{-k} = 1/4$, or
$R^{-k} - 1 = 0$, i.e. $R^{-k} = 1$
$R^{-k} = 1$ is not possible since $R > 1$.

Substituting $R^{-k} = 1/4$ in the equation (i), we get

$$20 = \frac{1 - \frac{1}{4}}{R - 1}$$

or $R - 1 = i = \dfrac{3}{4 \times 20}$

$= 0.0375$ or 3.75%

(c) The information can be presented as:

$$V = \frac{100}{(1 + 0.05)^1} + \frac{200}{(1 + 0.05)^2} + \frac{300}{(1 + 0.05)^3} + \frac{400}{(1 + 0.05)^4} + \ldots$$

The series on the RHS is an arithmetic-geometric series. We can rewrite it as follows:

$$\frac{V}{100} = \frac{1}{(1.05)^1} + \frac{1}{(1.05)^2} + \frac{1}{(1.05)^3} + \frac{1}{(1.05)^4} + \ldots$$

If we let $(1.05)^{-1} = x$, and denote the sum of the series by S, we get

$S = x + 2x^2 + 3x^3 + 4x^4 + \ldots$, and

$Sx = x^2 + 2x^3 + 3x^4 + \ldots$

By subtraction of the second equation from the first,

$S - Sx = x + x^2 + x^3 + x^4 + \ldots$

or $S(1 - x) = x(1 + x + x^2 + x^3 + \ldots)$

or $$S(1-x) = \frac{x}{1-x}$$

$$\therefore \quad S = \frac{x}{(1-x)^2}$$

Since we have put $S = V/100$, we have

$$V = \frac{100x}{(1-x)^2}$$

Substituting the value of x, we get

$$V = \frac{100(1.05)^{-1}}{[1-(1.05)^{-1}]^2}$$

$$= \frac{100 \times 1.05}{0.05^2}$$

$$= \text{Rs } 42{,}000$$

11. Amount required to pay Biren,

$$S = 6{,}000\left(\frac{1-1.10^{-10}}{1.10-1}\right)$$

$$= 6{,}000 \times 6.144567 = \text{Rs } 36{,}868$$

Let annual payment to pension Fund Trust be x.
With $n = 5$ and $i = 0.10$, we have

$$36{,}868 = \frac{x(1.10^5-1)}{(1.10-1)}$$

$$\therefore \quad x = \frac{36{,}868}{6.1051} = \text{Rs } 6{,}039$$

12. To choose between the two alternatives, we shall compare their present values. For the proposal of renting a building,

Net rental payable p.a., $A = 2{,}00{,}000 - 24{,}000 = \text{Rs } 1{,}76{,}000$
No. of years, $n = 20$
Rate of interest, $i = 8\%$
Price of building after 20 years = Rs 5,00,000

Total present value = PV of annuities + PV of building price

PV factor for annuities, for $i = 8\%$ and $n = 20$, is 9.8181 (Table B4), and PV factor for a rupee due after 20 years @ 8% = 0.2145 (Table B3). Thus,

Total present value = 1,76,000 × 9.8181 + 5,00,000 × 0.2145
= 17,27,985.6 + 1,07,250
= Rs 18,35,235.6

Since this value is less than Rs 20 lakh, the cost of building own premises, the company will do better to rent the facility.

13. (a) PV factor for an annuity for 7 years @ 15% = 4.1604
$\therefore$ NPV of the machine = 18,000 × 4.1604 – 72,000 = Rs 2,887
(b) NPV @ 16% = 4.0386 × 18,000 – 72,000 = Rs 694.8
NPV @ 17% = 3.9224 × 18,000 – 72,000 = (Rs 1397.2)
By interpolation, IRR $\simeq$ 16.2%
(c) Acceptable if the required rate of return is 16% or less.

14. (a) **Calculation of Net Present Value**

Year	*Cash inflow ('000 Rs)*	*PVF @ 14%*	*Present Value*
1	160	0.8772	140.35
2	190	0.7695	146.20
3	170	0.6750	114.75
4	150	0.5921	88.81
5	150	0.5194	77.91
6	150	0.4556	68.34
7	150	0.3996	59.95
8	200	0.3506	70.11
		PV of inflows	= 766.41

NPV = 766.41 – 1,000 = (Rs 233.59 thousand)

(b) IRR of the project works out to be 6.6% app.
The proposal is not an acceptable one.

15. *Proposal (a)*
Cash outlay = Rs 25,000
Savings @ Rs 8,000 p.a. for 6 years
NPV = 8,000 × 3.6847 – 25,000
= Rs 4,478

Proposal (b)
Cash outlay = Rs 70,000
Savings @ Rs 22,000 p.a. for 6 years
NPV = 70,000 × 3.6847 –70,000
= Rs 11.063

Proposal (b) is preferable although both have positive *NPV*s.

16. $\frac{\text{Investment}}{\text{Annual Cash Saving}} = \frac{50,000}{10,500} = 4.7619$

From the annuity PVF table (for n = 7),
PVF @ 10% = 4.8684 and PVF @ 11% = 4.7122
By interpolation, IRR = 10.7%
The company should not buy the machine since IRR < 12%.

17. *Project X:*

PVF for n = 10 @ 5% = 7.7217

∴ *NPV* = 30,000 × 7.7217 – 2,00,000 = Rs 31,651

Project Y:

PVF for n = 20 @ 5% = 12.4622

∴ *NPV* = 20,000 × 12.4622 – 2,00,000 = Rs 49.244

Since projects are mutually exclusive with different lives, we should compute and compare equivalent annuity for both projects at the require rate of return, 5%.
Equivalent (annual) annuity for *X*: 31,651/7.7217 = Rs 4,099
Equivalent (annual) annuity for *Y*: 49,244/12.4622 = Rs 3,951
Project *X* should be preferred.

18. The after-tax cash flows are used for each of the years in respect of both the projects, to calculate *NPV* and *IRR* values. To illustrate, for year 1 in case of project *A*:

Cash flow before depreciation and taxation	= 7,00,000
– Depreciation	= 2,00,000
	5,00,000
– Taxation @ 50%	= 2,50,000
	= 2,50,000

+ Depreciation	= 2,00,000
After-tax cash inflow	= 4,50,000

Project A

Year	*Cash flow*	*PVF*	*Present Value*
1	4,50,000	0.8696	3,91,320
2	5,00,000	0.7561	3,78,050
3	5,00,000	0.6575	3,28,750
4	5,50,000	0.5718	3,14,490
5	4,00,000	0.4972	1,98,880
			16,11,490
		less Outflow	10,00,000
		NPV =	6,11,490

Project B

Year	*Cash flow*	*PVF*	*Present Value*
1	5,00,000	0.8696	4,34,800
2	4,00,000	0.7561	3,02,440
3	6,00,000	0.6575	3,94,500
4	4,50,000	0.5718	2,57,310
5	4,00,000	0.4972	1,98,880
			15,87,930
		less Outflow	10,00,000
		NPV =	5,87,930

IRR: Project *A* 38.6%, Project *B* 38.3%
Project *A* is better of the two.

19. The calculation of net present value is given in the following table. The given cash flows are converted into their equivalents by multiplying them by their respective certainty-equivalent coefficients.

Calculation of Net Present Value

Year t	*Cash flow* *('000 Rs),* C_t	*C.E. Coeff.* α_t	$C_t\alpha_t$	*PVF* $(1.12)^{-1}$	*Present Value ('000 Rs)*
1	18	0.95	17.10	0.8929	15.26859
2	20	0.90	18.00	0.7972	14.34960
3	21	0.85	17.85	0.7118	12.70563
4	22	0.85	18.70	0.6355	11.88385
5	12	0.70	8.40	0.5674	4.76616
				Total	58.97383
				Less Cash outlay	64.00000
				NPV	(5.02617)

Thus, the project has an *NPV* = –Rs 5,026.17.

20. (a) *Alternative 1:*

Present value of Rs 2,50,000 (= Rs 3,00,000 – 50,000) received annually for five years @ 20% p.a.:

2,50,000 × 2.9906 (Table B4) = Rs 7,47,650

less Present value of outflow of Rs 1,00,000 at the end of five years @ 20% p.a.

1,00,000 × 0.4019 (Table B3) = Rs 40,190

Present value of cash inflows	Rs 7,07,460
less Initial outflows	Rs 5,00,000
Net Present value	Rs 2,07,460

Alternative 2:
Present value of Rs 1,00,000 (Rs 1,50,000 – 50,000)
Received annually for five years @ 20% p.a.

1,00,000 × 2.9906	= Rs 2,99,060
less Initial outflow	= Rs 2,50,000
Net Present Value	Rs 49,060

Since the *NPV* of alternative 1 is much higher than that of alternative 2, the management should adopt the first method of promotion.

Note: Allocation of fixed cost to the extent of Rs 20,000 per annum is not to be taken into account for computing the cash flows.

(b) For obtaining the *IRR* for alternative 2, we determine the ratio of the cost of the project to the annual net cash inflows, as Rs 2,50,000/Rs 1,00,000 = 2.5.

From the Table B4, we observe that corresponding to the period of five years, the *PV* values closest to 2.5 are those corresponding to i = 28% (2.5320) and i = 32% (2.3452).

PV of cash flows @ 28% = 1,00,000 = Rs 2,53,200
PV of cash flows @ 32% = 1,00,000 = Rs 2,34,520
Difference 18,680

The *IRR* can be interpolated as follows:

$$IRR = 28 + \frac{2{,}53{,}200 - 2{,}50{,}000}{18{,}680}\ (32 - 28)$$

$$= 28 + 0.68 = 28.68\%$$

21. **Calculation of Expected Values and Variances**

Year	*Cashflow (X)*	*Prob. (p)*	*pX*	$p(X - \bar{X})^2$
1	11,000	0.3	3,300	8,67,000
	12,000	0.1	1,200	49,000
	13,000	0.2	2,600	18,000
	14,000	0.4	5,600	6,76,000
			12,700	16,10,000
2	11,000	0.4	4,400	4,84,000
	12,000	0.2	2,400	2,000
	13,000	0.3	3,900	2,43,000
	14,000	0.1	1,400	3,61,000
			12,100	10,90,000
3	11,000	0.2	2,200	3,92,000
	12,000	0.3	3,600	48,000
	13,000	0.4	5,200	1,44,000
	14,000	0.1	1,400	2,56,000
			12,400	8,40,000

Calculation of Expected NPV and Standard Deviation

Year (n)	*Expected Cash flow*	*PVF $(1.10)^{-n}$*	*Present Value*	*Variance σ^2*	*PVF $(1.10)^{-2n}$*	*Present Value*
1	12,700	0.9091	11,545.45	16,10.000	0.8264	13,30,578.5
2	12,100	0.8264	10,000.00	10,90,000	0.6830	7,44,484.7
3	12,400	0.7513	9,316.30	8,40,000	0.5645	4,74,158.1
			30,861.75			σ^2 = 25,49,221.3
		less outlow	25,000.00			
		NPV =	5,861.75			

Expeced *NPV* = Rs 5,862, Standard deviation = $\sqrt{25{,}49{,}221.3}$

= Rs 1,597

22. (a) Expected cash flow (ECF) (in Rupees)

P_1: Year 1 ECF = 45,000 × 0.35 + 50,000 × 0.40 + 60,000 × 0.25 = 50,750
Year 2 ECF = 50,000 × 0.25 + 60,000 × 0.50 + 70,000 × 0.25 = 60,000

P_2: Year 1 ECF = (2,000) × 0.20 + 40,000 × 0.30 + 50,000 × 0.30 + 72,000 × 0.20 = 41,000
Year 2 ECF = 10,000 × 0.10 + 45,000 × 0.30 + 65,000 × 0.35 + 80,000 × 0.25 = 57,250

(b) From the cash flow probability distributions given, it is evident that there is much greater variability in case of project P_2 than in P_1. This would be reflected in their variances as well.

NPV for P_1 (using $r = 12\%$)
= 50,750 × 0.8929 + 60,000 × 0.7972 – 80,000 = Rs 13,147

NPV for P_2 (using $r = 14\%$)
= 41,100 × 0.8772 + 57,250 × 0.7695 – 80,000 = Rs 19

(c) IRR: for P_1 23.9%, for P_2 14.0%

23. **Calculation of Expected Values and Variances**

Year	*Cash flow (X)*	*Prob. (p)*	*pX*	$p(X - \bar{X})^2$
1	1,000	0.10	100	1,00,000
	1,500	0.20	300	50,000
	2,000	0.40	800	0
	2,500	0.20	500	50,000
	3,000	0.10	300	1,00,000
			2,000	3,00,000
2	1,900	0.20	380	1,05,125.0
	2,500	0.30	750	4,687.5
	2,750	0.20	550	3,125.0
	3,150	0.30	945	82,687.5
			2,645	1,95,625.0
3.	1,500	0.10	150	60,062.5
	2,250	0.70	1,575	437.5
	2,500	0.10	250	5,062.5
	3,000	0.10	300	52,562.5
			2,275	1,18,125.0

Calculation of Expected DCF and Standard Deviation

Year (n)	*Expected Cash flow*	*PVF $(1.10)^{-n}$*	*Present Value*	*Variance σ^2*	*PVF $(1.10)^{-2n}$*	*Present Value*
1	2,000	0.9091	1,818.18	3,00,000	0.8264	2,47,933.9
2	2,645	0.8264	2,169,42	1,95,625	0.6830	1,33,614.5
3	2,275	0.7513	1,709.24	1,18,125	0.5645	66,678.5
			DCF = 5,696.84			σ^2 = 4,48,226.9

Thus, DCF = Rs 5,696.84 and $\sigma = \sqrt{4,48,226.9}$ = Rs 669.5.

24. For each of the three alternatives, the expected value and standard deviation are shown calculated in table here.

Calculation of Expected Value and Standard Deviation

Outcome (X)	*Probability (p)*	*pX*	*$p(X - \bar{X})^2$*
Alternative A_1			
125	0.2	25	2,000
200	0.4	80	250
300	0.4	120	2,250
	Total	225	4,500
Alternative A_2			
225	0.3	67.5	6,091.875
400	0.5	200.0	14,028.125
500	0.2	100.0	3,511.250
	Total	367.5	23,631.125
Alternative A_3			
200	0.4	80	21,160
500	0.5	250	2,450
1,000	0.1	100	32,490
	Total	430	56,100

Thus, expected value for A_1 = Rs 225, for A_2 = Rs 367.5 and for A_3 = Rs 430.

Standard deviation for $A_1 = \sqrt{4,500}$ = Rs 67.08

for $A_2 = \sqrt{23,631.125}$ = Rs 153.725

for $A_3 = \sqrt{56,100}$ = Rs 236.854

$$\text{Coefficient of variation} = \frac{\sigma}{\text{Mean}} \times 100$$

for $A_1 = \dfrac{67.08}{225} \times 100$ = 29.81%

for $A_2 = \dfrac{153.725}{367.5} \times 100$ = 41.83%

for $A_3 = \dfrac{236.854}{430} \times 100$ = 55.08%

On the basis of the values computed, rankings are done here:
(i) A_3, A_2, A_1; (ii) A_1, A_2, A_3.
Note: (Mean and standard deviation values are in thousands of rupees).

25. *Project 1*

Calculation of Expected Cash Flow and Variance

Cash flow (X)	*Prob. (p)*	*pX*	$p(X-\bar{X})^2$
20,000	0.1	2,000	160×10^6
30,000	0.2	6,000	180×10^6
60,000	0.4	24,000	0
90,000	0.2	18,000	180×10^6
100,000	0.1	10,000	160×10^6
		60,000	680×10^6

Expected NPV = 60,000 × 4.3553 (PVF for annuity n = 6 @ 10%) – 2,00,000
= Rs 61,318
Variance = $680 \times 10^6[0.8264 + 0.6830 + 0.5645 + 0.4665 + 0.3855 + 0.3186]$
= $680 \times 10^6(3.2445)$
= 2206.26×10^6
Standard deviation = 46.971×10^3 = Rs 46,971

Project 2

Calculation of Expected Cash Flow and Variance

Cash flow (X)	*Prob. (p)*	*pX*	$p(X-\bar{X})^2$
30,000	0.20	6,000	156.8×10^6
50,000	0.30	15,000	19.2×10^6
70,000	0.40	28,000	57.6×10^6
90,000	0.10	9,000	102.4×10^6
		58,000	336×10^6

Expected NPV = 58,000 × 4.3553 – 2,00,000 = Rs 52,607
Variance = $336 \times 10^6 \times 3.2445$
= $1,090.152 \times 10^6$
Standard deviation = 33.017×10^3 = 33,017

26. (a) Calculation of expected return and variance is given in table here.

Calculation of Expected Return and Variance

Cash flow X	*Prob. p*	*pX*	$p(X-\bar{X})^2$
Year 1			
50,000	0.2	10,000	156.8×10^6
80,000	0.7	56,000	2.8×10^6
1,20,000	0.1	12,000	176.4×10^6
	Total	78,000	336×10^6

(*Contd.*)

(*Contd.*)

Year 2			
70,000	0.2	14,000	217.8×10^6
1,00,000	0.5	50,000	4.5×10^6
1,30,000	0.3	39,000	218.7×10^6
	Total	1,03,000	441×10^6
Year 3			
1,40,000	0.3	42,000	43.2×10^6
1,50,000	0.3	45,000	1.2×10^6
1,62,500	0.4	65,000	44.1×10^6
	Total	1,52,000	88.5×10^6

Thus, we have

$$\mu_1 = \text{Rs } 78{,}000,\ \mu_2 = \text{Rs } 1{,}03{,}000, \text{ and } \mu_3 = \text{Rs } 1{,}52{,}000;$$

$$\sigma_1 = \sqrt{336 \times 10^6} = \text{Rs } 18{,}330.3,\ \sigma_2 = \sqrt{441 \times 10^6} = \text{Rs } 21{,}000, \text{ and}$$

$$\sigma_3 = \sqrt{88.5 \times 10^6} = \text{Rs } 9{,}407.4.$$

(b) The expected *NPV* is Rs 39,951.4 as shown below.

Calculation of Expected *NPV*

Year t	*Expected Value* m_t	*PVF,* $(1.12)^{-t}$	*Present Value*
1	78,000	0.8929	69,646.2
2	1,03,000	0.7972	82,111.6
3	1,52,000	0.7118	1,08,193.6
			2,59,951.4
		less Cash outflow	2,20,000.0
		Expected *NPV*	39,951.4

(c) The calculation of variance is given here:

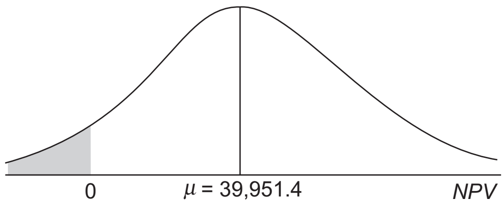
Calculation of Variance

Year t	*Variance* σ_t^2	*PVF,* $(1.12)^{-2t}$	*Present Value*
1	336×10^6	0.7972	267.8592×10^6
2	441×10^6	0.6355	280.2555×10^6
3	88.5×10^6	0.5066	44.8341×10^6
		Variance =	592.9488×10^6

Thus, Standard deviation, $\sigma = \sqrt{592.9488 \times 10^6}$

$= \text{Rs } 24{,}350.5$

(d) To determine the required probability, we shall find area to the left of $X = 0$ under the normal curve with $\mu = 39{,}951.4$ and $\sigma = 24{,}350.5$.



Determination of Probability

For $X = 0$, we have

$$Z = \frac{0 - 39{,}951.4}{24{,}350.5} = -1.64$$

From Table B1, area between μ and $Z = -1.64$ is 0.4495.

$\therefore$ $P(X < 0) = 0.5 - 0.4495 = 0.0505$

(e) From Table B2, $L_N(1.64) = 0.02114$. Thus,

$$\begin{aligned} EVPI &= \sigma \times L_N(1.64) \\ &= 24{,}350.5 \times 0.02114 \\ &= \text{Rs } 514.8 \end{aligned}$$

27. **Calculation of Expected Return and Variance**

Year 1				*Year 2*				*Year 3*			
X	p	pX	$p(X - \bar{X})^2$	X	p	pX	$p(X - \bar{X})^2$	X	p	pX	$p(X - \bar{X})^2$
50	0.10	5	40	20	0.10	2	160	–40	0.1	–4	810
60	0.20	12	20	40	0.25	10	100	30	0.3	9	120
70	0.40	28	0	60	0.30	18	0	50	0.3	15	0
80	0.20	16	20	80	0.25	20	100	80	0.2	16	180
90	0.10	9	40	100	0.10	10	160	140	0.1	14	810
Total		70	120			60	520			50	1,920

Thus, $\mu_1 = 70,\ \sigma_1^2 = 120$ and $\sigma_1 = \sqrt{120} = 10.95;$

$\mu_2 = 60,\ \sigma_2^2 = 520$ and $\sigma_2 = \sqrt{520} = 22.80;$ and

$\mu_3 = 50,\ \sigma_3^2 = 1{,}920$ and $\sigma_3 = \sqrt{1{,}920} = 43.82.$

The present values are shown calculated in table below.

From the calculations, it is evident that expected value of the project = 150.786 and Standard deviation = $\sqrt{\text{Variance}} = \sqrt{1{,}538.168} = 39.22$. It may be noted that cash flows are assumed to be independent.

Calculation of Present Values

Year (t)	*Expected value* μ_t	*PVF @ 10%*	*Present Value* $(1 + 0.10)^{-t}$	*Variance* σ_1^2	*PVF @ 10%* $(1 + 0.10)^{-2t}$	*Present Value*
1	70	0.9091	63.637	120	0.8264	99.168
2	60	0.8264	49.584	520	0.6830	355.160
3	50	0.7513	37.565	1,920	0.5645	1,083.840
		EMV	150.786		Variance	1,538.168

28. **Calculation of Expected Values and Variances**

Year 1 Cash Flow Prob.				Year 2 Cash Flow Prob.				Year 3 Cash Flow Prob.			
(X)	*(p)*	*pX*	*$p(X-\bar{X})^2$*	*(X)*	*(p)*	*pX*	*$p(X-\bar{X})^2$*	*(X)*	*(p)*	*pX*	*$p(X-\bar{X})^2$*
100	0.10	10	4,000	300	0.2	60	3,380	500	0.1	50	5,760
200	0.20	40	2,000	400	0.4	160	360	600	0.3	180	5,880
300	0.30	90	0	500	0.3	150	1,470	700	0.3	210	480
400	0.40	160	4,000	600	0.1	60	2,890	1,000	0.3	300	20,280
	Total	300	10,000			430	8,100			740	32,400

Thus, we have

$$\mu_1 = 300,\ \sigma_1 = \sqrt{10{,}000} = 100;\ \mu_2 = 430,\ \sigma_2 = \sqrt{8{,}100} = 90;\ \text{and}\ \mu_3 = 740,$$

$$\sigma_3 = \sqrt{32{,}400} = 180.$$

(b) The calculation of expected *NPV* and standard deviation is given here.

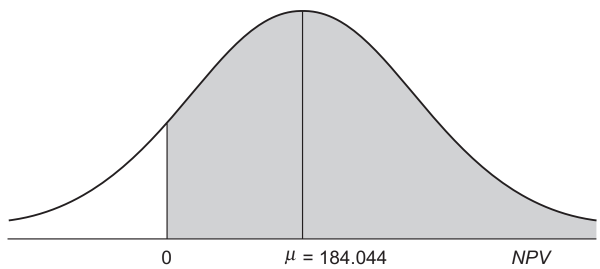
Calculation of Expected *NPV* and Standard Deviation

Year t	*PV Factor $(1 + 0.10)^{-t}$*	*Exp. Value*	*Present Value*	σ_t	*Present Value*
1	0.9091	300	272.730	100	90.910
2	0.8264	430	355.352	90	74.376
3	0.7513	740	555.962	180	135.234
		Total	1,184.044		σ = 300.520
		Less outflow	1,000.000		
		Expected *NPV*	184.044		

Thus, expected *NPV* of the project is 184.044. The standard deviation is calculated for use in part (c).

(c) To determine the required probability, we shall find the area to the right of $X = 0$ under the normal curve with $\mu = 184.044$ and $\sigma = 300.520$. This is shown in the figure. For $X = 0$,

$$Z = \frac{0 - 184.044}{300.520}$$
$$= -0.61$$



Calculation of Probability

Area corresponding to $Z = -0.61$ is 0.2291. Thus, $P(X > 0) = 0.2291 + 0.5 = 0.7291$.

(d) If cash flows are assumed to be independent, the standard deviation would be equal to 179.12, determined as follows.

$$\sigma^2 = 90.910^2 + 74.376^2 + 135.234^2 = 32,084.65$$

$$\therefore \quad \sigma = \sqrt{32,084.65} = 179.12$$

Now, to find the area under the curve to the right of $X = 0$, we have,

$$Z = \frac{0 - 184.044}{179.12}$$

$$= -1.03$$

From the normal area table (B1), area corresponding to $Z = -1.03$ is 0.3485.
Thus, $P(X > 0) = 0.3485 + 0.5 = 0.8485$.

(e) (i) When cash flows are perfectly correlated,

$L_N(0.61) = 0.1659$ (from Table B2)

Thus, $EVPI = 300.52 \times 0.1659 =$ Rs 49.86

(ii) When cash flows are independent,

$L_N(0.03) = 0.07866$ (from Table B2)

Thus, $EVPI = 179.12 \times 0.07866 =$ Rs 14.09.

29. **Calculation of Expected *NPV* and Standard Deviation**

Year (n)	*Expected Cash Flow*	*PVF $(1.10)^{-n}$*	*Present Value*	*Standard Deviation*	*Variance σ^2*	*PVF $(1.10)^{-2n}$*	*Present Value*
1	12,000	0.9091	10,909.09	5,000	25×10^6	0.8264	20.66×10^6
2	12,000	0.8264	9,917.36	4,000	16×10^6	0.6830	10.93×10^6
3	12,000	0.7513	7,513.15	5,000	25×10^6	0.5645	14.11×10^6
			28,339.60				45.70×10^6
		less outflow	20,000.00				
		Expected *NPV* =	8,339.60				

$\therefore$ Expected NPV = Rs 8,339.60 and $\sigma = \sqrt{45.70 \times 10^6}$ = Rs 6,760.27

To calculate required probability,

$$Z_1(X = 0) = \frac{0 - 8,339.60}{6,760.27} = -1.23 \quad \therefore \text{Area} = 0.3907$$

$$Z_2(X = 10,000) = \frac{10,000 - 8,339.60}{6,760.27} = 0.25 \quad \therefore \text{Area} = \underline{0.0987}$$

Probability = 0.4894

30. To calculate expected net present value, we add the mean cash flows of the two components. The values are shown in table below where present value calculations are also given.

Calculation of Expected *NPV*

Year t	*Expected Value*	*PVF, (1.15)$^{-t}$*	*Present Value*
1	Rs 82,000	0.8696	71,307.2
2	1,00,000	0.7561	75,610.0
3	98,000	0.6675	64,435.0
4	98,000	0.5718	56,036.4
5	1,07,000	0.4972	53,200.4
6	1,12,000	0.4323	48,4176
			3,69,006.6
		less Cash outflow	3,40,000.0
		Expected *NPV*	29,006.6

To calculate standard deviation for the project, we first obtain variance for each of the two components and then sum the two. From the overall variance, we get the standard deviation. The calculations are shown below in (a) and (b).

(a) Calculation of Standard Deviation (Correlated Component)

Year t	*Standard Deviation* σ_t	*PVF, (1.15)$^{-t}$*	*Present Value*
1	4,400	0.8696	3,826.24
2	4,500	0.7561	3,402.45
3	3,000	0.6575	1,972.50
4	3,200	0.5718	1,829.76
5	4,000	0.4972	1,988.80
6	4,000	0.4323	1,729.20
			σ = 14,748.95

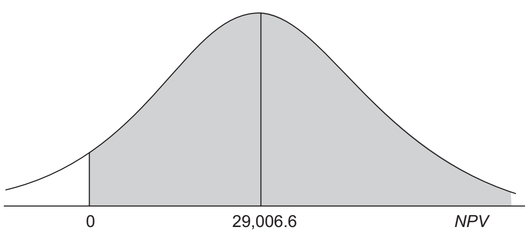
(b) Calculation of Variance (Independent Component)

Year t	*Standard Deviation* σ_t	*PVF, (1.15)$^{-t}$*	*Present Value PV*	*(PV)^2*
1	4,400	0.8696	3,478.40	1,20,99,266.56
2	4,400	0.7561	3,326.84	1,10,67,864.38
3	4,800	0.6575	3,156.00	9,960,336.00
4	4,000	0.5718	2,287.20	52,31,283.84
5	4,000	0.4972	1,988.80	39,55,325.44
6	3,600	0.4323	1,556.28	24,22,007.44
				σ = 4,47,36,083.66

Total variance = $(14{,}748.95)^2 + (4{,}47{,}36{,}083.66) = 26{,}22{,}67{,}609.7$

Standard deviation = $\sqrt{26{,}22{,}67{,}609.7}$ = 16,194.7

To calculate the probability that the project would be successful, we determine the area under the normal curve to the right of $X = 0$ (where X is the *NPV*), the parameters of the curve being μ = 29,006.6 and σ = 16,194.7.



Determination of Probability

Thus,

$$Z = \frac{0 - 29{,}006.6}{16{,}194.7} = -1.79$$

Area corresponding to $Z = 1.79$ is 0.4633.

Thus, $P(X > 0) = 0.4633 + 0.5 = 0.9633$, which is the probability that the project would be successful. Further from Table B2, $L_N(1.79) = 0.01464$. Therefore, $EVPI = \sigma \times L_N = 16{,}194.7 \times 0.01464 =$ Rs 237.09.

31. **Calculation of Expected *NPV* and Standard Deviation**

Year (n)	*Expected Cash flow*	*PVF $(0.10)^{-n}$*	*Present value*	*Standard deviation*	*Present value*
1	36,000	0.9091	32,727.27	3,200	2909.09
2	42,000	0.8264	34,710.74	3,600	2975.21
3	50,000	0.7513	37565.74	3,600	2704.73
4	45,000	0.6830	30,735.81	3,300	2253.94
			1,05,003.80		10,842.98
		less outflow	1,00,000.00		
		Expected *NPV* =	5,003.80		

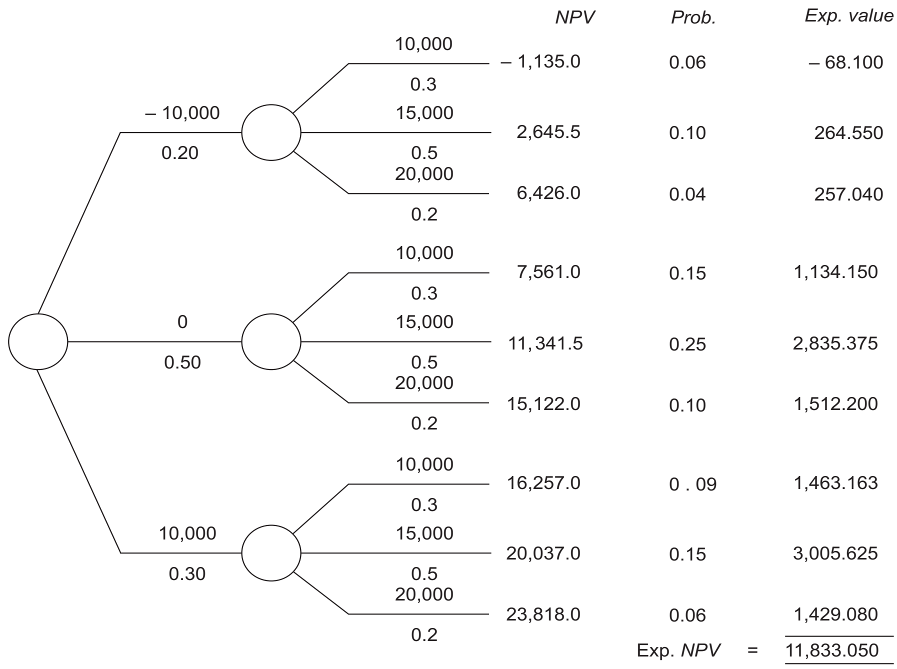
With $\mu = 5{,}003.80$ and $\sigma = 10{,}842.98$, $Z(0) = \dfrac{0 - 5003.80}{10842.98} = -0.46$

From Table B2, $L_N(0.46) = 0.2104$

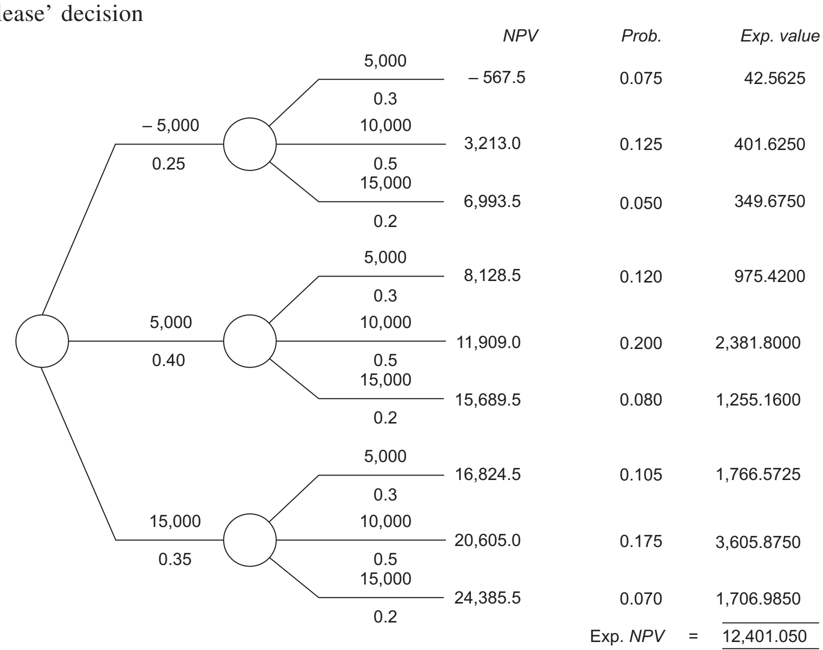
$\therefore\ EVPI = \sigma \times L_N = 10{,}842.98 \times 0.2104 =$ Rs 2281.36.

32. The *NPV* distributions for both the proposals are derived as shown in the figure below. Various NPVs are obtained using a discount-rate of 15% and their probabilities have been calculated by multiplying the probabilities on the relevant forks. The standard deviations are calculated from variances whose values are obtained in the table following.

For 'buy' decision



For 'lease' decision



Decision-tree: Calculation of *NPV*

Calculation of Variances

For 'Buy' Decision $P(X - \bar{X})^2$	*For 'Lease' Decision* $p(X - \bar{X})^2$
1,00,90,144.44	1,26,13,649.41
84,41,015.63	1,05,52,418.00
11,69,425.96	14,62,052.81
27,37,497.60	21,90,510.75
60,393.06	48,412.80
10,81,752.10	8,65,138.58
17,61,459.84	20,54,571.99
1,00,97,073.03	1,17,78,482.80
86,18,413.50	1,00,53,976.81
σ^2 = 4,40,57,172.15	5,16,19,213.94

'Buy' Decision: $\sigma = \sqrt{4{,}40{,}57{,}172.15} = 6{,}637.56$

Mean = 11,833.05

$\therefore$ Coefficient of variation = $\dfrac{\sigma}{\text{Mean}} \times 100$

$= \dfrac{6637.56}{11{,}833.05} \times 100 = 56.08\%$

'Lease' Decision: $\sigma = \sqrt{5{,}16{,}19{,}213.94} = 7{,}184.65$

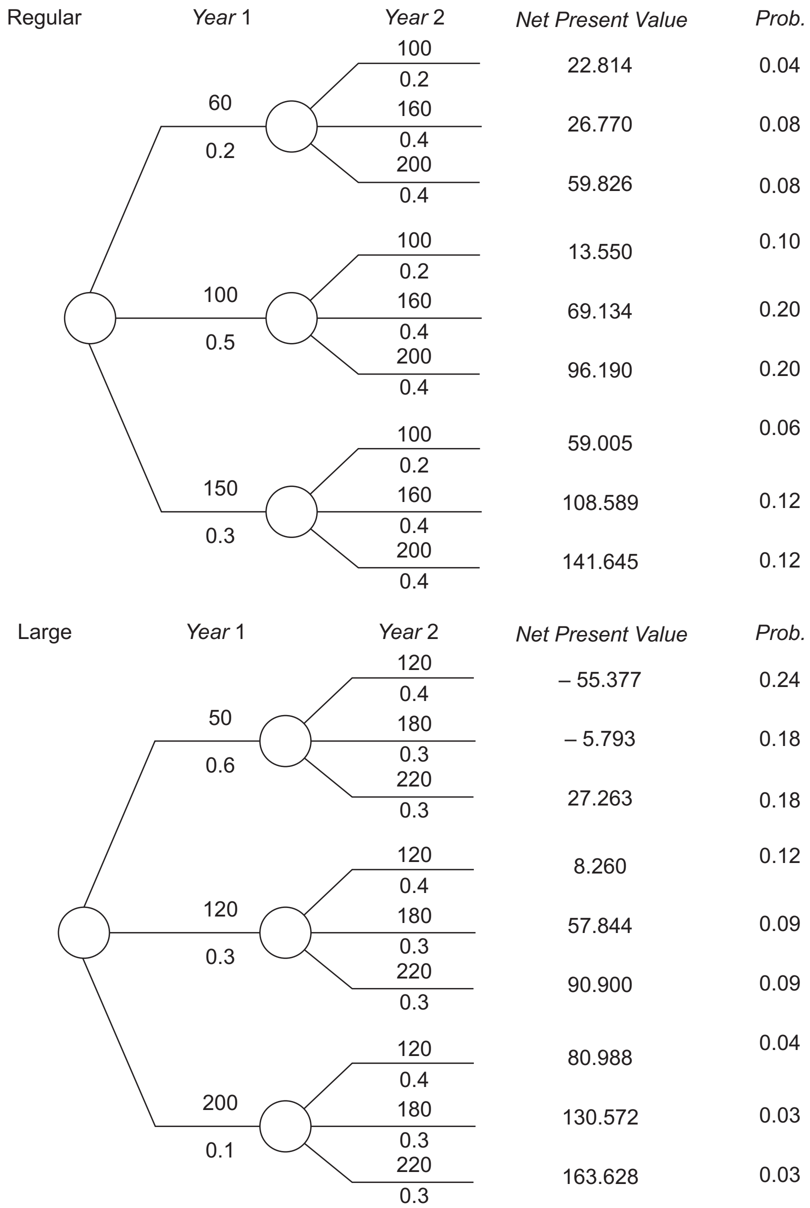
Mean = 12,401

$\therefore$ Coefficient of variation = $\dfrac{7{,}184.65}{12{,}401} \times 100 = 57.94\%$

Thus, lease may be preferred by the management because it has a higher expected *NPV*, although it is marginally riskier alternative.

33. The decision tree depicts the various possible *NPVs* for each of the alternatives. To illustrate, for the 'regular' size, when demand situation is low in each of the two years, with profits as 60 and 100 thousand rupees, the NPV is –22.814 (thousand rupees) as shown below:

$$NPV = 60 \times 0.9091 + 100 \times 0.8264 - 160$$
$$= 54.546 + 82.64 - 160 = -22.814$$



Decision Tree: Obtaining *NPVs*

The calculation of mean and standard deviation is given below in respect of regular store.

Calculation of Mean and Standard Deviation

NPV, X	*Prob. P*	*pX*	$p(X - \bar{X})^2$
–22.814	0.04	–9.91256	374.91802
26.770	0.08	2.14160	178.45383
59.826	0.08	4.78608	16.07218
13.550	0.10	1.35500	65.42025
69.134	0.20	13.82680	4.73559
96.190	0.20	19.23800	98.47922
59.005	0.06	3.54030	13.49100
108.589	0.12	13.03068	143.56786
141.645	0.12	16.99740	549.10152
	Total	74.0033	1,744.23947

The mean and standard deviation calculations for large store are given here.

Calculation of Mean and Standard Deviation

NPV, X	*Prob. P*	*pX*	$p(X - \bar{X})^2$
–55.377	0.24	–13.29048	1,257.9181
–5.793	0.18	–1.04274	93.6779
27.263	0.18	4.90734	18.8854
8.260	0.12	0.99120	9.2085
57.844	0.09	5.20596	149.9939
90.900	0.09	8.18100	491.2429
80.988	0.04	3.23952	163.6762
130.572	0.03	3.91716	386.8217
163.628	0.03	4.90884	644.8172
	Total	17.01780	3,216.2418

For regular store:

Expected value = 74, Standard deviation = $\sqrt{1{,}744.23947}$ = 41.764

For large store:

Expected value = 17.02, Standard deviation = $\sqrt{3{,}216.2418}$ = 56.712

34. Fixed cost = Rs 96,000

$$\text{P/V ratio} = \frac{\text{SP} - \text{VC}}{\text{SP}} = \frac{20 - 4}{20} = 80\%$$

$$\text{Break-even Sales} = \frac{\text{Fixed cost}}{\text{P/V Ratio}}$$

$$= \frac{96{,}000}{80\%} = \text{Rs } 1{,}20{,}000$$

To find profit at Sales = Rs 2,00,000

$$\text{Sales} = \frac{\text{Fixed cost + Profit}}{\text{P/V Ratio}}$$

$$2,00,000 = \frac{96,000 + \text{Profit}}{80\%}$$

∴ Profit = 2,00,000 × 0.80 – 96,000 = Rs 64,000

35. (a)

	AB Ltd.	*CD Ltd.*
Sales	Rs 1,50,000	Rs 1,50,000
Less: Variable cost	1,20,000	1,00,000
Contribution margin	30,000	50,000
P/V Ratio	$\frac{30,000}{1,50,000} \times 100$	$\frac{50,000}{1,50,000} \times 100$
	= 20%	= 33.33%
Break-even Sales		
$= \frac{\text{Fixed cost}}{\text{P/V Ratio}}$	$= \frac{15,000}{20\%}$	$= \frac{35,000}{33.33\%}$
	= Rs 75,000	= Rs 1,05,000

(b) (i) Once the fixed cost is recovered, a firm with higher P/V ratio would earn higher profits. Thus, *CD* Ltd., would be better placed when there is heavy demand.

(ii) When demand for the product is low, a firm with lower fixed cost would be better placed. Thus, in the given problem, *AB* Ltd., would start earning profit once it recovers Rs 15,000 of fixed cost while *CD* Ltd., can earn profit only after earning Rs 35,000 to meet its fixed cost. Thus, *AB* Ltd., is likely to earn higher profits in periods of low demand.

36. Given, Sales 1,00,000 units @ Rs 20 per unit,

Fixed cost = Rs 7,92,000

Variable cost = Rs 14/unit

Thus, Contribution margin = Rs 20 –14 – Rs 6/unit

(i) $$\text{Break-even point (units)} = \frac{\text{Fixed cost}}{\text{Contribution margin}}$$

$$= \frac{7,92,000}{6}$$

= 1,32,000

Also, $$P/V \text{ ratio} = \frac{\text{Contribution margin}}{\text{Selling price}} \times 100$$

$$= \frac{6}{20} \times 100 = 30\%$$

$$\text{Break-even sales (Rs)} = \frac{\text{Fixed cost}}{\text{P/V Ratio}}$$

$$= \frac{7,92,000}{30\%}$$

= Rs 26,40,000

(ii) Required sales = $\dfrac{\text{Fixed cost + Desired profit}}{\text{Contribution margin}}$

$= \dfrac{7{,}92{,}000 + 60{,}000}{6}$

= 1,42,000 units

(iii) Sales = $\dfrac{\text{Fixed cost + Desired after – Tax profit}\left(1 - \frac{1}{t}\right)}{\text{Contribution margin}}$

$= \dfrac{7{,}92{,}000 + 90{,}000\left(\frac{1}{1 - 0.50}\right)}{6}$

= 1,62,000 units

(iv) New fixed cost = 5,00,000 × $\dfrac{20}{100} \times \dfrac{110}{100}$ + 4,00,000 + 2,92,000

= 1,10,000 + 4,00,000 + 2,92,000

= Rs 8,02,000

Contribution margin = 20 – $\left[\dfrac{11}{2} \times \dfrac{110}{100} + \dfrac{11}{2} + 3\right]$

= Rs 5.45

Revised *BEP* = $\dfrac{8{,}02{,}000}{5.45}$

= 1,47,156 units

37. (a) The contribution margin ratio (*P*/*V* ratio) is calculated here:

	P_1	P_2	P_3	P_4	*Total*
Sales (units)	3,800	4,800	6,000	2,000	
Unit price (Rs)	20	25	15	40	
Sales (Rs)	76,000	1,20,000	90,000	80,000	3,66,000
Variable Cost					
Per unit (Rs)	10	10	12	24	
Total *VC* (Rs)	38,000	48,000	72,000	48,000	2,06,000
Contribution (Rs)	38,000	72,000	18,000	32,000	1,60,000

Contribution margin ratio = $\dfrac{\text{Contribution}}{\text{Sales}} \times 100$

$= \dfrac{1{,}60{,}000}{3{,}66{,}000} \times 100 = 43.72\%$

Break-even sales = $\dfrac{\text{Fixed cost}}{CM \text{ ratio}}$

$= \dfrac{80{,}000}{43.72\%}$ = Rs 1,83,000

$$\text{Margin of safety} = \frac{\text{Actual sales} - \text{Break-even sales}}{\text{Actual sales}}$$

$$= \frac{3{,}66{,}000 - 1{,}83{,}000}{3{,}66{,}000}$$

$$= 50\%$$

38. Here Sales = Rs 8,00,000 and Margin of safety = 50%. Thus, Break-even sales = 8,00,000 – 50% of 8,00,000 = Rs 4,00,000

Now,

$$\text{Break-even sales} = \frac{\text{Fixed cost}}{\text{P/V Ratio}}$$

$$4{,}00{,}000 = \frac{\text{Fixed cost}}{40\%}$$

∴ Fixed Cost = 4,00,000 × 40% = Rs 1,60,000

To find profit at sales = Rs 8,00,000:

$$8{,}00{,}000 = \frac{160{,}000 + \text{Profit}}{\text{P/V Ratio}}$$

∴ Profit = 8,00,000 × 40% – 1,60,000
= Rs 1,60,000

39. Here the output in year 1 and 2 is given to be 7,000 and 9,000 units respectively, while the unit price is Rs 100. Now, let the variable cost be Rs x per unit and the total fixed cost be Rs F. From the given information, we have

	Year 1	*Year 2*
Sales (Rs)	7,00,000	9,00,000
Variable cost	7,000x	9,000x
Contribution	7,00,000 – 7,000x	9,00,000 – 9,000x
Fixed cost	F	F
Total profit (given)	(10,000)	10,000

From these, we have

$$7{,}00{,}000 - 7{,}000x - F = -10{,}000 \quad \text{(i)}$$

$$9{,}00{,}000 - 9{,}000x - F = 10{,}000 \quad \text{(ii)}$$

Solving (i) and (ii) simultaneously, we get x = 90 and F = 80,000. Thus, variable cost per unit = Rs 90 and fixed cost = Rs 80,000. Now,

$$\text{Break-even point (units)} = \frac{\text{Fixed cost}}{\text{Contribution margin}}$$

$$= \frac{80{,}000}{100 - 90}$$

$$= 8{,}000$$

$$\text{Total sales (profit = Rs 50,000)} = \frac{\text{Fixed cost} + \text{Profit}}{\text{Contribution margin}}$$

$$= \frac{80{,}000 + 50{,}000}{10}$$

$$= 13{,}000$$

Accordingly,

(a) Fixed cost = Rs 80,000

(b) Number of units to break-even = 8,000

(c) Number of units to earn a profit of Rs 50,000 = 13,000

40. (a) We first calculate the overall *P*/*V* ratio as follows:

	Ace	*Utility*	*Luxury*	*Supreme*	*Total*
Sales-mix	$\frac{100}{3}\%$	$\frac{125}{3}\%$	$\frac{50}{3}\%$	$\frac{25}{3}\%$	100%
Sales	2,00,000	2,50,000	1,00,000	50,000	6,00,000
Less:					
Variable cost	1,20,000	1,70,000	80,000	20,000	3,90,000
Contribution margin	80,000	80,000	20,000	30,000	1,10,000

$$\text{Overall } P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}} \times 100$$

$$= \frac{2{,}10{,}000}{6{,}00{,}000} \times 100 = 35\%$$

$$\text{Break-even point} = \frac{\text{Fixed cost}}{P/V \text{ Ratio}}$$

$$= \frac{1{,}59{,}000}{35\%} = \text{Rs } 4{,}54{,}285.71$$

(b) Break-even point under new proposal is shown calculated here.

	Ace	*Utility*	*Luxury*	*Supreme*	*Total*
Sales-mix	25%	40%	30%	5%	100%
Sales	1,50,000	2,40,000	1,80,000	30,000	6,00,000
Less:					
Variable cost	90,000	1,63,200	1,44,000	12,000	4,09,200
Contribution margin	60,000	76,800	36,000	18,000	1,90,800

$$\text{New } P/V \text{ ratio} = \frac{1{,}90{,}800}{6{,}00{,}000} \times 100 = 31.8\%$$

$$\text{Break-even point} = \frac{1{,}59{,}000}{31.8\%}$$

$$= \text{Rs } 5{,}00{,}000$$

CHAPTER 19

1. The required values are given in third to fifth columns of table. The three-monthly values are obtained as (220 + 228 + 217)/3 = 221.67, (228 + 217 + 219)/3 = 221.33 and so on. Similarly, five-monthly values are obtained by considering five monthly-data. The last column contains moving averages calculated by using weights in the given ratio.

Calculation of Forecasted Demand

Month	*Demand Y*	*3-monthly Moving Average*	*5-monthly Moving Average*	*4-monthly Moving Average (weighted)*
1	220			
2	228			
3	217			
4	219	221.67		
5	258	221.33		157.20
6	241	231.33	228.40	199.20
7	239	239.33	232.60	220.30
8	244	246.00	234.80	235.10
9	256	241.33	240.20	239.30
10	260	246.33	247.60	241.40
11	265	253.33	248.00	243.30
12		260.33	252.80	247.50

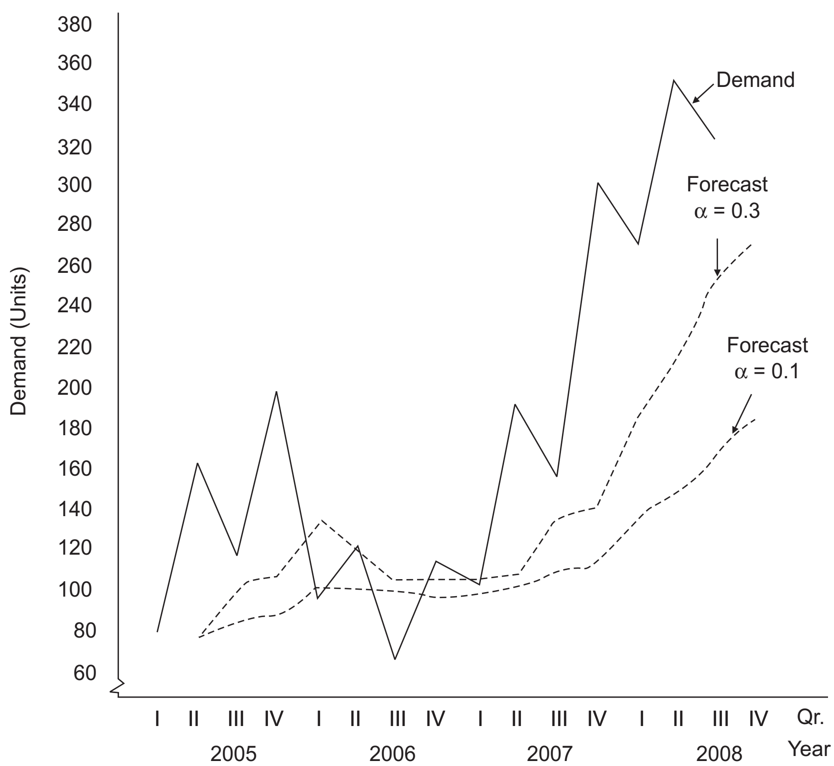
2. **Calculation of Moving Averages**

Year	*Profit*	*4-yearly Moving Average*	*5-yearly Moving Average*
1994	48		
1995	53		
1996	55		
1997	56		
1998	58	53.00	
1999	63	55.50	54.00
2000	68	58.00	57.00
2001	60	61.25	60.00
2002	61	62.25	61.00
2003	68	63.00	62.00
2004	58	64.25	64.00
2005	63	61.75	63.00
2006	70	62.50	62.00
2007	76	64.75	64.00
2008	83	66.75	67.00
2009	88	73.00	70.00
2010		79.25	76.00

3. **Forecasting of Demand: Exponential Smoothing**

	Year and Quarter, t	Demand Y_t	Forecast (F_t) $\alpha = 0.1$	$\alpha = 0.3$
2005	I	70		
	II	160	70.00	70.00
	III	110	79.00	97.00
	IV	200	82.10	100.90
2006	I	90	93.89	130.63
	II	120	93.50	118.44
	III	60	96.15	118.91
	IV	110	92.54	101.24
2007	I	100	94.28	103.87
	II	190	94.85	102.71
	III	150	104.37	128.89
	IV	300	108.93	135.23
2008	I	270	128.04	184.66
	II	350	142.23	210.26
	III	320	163.01	252.18
	IV		178.71	272.53

The actual and forecasted demand values are shown plotted in the figure.



Demand Forecasting—Exponential Smoothing

Calculation of MAD: With the help of actual and forecasted demand values, forecast errors, defined as absolute differences between various pairs of such values $|Y_t - F_t|$, are calculated. These are presented in table below. The mean absolute difference (MAD) when $\alpha = 0.1$ is found to be 83.37, and when $\alpha = 0.3$, it is 62.99. Thus, $\alpha = 0.3$ is more appropriate of the two.

Calculation of Forecast Error, *MAD*

Year	*Quarter*	*Demand* Y_t	*Forecast* $\alpha = 0.1$	$\alpha = 0.3$	*Error,* $|Y_t - F_t|$ $\alpha = 0.1$	$\alpha = 0.3$
2005	I	70				
	II	160	70.00	70.00	90.00	90.00
	III	110	79.00	97.00	31.00	13.00
	IV	200	82.10	100.90	117.90	99.10
2006	I	90	93.89	130.63	3.89	40.63
	II	120	93.50	118.44	26.50	1.56
	III	60	96.15	118.91	36.15	58.91
	IV	110	92.54	101.24	17.46	8.76
2007	I	100	94.28	103.87	5.72	3.87
	II	190	94.85	102.71	95.15	87.29
	III	150	104.37	128.89	45.63	21.11
	IV	300	108.93	135.23	191.07	164.77
2008	I	270	128.04	184.66	141.96	85.34
	II	350	142.23	210.26	207.77	139.74
	III	320	163.01	252.18	156.99	67.82
	IV		178.71	272.53		
				MAD	83.37	62.99

The exponential smoothing method does not appear to be appropriate method of forecasting in this case in view of relatively large forecasting error observed.

4. **Forecasting: Exponential Smoothing**

Year	*Quarter*	*Value* Y_t	*Forecast* F_t	α	*Error* $|Y_t - F_t|$	*Cumulative Error*	*MAD*
2005	I	70	—	—	—	—	—
	II	160	70.00	—	90.00	90.00	90.00
	III	110	142.00	0.8	32.00	122.00	61.00
	IV	200	116.40	0.8	83.60	205.60	68.53
2006	I	90	183.28	0.8	93.28	298.88	74.72
	II	120	108.66	0.8	11.34	310.22	62.04
	III	60	117.73	0.8	57.73	367.95	61.33
	IV	110	71.55	0.8	38.45	406.42	58.06
2007	I	100	83.09	0.3	16.91	423.31	52.91
	II	190	88.16	0.3	101.84	525.15	58.35
	III	150	118.71	0.3	31.29	556.44	55.64
	IV	300	128.10	0.3	171.90	728.34	66.22
2008	I	270	365.62	0.8	4.38	732.72	61.06
	II	350	269.12	0.8	80.88	813.60	62.58
	III	320	333.82	0.8	13.82	827.42	59.10
	IV	—	329.67	0.3	—	—	—

5. (i) The constants a and b for the trend line $Y_t = a + bX$ can be obtained by solving the following pair of equations simultaneously.

$$\Sigma Y = na + b\Sigma X \quad \text{(i)}$$
$$\Sigma XY = a\Sigma X + b\Sigma X^2 \quad \text{(ii)}$$

Obtaining Trend Equation

Year	*X*	*Demand (Y)*	*XY*	X^2	Y_t
2002	0	77	0	0	83
2003	1	88	88	1	85
2004	2	94	188	4	87
2005	3	85	255	9	89
2006	4	91	364	16	91
2007	5	98	490	25	93
2008	6	90	540	36	95
Total	21	623	1,925	91	

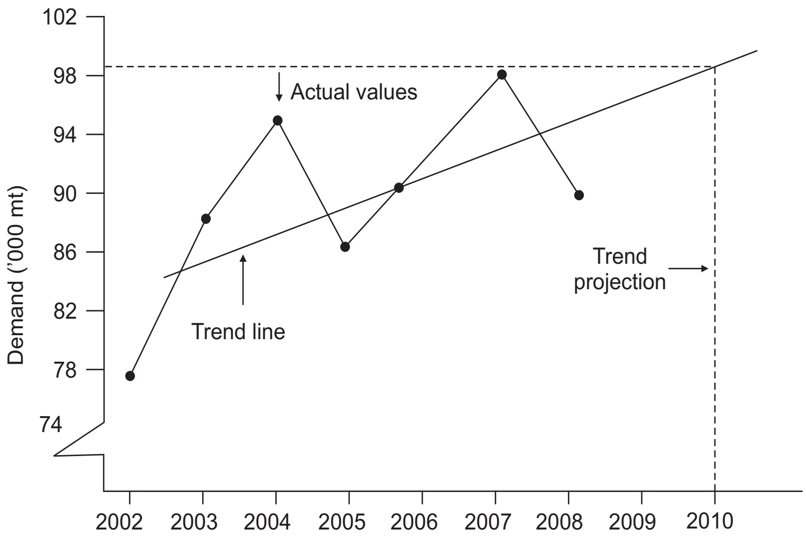
Substituting the calculated value in the two equations, we get

$$623 = 7a + 21b$$
$$1{,}925 = 21a + 91b$$

Solving these equations simultaneously, we get $a = 83$ and $b = 2$. Accordingly, the trend equation is:

$Y_t = 83 + 2X$
Origin: 2002
X unit: 1 Year
Y unit: Annual demand ('000 mt)

(ii) The trend values for various years may be obtained by substituting the relevant X values in the trend equation. These are given in the last column of the table. Further, the actual and the trend values are shown graphically here.



Fitting the Straight Line Trend

(iii) *Forecast for year 2010*

With 2002 = 0, the X-value for 2010 is 8. Thus,

$Y_t(2010) = 83 + 2 \times 8$

$= 99$ ('000 mt)

6. **Obtaining the Trend Equation**

Year	*X*	*Demand (Y)*	*XY*	X^2
2002	–3	80	–240	9
2003	–2	84	–168	4
2004	–1	90	–90	1
2005	0	93	0	0
2006	1	98	98	1
2007	2	100	200	4
2008	3	104	312	9
Total	0	649	112	28

Since $\Sigma X = 0$, we have

$$a = \frac{\Sigma Y}{n}, \quad \text{and} \quad b = \frac{\Sigma XY}{\Sigma Y^2}$$

$$= \frac{649}{7} = 92.7 \qquad = \frac{112}{28} = 4$$

Accordingly, the trend equation is:

$Y_t = 92.7 + 4X$

Origin: 2005

X: 1 Year

Y unit: Annual demand of steel ingots (in millions)

$Y_t(2010) = 92.7 + 4 \times 4 = 108.7$ m

7. Here the data given are as monthly demand for motor fuel. For obtaining straight line trend, they are first converted into yearly totals. The calculations for obtaining a and b are given in the table.

Obtaining Straight Line Trend

Year	*X*	*Demand (Y)*	*XY*	X^2
1998	–5	732	–3,660	25
1999	–4	792	–3,168	16
2000	–3	864	–2,592	9
2001	–2	912	–1,824	4
2002	–1	984	–984	1
2003	0	1,080	0	0
2004	1	1,152	1,152	1
2005	2	1,200	2,400	4
2006	3	1,236	3,708	9
2007	4	1,320	5,280	16
2008	5	1,368	6840	25
Total	0	11,640	7,152	110

Here $\Sigma X = 0$. Thus, we have,

$$a = \frac{\Sigma Y}{n}, \qquad b = \frac{\Sigma XY}{\Sigma Y^2}$$

$$= \frac{11,640}{11} \qquad = \frac{7,152}{110}$$

$$= 1,058.18 \qquad = 65.02$$

The trend equation is: $Y_t = 1,058.18 + 65.02X$

Origin: 2003

X unit: 1 Year

Y unit: Annual Demand

$Y_t(2009) = 1,058.18 + 65.02 \times 7$

$= 1,513.32$ million barrels

8. The given data are reproduced in the table where the average values for various months are also given. Thus, for January the average sales is (46 + 45 + 42)/3 = 44.33 thousand rupees. The overall average for the twelve months works out to be 43.67 thousand rupees. Seasonal indices for various months are calculated as the ratio of the monthly averages to overall average, expressed as percentages. Finally, the sales estimates are obtained as: seasonal index × average monthly sales/100. Thus, for January, we have (101.53 × 5,60,000/12) × 100 = Rs 47,379.

Seasonal Indices and Monthly Sales Schedule

Month	*Year*			*Average*	*Seasonal Index*	*Expected Sales (Rs)*
	2006	*2007*	*2008*			
Jan	46	45	42	44.33	101.53	47.379
Feb	45	44	41	43.33	99.24	46,310
Mar	44	43	40	42.33	96.95	45,242
Apr	46	46	44	45.33	103.82	48,448
May	45	46	45	45.33	103.82	48,448
Jun	47	45	45	45.67	104.58	48,804
Jul	46	47	46	46.33	106.11	49,517
Aug	43	42	43	42.67	97.71	45,598
Sep	40	43	41	41.33	94.66	44,173
Oct	40	42	40	40.67	93.13	43.461
Nov	41	43	42	42.00	96.18	44,886
Dec	45	44	45	44.67	102.29	47,735

9. The least square regression equation of Y on X is given by $Y = a + bX$. The constants a and b for this equation may be obtained as follows:

$$b = \frac{\Sigma XY - n\bar{X}\bar{Y}}{\Sigma X^2 - n\bar{X}^2}\text{; and}$$

$$a = \bar{Y} - b\bar{X}$$

Obtaining Regression Equation

	X	Y	XY	X^2
	89	92	8,188	7,921
	86	91	7,826	7,396
	74	84	6,216	5,476
	65	75	4,875	4,225
	64	73	4,672	4,096
	63	72	4,536	3,969
	66	71	4,686	4,356
	67	75	5,025	4,489
	72	78	5,616	5,184
	79	84	6,636	6,241
Total	725	795	58,276	53.353

Here, $\bar{X} = \Sigma X/n = 725/10 = 72.5$, and

$\bar{Y} = \Sigma Y/n = 795/10 = 79.5$.

Thus,

$$b = \frac{58{,}276 - 10 \times 72.5 \times 79.5}{53{,}353 - 10 \times 72.5^2}$$

$$= \frac{638.5}{790.5} = 0.8077$$

$$a = 79.5 - 0.8077 \times 72.5$$
$$= 20.9405$$

The regression equation, therefore, is

$$Y = 20.9405 + 0.8077X$$

Forecasts:

For $X = 70$, $Y = 20.9405 + 0.8077 \times 70$
$= 77.48$

For $X = 85$, $Y = 20.9405 + 0.8077 \times 85$
$= 89.60$

10. To fit the required regression equations, we first calculate the returns on indices and on the share. For example, the index moves from 1376.15 to 1388.75 in the first instance. We have the return as (1388.75 – 1376.15)/1376.15 = 0.9156 per cent. The index returns are denoted as *X*-variable while the share returns as *Y*-variable.

Obtaining Regression Coefficients

Day	*Index*	*Share Prince*	*Index Returns, X*	*Share Returns, Y*	*XY*	X^2
1	1,376.15	818.35				
2	1,388.75	811.75	0.9156	–0.8065	–0.738430	0.838319
3	1,408.85	819.85	1.4473	0.9978	1.444224	2.094807
4	1,418.00	836.05	0.6495	1.9760	1.283326	0.421806
5	1,442.85	815.65	1.7525	–2.4400	–4.276102	3.071145
6	1,445.15	804.30	0.1594	–1.3915	–0.221819	0.025411
7	1,438.65	801.30	–0.4498	–0.3730	0.167766	0.202302
8	1,447.55	792.30	0.6186	–1.1232	–0.694836	0.382710
9	1,439.70	778.30	–0.5423	–1.7670	0.958240	0.294085
10	1,427.65	740.95	–0.8370	–4.7989	4.016600	0.700535
11	1,398.25	718.35	–2.0593	–3.0501	6.281236	4.240833
12	1,401.40	737.50	0.2253	2.6658	0.600563	0.050752
13	1,419.70	735.55	1.3058	–0.2644	–0.345272	1.705210
		Total	3.1857	–10.3751	8.475496	14.027915

Further,

$$\bar{X} = \frac{\Sigma X}{n}, \qquad \text{and} \qquad \bar{Y} = \frac{\Sigma Y}{n}$$

$$= \frac{3.1857}{12} = 0.2655 \qquad\qquad = \frac{-10.3751}{12} = -0.8646$$

Now, the constants a and b for the regression equation $Y = a + bX$ may be obtained as follows:

$$b = \frac{\Sigma XY - n\bar{X}\bar{Y}}{\Sigma X^2 - n\bar{X}^2}$$

$$= \frac{8.475496 - 12 \times 0.2655 \times (-0.8646)}{14.027915 - 12 \times 0.2655^2}$$

$$= 0.8519$$

$$a = \bar{Y} - b\bar{X}$$

$$= -0.8646 - 0.8519 \times 0.2655$$

$$= -1.0907$$

Accordingly, the regression equation is:

$$Y = -1.0907 + 0.8519X$$

The regression coefficient 0.8519 implies that a 1% increase in index would cause 0.8519% increase in the share price.

Estimation:

For $X = 12$, $Y = -1.0907 + 0.8519 \times 12$

$= 9.1321\%$

11. (i) Let Y, X_1, and X_2 represent sales, advertising, and price respectively. The required regression equation is:

$$Y = a + b_1X_1 + b_2X_2$$

The parameters a, b_1, and b_2 for this can be obtained from the following normal equations:

$$\Sigma Y = na + b_1\Sigma X_1 + b_2\Sigma X_2$$

$$\Sigma X_1Y = a\Sigma X_1 + b_1\Sigma X_1^2 + b_2\Sigma X_1X_2$$

$$\Sigma X_2Y = a\Sigma X_2 + b_1\Sigma X_1X_2 + b_2\Sigma X_2^2$$

Calculation of Regression Coefficients

	Y	X_1	X_2	X_1Y	X_2Y	X_1Y_2	X_1^2	X_2^2
	33	3	125	99	4,125	375	9	15,625
	61	6	115	366	7,015	690	36	13,225
	70	10	140	700	9,800	1,400	100	19,600
	82	13	130	1,066	10,660	1,690	169	16,900
	17	9	145	153	2,465	1,305	81	21,025
	24	6	140	144	3,360	840	36	19,600
Total	287	47	795	2,528	37,425	6,300	431	1,05,975

Substituting the calculated values in the equations given earlier, we get

$$6a + 47b_1 + 795b_2 = 287$$

$$47a + 431b_1 + 6{,}300b_2 = 2{,}528$$

$$795a + 6{,}300b_1 + 1{,}05{,}975b_2 = 37{,}425$$

Solving the three equations simultaneously, we get
$a = 219.23$, $b_1 = 6.3815$, and $b_2 = -1.6708$. The regression equation, therefore, is:
$Y = 219.23 + 6.3815X_1 - 1.6708X_2$

(ii) For $X_1 = 7$ and $X_2 = 132$;

$$Y = 219.23 + 6.3815 \times 7 - 1.6708 \times 132$$
$$= 43.25 \text{ or } 43 \text{ approx.}$$

12. The regression equation is $Y = a + b_1X_1 + b_2X_2$.
The parameters a, b_1, and b_2 are obtainable as follows:

$$\Sigma Y = na + b_1\Sigma X_1 + b_2\Sigma X_2$$
$$\Sigma X_1 Y = a\Sigma X_1 + b_1\Sigma X_1^2 + b_2\Sigma X_1X_2$$
$$\Sigma X_2 Y = a\Sigma X_2 + b_1\Sigma X_1X_2 + b_2\Sigma X_2^2$$

Obtaining of Regression Parameters

Y	X_1	X_2	X_1Y	X_2Y	X_1Y_2	X_1^2	X_2^2
72	12	5	864	360	60	144	25
76	11	8	836	608	88	121	64
78	15	6	1,170	468	90	225	36
70	10	5	700	350	50	100	25
68	11	3	748	204	33	121	9
80	16	9	1,280	720	144	256	81
82	14	12	1,148	984	168	196	144
65	8	4	520	260	32	64	16
62	8	3	496	186	24	64	9
90	18	10	1,620	900	180	324	100
743	123	65	9,382	5,040	869	1,615	509

From the calculations, we have

$$10a + 123b_1 + 65b_2 = 743$$
$$123a + 1{,}615b_1 + 869b_2 = 9{,}382$$
$$65a + 869b_1 + 509b_2 = 5{,}040$$

Solving these equations, we get $a = 47.1649$, $b_1 = 1.599$, and $b_2 = 1.1487$. The regression equation is:

$$Y = 47.1649 + 1.599X_1 + 1.1487X_2$$

Estimation: For $X_1 = 13$ and $X_2 = 7$ (since original values are in thousands), we have

$$Y = 47.1649 + 1.599 \times 13 + 1.1487 \times 7$$
$$= 76$$

Thus, approximate sales = Rs 76,000.